

# Energy-Aware Models for Warehousing Operations

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**Abstract.** There is a growing need in industries worldwide to become more sustainable and energy efficient. Due to rapid increase in demand of goods, there has been a rise in demand of logistics and operational services. This necessitates needs for a large number of warehouses and distribution centers to satisfy demand. It is imperative that warehouses follow the same sustainable development model practiced in other industries. This paper extends energy efficiency techniques suggested for manufacturing to warehousing. Specifically, warehouses are modeled as  $M/M/c$  queues where forklifts are servers and this model is used to evaluate performance of energy control policies. The model is then extended to general distribution queues. Experiments based on real-world data yield results that indicate that for system utilization values between 40% and 100%, as the number of servers in the system increases by a factor of 4, energy consumption increases by a factor of 3.78.

**Keywords:** Energy control, Queuing, Warehousing, Forklifts.

## 1 Introduction

The world today has incorporated a more ecological approach in the utilization of energy resources, targeted toward the exploration of “green” options and renewable sources of energy. Various efforts have been taken to understand and reduce the consumption of energy in the manufacturing sector [1]. From a logistics standpoint, research has been undertaken to find more sustainable choices due to increase in demand of various products. To put this into perspective, the amount of money invested in business logistics in the U.S. in 2012 was \$1.33 trillion, which is 8.5% of the GDP [2]. Warehousing, a critical element in the logistics sector, accounts for 8% of the total energy consumption of all commercial buildings in the country [3]. According to an analysis of the energy consumed in warehouses/distribution centers (DCs) [4], HVAC and lighting are critical components for both non-refrigerated and refrigerated warehouses, accounting for a significant portion of energy utilization.

Discounting the smaller consumers of energy such as office equipment, it is safe to assume that other than heating, ventilation and air conditioning (HVAC), a predominant contributor to energy usage is the movement of material from place to place within the DC. Material movement contributes to a significant portion of the final product’s cost, and warehouses contain specific “Warehouse Management Systems” (WMS) to aid in handling the material, including non-automated and automated sys-

tems [5]. Automated Storage/Retrieval Systems (AS/RS) are an important tool used in material handling in warehouses and most modern factories for work-in-process storage [6]. However, manually operated forklifts continue to play a major role in the efficient functioning of a warehouse. In 2012 alone, the top twenty manufacturers of lift trucks worldwide shipped \$30.4 billion worth of forklifts, and the forklift domain is increasingly becoming a platform to enable better technology for more productivity. Forklifts, in performing the functions of order picking and put-away in warehouses, contribute to most of the energy consumption among all other material handling systems in terms of fuel cells and electric charge required [7]. Warehouses may be roughly modeled as a queueing system in which stock-keeping units (SKU's) are customers that arrive at the receiving dock, where they join a queue usually serviced by forklifts for storage, until they are shipped out [8]. The primary concern of this paper is to establish a queueing system considering forklifts as servers and relate the same to an energy model with the inclusion of energy waste reduction controls during forklift idling, similar to idling of machines in a manufacturing unit [9].

A majority of existing literature is devoted to research in the modeling of energy aware manufacturing systems, useful in the shop floor. Reduction of wastage of energy by using heuristics for various dispatch rules has also been proposed [9]. In terms of warehousing, literature is focused on the development of energy efficient material handling applications by finding an optimized travel path sequence, considering the Traveling Salesman Problem (TSP) to address the problem of order picking [10]. Algorithms have been designed for effective performance end-of-aisle order picking systems, and travel times have been analyzed considering queueing models for item location in warehouses [11]. Queueing models for centralized inventory information in warehouses and warehouses with autonomous vehicles have been developed to evaluate congestion effects in storage and retrieval transactions [12], and there has been recent research dedicated to the development of queueing models for warehouse AS/RS [6], [13]. There is a requirement to develop computationally intuitive energy-aware warehousing models as it relates to queueing theory, which effectively captures the consumption of energy on a large scale, to facilitate more productive insights during warehouse planning and design stages.

## 2 Warehouse Layout

The design of work-in-process warehouse layouts are heavily influenced by response times of the material handling system. Pandit and Palekar (1993) propose a warehouse layout for a multi-vehicle handling system. The warehouse is considered to be rectangular in shape to facilitate ease of storage of rectangular units in the form of pallets, stored within the warehouse on racks [14]. The racks are arranged back-to-back in the form of blocks, and space between blocks form aisle, creating a guide-way network which expedites forklift movement and reduces congestion. The layout suggested is a probable aisle arrangement in large warehouses, in which the rack accessibility of forklifts is maximized. Fig. 1 details the warehouse layout considered for the purpose of this paper.

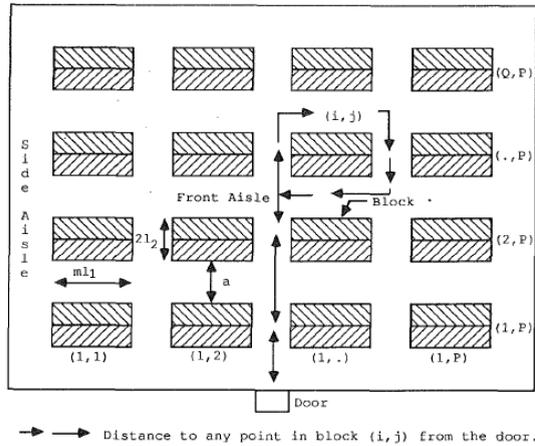


Fig. 1. Layout details (Source: Pandit & Palekar, 1993)

The warehouse operates under the following assumptions: storage and retrieval requests arrive at a door whenever a truck moves into the loading/unloading dock. Immediate fulfillment of the request is done if a forklift is free. If the order is waiting on a forklift, it enters itself into a queue which empties on a first-come, first-serve (FCFS) basis rather than a priority based emptying of the queue, for ease of calculation. The system is modeled as  $M/M/c$  queue, in which  $c$  indicates total number of forklifts operated in the warehouse.

### 3 Warehousing Energy Control Model

The recently proposed machine-level EC1 energy control policy is considered to be the basis for the model proposed in this paper. The main objective of the policy is the reduction of energy consumption during machine idling, thus reducing wasted energy. The policy states that this can be achieved if the machine is switched to a lower power consumption state if its idle time exceeds a minimum threshold value  $\tau$  [1], [9]. Determining an optimal value of  $\tau$  could involve a trade-off between energy savings and loss of production [1]. The concept of utilization of a minimum threshold value of time to conserve energy has been explored in terms of manufacturing systems where production schedules are fixed at least  $\tau$  ahead of time [9]. The understanding of interplay between energy control (EC) policies and production control (PC) policies as it relates to key performance indicators (KPIs) becomes necessary. It is safe to assume that the same model can be applied to a distribution center as well, seeing as the loading or unloading of trucks in loading docks to pick material for packing and shipping or to put-away material in storage respectively takes place on a fixed schedule, with planned shipments [15].

In the context of this paper, we consider  $\tau$  to represent the average idle time threshold of the forklift system in the warehouse. The EC1 energy control policy serves to link warehousing decisions which would influence KPIs of the system with EC and

PC policies. Major warehousing decisions include configuration issues viz., how material flow should be organized, design of the order picking and shipping process, sizing of the warehouse, allocation of storage capacity, determining lighting and electricity requirements, batch sizing and scheduling, and level of automation required [5]. The KPIs of interest include energy consumption per forklift, total system energy consumption over the long term, cycle time for each forklift, throughput of the facility, and utilization and availability of the forklift system.

## 4 Multi-server Queueing Model with Energy Control

A key characteristic of discrete manufacturing is that the energy consumed by a machine tool while it is idling or busy is quite similar [10]. For any system, energy can be expressed as the product of power and time. Power consumed by any machine in the system, in this case  $c$  forklifts, changes according to the state of the system. For practical purposes, systems tend to consist of two states on average: *busy* and *idle*. We define an intermediate third state for the system which occurs when forklifts perform non-value added activities. We consider this to be the *apparent idling* condition of the forklift system. Average power values for the 3 system states are defined as  $W_0$  for idle condition,  $W_1$  for apparent idling and  $W_p$  for busy state. The idle time of the system is composed of time taken by the forklifts to perform non-value added activities (considered as *apparent idle time* of the system) and time in which the forklift is idle (*real idle time* of the system). An average idle time threshold  $\tau$  is specified for the system. We define power consumption during real idle time ( $W_0$ ) to be zero, i.e., the forklift is switched off when not performing any activity.

Probabilistic models will be constructed in this section to find out the state of the system at any given time, enabling us to estimate consumption of energy in the system. Consider a multi-server machine system. Orders arrive at the loading dock at a rate  $\lambda$  according to a Poisson process with exponential inter-arrival times and get serviced at a rate  $\mu$  according to an exponential distribution, with  $c$  forklifts being the servers. The queue is processed on a FCFS basis, with jobs departing the system after they have been processed by any of the servers. This is the M/M/c model under consideration.

For M/M/1 queue, utilization of the machine is given by the following relation [16]:

$$\text{Utilization } \rho = \text{Arrival rate/Service rate} = \lambda/\mu \quad (1)$$

For the M/M/c queue, average utilization of the system is expressed as:

$$\rho = \lambda/c\mu \quad (2)$$

For stability and to provide bounds to the system, the utilization value should be lower than 1. Utilization parameter  $\rho$  represents the average fraction of time during which each of the  $c$  servers is occupied with a task. The fraction of time that the system will be idle is expressed as [17]:

$$\delta = \left[ \frac{(c\rho)^c}{c!(1-\rho)} + \sum_{n=0}^{c-1} \frac{(c\rho)^n}{n!} \right]^{-1} \quad (3)$$

Prabhu et al. (2012) calculated the probability that inter-arrival time is more than the average threshold idle time  $\tau$  by the following equation:

$$P(x > \tau) = \int_{\tau}^{\infty} f(x)dx = e^{-\lambda\tau} \tag{4}$$

where  $f(x)$  for an exponential distribution =  $\lambda e^{-\lambda x}$ . By logic,  $(1 - e^{-\lambda\tau})$  will be the probability of forklifts doing non-value added activities. Consequently, the probability that the system is in the idle state and time between arrivals is greater than  $\tau$  is calculated as:

$$\left[ \frac{(c\rho)^c}{c!(1-\rho)} + \sum_{n=0}^{c-1} \frac{(c\rho)^n}{n!} \right]^{-1} P(x > \tau) = \left[ \frac{(c\rho)^c}{c!(1-\rho)} + \sum_{n=0}^{c-1} \frac{(c\rho)^n}{n!} \right]^{-1} e^{-\lambda\tau} \tag{5}$$

Under steady state condition, arrivals are independent of the state of the system. Thus the long term energy consumption equation for the system over time T is:

$$E = \left\{ cW_p\rho + W_0 \left[ \frac{(c\rho)^c}{c!(1-\rho)} + \sum_{n=0}^{c-1} \frac{(c\rho)^n}{n!} \right]^{-1} e^{-\lambda\tau} + W_1 \left[ \frac{(c\rho)^c}{c!(1-\rho)} + \sum_{n=0}^{c-1} \frac{(c\rho)^n}{n!} \right]^{-1} (1 - e^{-\lambda\tau}) \right\} T \tag{6}$$

Substituting the value of  $W_0$  as 0 in Equation (6),

$$E = \left\{ cW_p\rho + W_1 \left[ \frac{(c\rho)^c}{c!(1-\rho)} + \sum_{n=0}^{c-1} \frac{(c\rho)^n}{n!} \right]^{-1} (1 - e^{-\lambda\tau}) \right\} T \tag{7}$$

The ratio of energy wasted during idle time ( $E_w$ ) to the energy that is actively used when the forklifts are accomplishing value-added activities ( $E_p$ ) is given by the following relationship:

$$\frac{E_w}{E_p} = \frac{W_1 \left[ \frac{(c\rho)^c}{c!(1-\rho)} + \sum_{n=0}^{c-1} \frac{(c\rho)^n}{n!} \right]^{-1} (1 - e^{-\lambda\tau})}{cW_p\rho} \tag{8}$$

The model presented has assumed a Poisson input process. However, this assumption would be violated if arrivals definitely do not occur randomly for the warehousing system. This necessitates the use of an arbitrary distribution queueing model [18]. The queue is classified as a G/G/c model, where arrival and service distributions usually follow different processes but can be the same as well. Using Equation (7) as a basis, different energy relations based on various inter-arrival distributions can be calculated using the following general equation:

$$E = \left\{ cW_p\rho + W_1 \left[ \frac{(c\rho)^c}{c!(1-\rho)} + \sum_{n=0}^{c-1} \frac{(c\rho)^n}{n!} \right]^{-1} F(\tau) \right\} T \tag{9}$$

where  $\int_{\tau}^{\infty} f(x)dx = 1 - F(\tau)$  for any distribution.

When the utilization of the forklift is high,  $W_1 \rightarrow 0$  and the only term that will affect the equation is  $W_p\rho$ . Similarly under low utilization condition, only the  $W_1$  term would affect the equation. It is to be noted that when utilization is high, i.e.,  $W_p \rightarrow 1$ ,

the EC1 energy control policy will not influence energy consumption. For a 100% utilization of the system, i.e., when all forklifts are in operation,

$$E = cW_p\rho T \quad (10)$$

Depending upon the type of warehouse system considered, certain constraints are introduced to the equation which is then differentiated with respect to  $c$  to find the optimal value yielding minimum energy consumption. Constraints could be the maximum amount of throughput a warehouse is capable of handling, layout of warehouse, capacity, response time, storage density and inventory availability.

## 5 Experimentation and Results

The analysis of the real time operations of one of the DCs of a personal care manufacturer located in Brazil was done for experimentation purposes. The layout of the DC in consideration is similar to the one suggested in Fig. 1 of this paper. Analysis of arrival rates of products over a three-day period showed that the products arrived with a beta distribution on day 1 with shape parameters  $\alpha = 1.87$  and  $\beta = 2.05$ ; a triangular distribution on day 2 with parameters  $a = 0.58$ ,  $b = 4.9$  and  $c = 0.62$ ; a beta distribution on day 3 with shape parameters  $\alpha = 2.17$  and  $\beta = 2.26$ .

Assuming average forklift speed of travel is 5 mph for all operations, the worst case scenario assumes that on average, each forklift travels to all rack positions in the DC to drop off orders and picks up orders to drop them off at the loading dock every day. Since average demand per day is a constant number, it is safe to assume that service rates follow the same arrival distribution over the three days observed.

Consider that  $W_1 = W_p$ , a fair assumption to make as forklifts would usually spend the same amount of power doing non-value added activities as they do for value-added work. By logic, average idle time threshold  $\tau$  decreases with an increasing number of forklifts in the system. The number of forklifts in the system would be dependent on the size of the warehouse. Let us consider 5 forklifts for a smaller warehouse and 20 forklifts for a larger one. Consider that for 5 forklifts,  $\tau$  value is 60 minutes, and for 20 forklifts, it is 10 minutes. Varying utilization, using Equation (9), surface plots of the daily energy values for the three distributions are illustrated in Fig. 2. For different utilization values, the energy consumption increases by an average of 278.8% as number of forklifts in the system increase from 5 to 20. Thus it can be seen that larger warehouses with  $n$  times the number of forklifts as smaller ones consume nearly  $n$  times more energy for any arrival distribution.

It is observed that the values obtained for the general distribution model do not vary significantly between distributions, because of the scaling down of the  $W_1$  term due to minute values of idle probability of the system. This is because the chance of the entire forklift system being idle at the same time is almost zero. In such situations, only the  $W_p$  term of the equation would affect the energy value. Energy varies proportionally with utilization for a constant  $c$  value and number of forklifts for a constant  $p$ .

Fig. 3(a) details the variation of daily energy with respect to change in utilization with 5 forklifts in the system. Fig. 3(b) illustrates energy variation with respect to number of forklifts at 70% utilization. It is observed that variation of energy is almost linear in both cases.

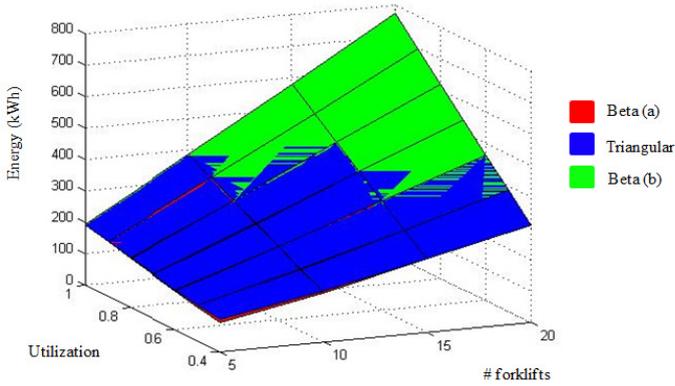
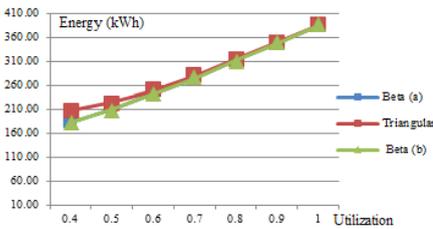


Fig. 2. Energy surface plot of different distributions



(a) Utilization for 5 forklifts



(b) Number of forklifts at 70% utilization

Fig. 3. Variation of daily energy

## 6 Conclusions

The motivation for this work was the need for an energy-aware warehousing model which leveraged the principles proposed by the EC1 energy control policy to forklift queues in warehouses and DCs. The paper suggests a model for which the recently proposed M/M/1 manufacturing model with energy control is the basis, and extends it to an M/M/c queue in a warehouse. Experimentation on the model yielded results which are largely dependent on the number of servers considered in the system. For a larger number of servers, the apparent idling energy of the system would tend to be on the lower side. The analytical model developed can be used to determine availability of forklifts to accomplish tasks and assignment of forklifts for the same, determination of energy spent by forklifts, and calculation of optimal number of forklifts to reduce energy in the system based on certain constraints. Future work can be extended to the determination of the energy consumption in other avenues in warehouses, especially newer ones with higher capital investment where automation will come into play, and implementation of other energy control policies for the same. Validation of the suggested model can be carried out by simulating the queuing of forklifts in Arena and employing the suggested equation to calculate energy.

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