

# Image Segmentation Using Active Contours and Evidential Distance

Foued Derraz<sup>1,3</sup>, Antonio Pinti<sup>4,5</sup>, Miloud Boussahla<sup>3</sup>,  
Laurent Peyrodie<sup>2</sup>, and Hechmi Toumi<sup>5</sup>

<sup>1</sup> Facult Libre de Mdicine  
Institut Catholique de Lille  
Universit Catholique de Lille  
46 rue du Port de Lille, France  
foued.derraz@ic1-lille.fr

<sup>2</sup> Hautes Etudes d'Ingenieur  
Universit Catholique de Lille  
46 rue du Port de Lille, France  
laurent.peyrodie@hei.fr

<sup>3</sup> Telecommunication Laboratory  
Technology Faculty

Abou Bekr Belkaid University, Tlemcen, Algeria  
BP 230, Tlemcen 13000, Algeria  
m\_boussahla@mail.univ-tlemcen.dz

<sup>4</sup> ENSIAME UVHC, Universite de Valenciennes, France  
antonio.pinti@univvalenciennes.fr

<sup>5</sup> A5 EA 4708, I3MTO, CHRO 1, rue Porte Madeleine, 45032, Orleans, France  
hechmi.toumi@univ-orleans.fr

**Abstract.** We proposed a new segmentation based on Active Contours (AC) for vector-valued image that incorporates evidential distance. The proposed method combine both Belief Functions (BFs) and probability functions in the Bhattacharyya distance framework. This formulation allows all features issued from vector-valued image and guide the evolution of AC using an inside/outside descriptor. The imprecision caused by the variation of the contrast issued from the multiple channels is incorporated in the BFs as weighted parameters. We demonstrated the performance of the proposed algorithm using some challenging color biomedical images.

**Keywords:** Active Contours, Characteristic function, Belief Function, Bhattacharyya distance ,Dempster Shafer rule.

## 1 Introduction

Segmentation based AC models presents several challenges that are mainly related to image noise, poor contrast, weak or missing boundaries between imaged objects, inhomogeneities, etc. One way to overcome these difficulties is to exploit the prior knowledge in order to constrain the segmentation process. Due occlusion or texture this is often not appropriate to delineate object regions.

Statistical knowledge [4,20] and additional information such as texture [4] can improved the segmentation based AC models for vector-valued image [3,22]. Another reason for failed segmentations is due local or global minimizer for AC models [2]. To overcomes these difficulties, the evidential framework appears to be a new way to improve segmentation based AC models for vector valued images [19,12,21]. The Dempster Shafer (DS) framework [7] has been combined with either a simple thresholding [19], a clustering algorithm [14], a region merging algorithm [12] or with an AC algorithm [21]. In this paper we propose to use the evidential framework [7] to combine several information sources and incorporates them in the formulation of the AC models. The fusion of this information from different feature channels, e.g., color channels and texture offers an alternative to the Bayesian framework. Instead to fuse separated probability densities, the evidential framework allows both inaccuracy and uncertainty. This concept is represented using BFs [7,5,8,1] which is particularly well suited to represent information from partial and unreliable knowledge. To use BFs as an alternative to probability in segmentation process can be very helpful in reducing uncertainties and imprecisions using conjunctive combination of neighboring pixels. First, it allows us to reduce the noise and secondly, to highlight conflicting areas mainly present at the transition between regions where the contours occurs. In addition, BFs has the advantage to manipulate not only singletons but also disjunctions. This gives the ability to explicitly to represent both uncertainties and imprecisions. The disjunctive combination allows transferring both uncertain and imprecise information on disjunctions [7,1]. Then, the conjunctive combination is applied to reduce uncertainties due to noise while maintaining representation of imprecise information at the boundaries between areas on disjunctions. In this paper, we proposed to incorporate the BFs in the formulation of the AC models. In Section 2, we review of the AC models based Vector-valued image segmentation in total variation framework, which is the basis for our segmentation framework. In section 3, we formulated our AC models in evidential framework. Experimental results in Section 4 demonstrate the advantages of the proposed method.

## 2 Globally Active Contours for Vector-Valued Image in Evidential Framework

The evidential framework is provided through the definition of the plausibility ( $Pl$ ) and belief ( $Bel$ ) function [7,8], which are both derived from a mass function ( $m$ ). For the frame of discernment  $\Omega_{II} = \{\Omega_1, \Omega_2, \dots, \Omega_n\}$ , composed of  $n$  single mutually exclusive subsets  $\Omega_i$ , the mass function is defined by  $m : 2^\Omega \rightarrow [0, 1]$ .

$$\begin{aligned}
 m(\emptyset) &= 0 \\
 \sum_{\Omega_i \subseteq \Omega} m(\Omega_i) &= 1; \quad Bel(\Omega) = \sum_{\Omega_i \subseteq \Omega_{II}} m(\Omega_i) = 1 \\
 Pl(\Omega) &= \sum_{\Omega_i \cap \Omega_{II} \neq \emptyset} m(\Omega_i)
 \end{aligned} \tag{1}$$

The relation between mass function, *Bel* and *Pel* can be described as follows:

$$m(\Omega_i) \leq Bel(\Omega_i) \leq p(\Omega_i) \leq Pl(\Omega_i) \tag{2}$$

When  $m(\Omega) > 0$ ,  $\Omega$  is a so called focal element [12,7]. The independent masses  $m_m$  are defined within the same frame of discernment as:

$$m(\Omega_{i=\{1,\dots,n\}}) = m_1(\Omega_{i=\{1,\dots,n\}}) \otimes m_2(\Omega_{i=\{1,\dots,n\}}) \dots \otimes m_m(\Omega_{i=\{1,\dots,n\}}) \tag{3}$$

The total belief assigned to a focal element  $\Omega_i$  is equal to the belief strictly placed on the foreground region  $\Omega_i$ . Then Belief Function (*Bel*) can expressed as:

$$Bel(\Omega_i) = m(\Omega_i) \tag{4}$$

This relation can be very helpful in the formulation of our AC model. The segmentation based AC for vector Valued image **I** consists in finding one or more regions  $\Omega$  from **I**. In this framework, we search for the domain  $\Omega$  or the partition of the image  $P(\Omega)$  that maximizes the Bhattacharyya distance [15,9] between *Bel* associated to the inside/outside region or minimizing the criterion:

$$\partial \hat{\Omega} = \arg \min \left\{ \underbrace{\log \left( \frac{1}{p(P(\Omega))} \right)}_{E_b(\partial \Omega)} + \underbrace{\sum_{j=1}^m \lambda_{in}^j \int_{R^+} \sqrt{m_{in}^j(\Omega) p_{out}^j}}_{E_{data}(I, \Omega)} \right. \\ \left. + \underbrace{\sum_{j=1}^m \lambda_{out}^j \int_{R^+} \sqrt{m_{out}^j(\Omega) p_{in}^j}}_{E_{data}(I, \Omega)} \right\} \tag{5}$$

Similairement as in [21], we used the definitions proposed in [1] to define mass function for all image channels  $I_j$  as:

$$m_{j=\{1,\dots,m\}}(\Omega_{in/out}) = p_{in/out}^{j=\{1,\dots,m\}} \\ m_{j=\{1,\dots,m\}}(\Omega) = 1 - p_{in}^{j=\{1,\dots,m\}} + p_{out}^{j=\{1,\dots,m\}} \tag{6} \\ m_{j=\{1,\dots,m\}}(\emptyset) = 0$$

The pdfs  $p_{in}^j$  and  $p_{out}^j$  are estimated for all channels using Parzen kernel [17]. Our proposed method uses the total belief committed to foreground or background region. In the next section we proposed a fast version of our segmentation algorithm.

### 3 Fast Algorithm Based on Split Bregman

The Split Bregman method [2,10] is an efficient optimization technique for solving  $L^1$  regularized problems and has good convergence properties [2,10]. In order

to find a contour minimizing AC energy functional, the Split Bregman method [11] will separate the  $L^1$  and  $L^2$  norm, by introducing a vectorial variable  $d$  and imposing constraints on the segmentation problem. This results in the following segmentation problem:

$$\min_{\chi, d} (E(\chi, d)) = \int_{\Omega} |d(\mathbf{x})| d\mathbf{x} + \sum_{j=1}^m \lambda_{in}^j \int_{\Omega} V_{Belief}^{in} \chi + \sum_{j=1}^m \lambda_{out}^j \int_{\Omega} V_{Belief}^{out} \chi \tag{7}$$

This constrained segmentation problem can be transformed to an unconstrained segmentation problem by adding a quadratic penalty function. This function only approximates the constraint  $d = \nabla\chi$ . However, by using a Split Bregman technique [11], this constraint can be enforced exactly in an efficient way. An extra vector,  $b^k$  is added to the penalty function (7). Then the following two unconstrained steps are iteratively solved by:

$$\left\{ \begin{array}{l} (\chi^{k+1}, d^{k+1}) = \arg \min \left\{ \begin{array}{l} \int_{\Omega_0} |d| + \sum_{j=1}^m \lambda_{in}^j \int_{\Omega} V_{Belief}^{in} \chi + \\ \sum_{j=1}^m \lambda_{out}^j \int_{\Omega} V_{Belief}^{out} \chi + \\ \frac{\mu}{2} \int_{\Omega_0} |d - \nabla\chi^k - b^k|^2 \end{array} \right\} \\ b^{k+1} = b^k + \nabla\chi^{k+1} - d^{k+1} \end{array} \right. \tag{8}$$

where  $\mu$  is a weighting parameter. The first step requires optimizing for two different vectors. We approximate these optimal vectors by alternating between  $\chi$  and  $d$  independently:

$$\left\{ \begin{array}{l} \chi^{k+1} = \arg \min \left\{ \begin{array}{l} \int_{\Omega_0} |d| + \sum_{j=1}^m \lambda_{in}^j \int_{\Omega} V_{Belief}^{in} \chi + \\ \sum_{j=1}^m \lambda_{out}^j \int_{\Omega} V_{Belief}^{out} \chi + \\ \frac{\mu}{2} \int_{\Omega_0} |d - \nabla\chi^k - b^k|^2 \end{array} \right\} \\ d^{k+1} = \arg \min_{d^k} \left\{ \int_{\Omega_0} |d^k| + \frac{\mu}{2} \int_{\Omega_0} |d^k - \nabla\chi^k - b^k|^2 \right\} \\ b^{k+1} = b^k + \nabla\chi^{k+1} - d^{k+1} \end{array} \right. \tag{9}$$

This problem can be optimized by solving a set of Euler-Lagrange equations. For each element  $\chi^{k+1}$  of the optimal  $\hat{\chi}$ , the following optimality condition should be satisfied :

$$\underbrace{\left( \Delta - \frac{1}{\mu} \sum_{j=1}^m \lambda_{in}^j V_{Belief}^{in} - \frac{1}{\mu} \sum_{j=1}^m \lambda_{out}^j V_{Belief}^{out} \right)}_A \chi^k = \underbrace{\text{div}(b^k - d^k)}_C \tag{10}$$

Note that this system of equations can be written as  $A\chi = C$ . In [6] they proposed to solve this linear system using the Kyrlov subspace method. The solution of equation (10) is unconstrained, i.e.  $\chi$  does not have to lie in the interval  $[0, 1]$ . If  $\chi \in [0, 1]$ , the constrained optimum  $\chi \in \{0, 1\}$ , since a quadratic function is monotonic in an interval which does not contain its extremum. Then the constrained optimum can be calculated as follows [10,6]:

$$\hat{\chi} = \max \{ \min \{ \chi, 1 \}, 0 \} \tag{11}$$

Finally, the minimizing solution  $d^{k+1}$  is given by soft-thresholding:

$$d^{k+1} = \frac{\nabla\chi^{k+1} + b^k}{|\nabla\chi^{k+1} + b^k|} \max \left( |\nabla\chi^{k+1} + b^k| - \frac{1}{\mu}, 0 \right) \tag{12}$$

Note that this results in a minimizer which values are between 0 and 1, and the final active contour curve is given by the boundary of:

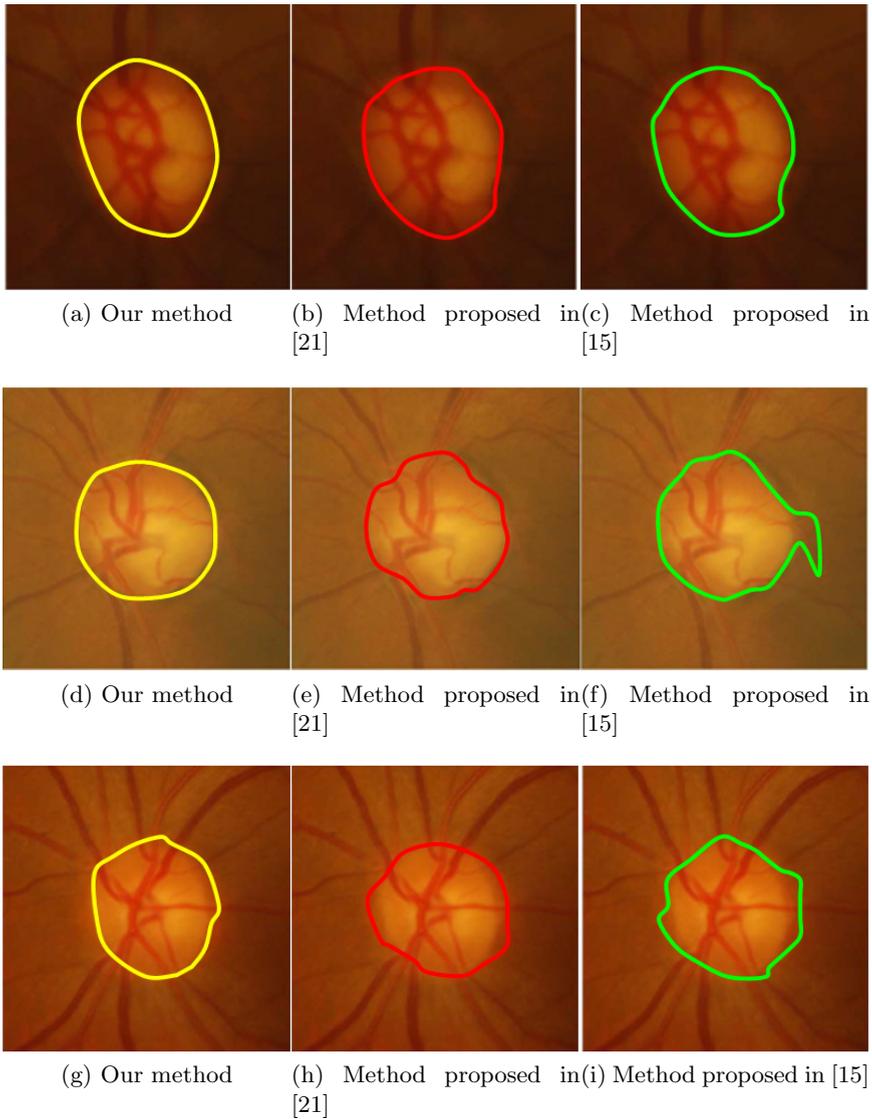
$$\eta_\sigma(\hat{\chi}) = \begin{cases} 1 & \text{if } \sigma < \hat{\chi}^{final} \leq 1 \\ 0 & 0 \leq \hat{\chi}^{final} \leq \sigma \end{cases} \tag{13}$$

In algorithm below an overview of the complete segmentation. Our segmentation model is initialized using an initial curve  $\chi^0$ . Given the parameters  $d^0, b^0, V_{Belief}^{in}$  and  $V_{Belief}^{out}$  and for the set parameters  $\lambda_{in}, \lambda_{out}, \mu,$  and  $\sigma$ , the KI function solves equation (10) to update primal variable  $\chi$ . The dual variable  $d^{k+1}$  is shrink iteratively. The final segmentation is given by  $\left\{ \mathbf{x} \in \Omega \mid \chi(\mathbf{x})^{final} \geq \frac{1}{2} \right\}$ .

## 4 Results

The proposed method was tested on a dataset [18,16] which contained both 914 color images acquired from 52 patients. These images have a definition of 1280 pixels/line for 1008 lines/image and are lossless compressed images. To illustrate and demonstrates the accuracy of our segmentation method, we present some results of our method and compare them to segmentation done by the traditional AC model based vector value image and the model proposed in [21]. The three methods are evaluated on 10 color images taken form the dataset [18,16] using F-measure criterion[13]. Traditional segmentation and method in [21] are initialized by contour curve around the object to be segmented, our method is free initialization and the segmentation done by the three method are presented for three challenging images (see Figure.1).

The accuracy of the segmentation is represented in term of Precision/Recall [13]. The proposed method give the best segmentation and the F-measure is better then the other methods (see Table.1).



**Fig. 1.** Images taken from the dataset [16]. The from the left to right, en yellow color segmentation done by our segmentation model, in red color segmentation done by the model proposed in [21] and green color, the segmentation done by the traditional model proposed in [15].

**Table 1.** Quantitative evaluation of the segmentation using F-measure

Image	Our method	Method in [21]	Method in [15]
Image 1	0.82	0.81	0.81
Image 2	0.79	0.76	0.77
Image 3	0.83	0.79	0.75

## 5 Conclusion

In this paper, we have investigate the use of the BFs in the Bhattacharyya distance framework. The results have shown that proposed approach give the best segmentation for biomedical images. The experimental results show that the segmentation performance is improved by using the three information sources to represent the same image with respect to the use information. Indeed, there are some drawbacks of our proposed method. The proposed is very high time consuming for calculating the mass functions. Furthermore, the research of other optimal models to estimate the mass functions in the DS theory and the imprecision coming from different images channels are an important perspective issue of our work.

## References

1. Appriou, A.: Generic approach of the uncertainty management in multisensor fusion processes. *Revue Traitement du Signal* 22(2), 307–319 (2005)
2. Bresson, X., Esedoglu, S., Vanderghenst, P., Thiran, J.P., Osher, S.: Fast global minimization of the active contour/snake model. *J. Math. Imaging Vis.* 28(2), 151–167 (2007)
3. Chan, T.F., Sandberg, B.Y., Vese, L.A.: Active contours without edges for vector-valued images. *Journal of Vis. Communi. and Image Repres.* 11, 130–141 (2000)
4. Cremers, D., Rousson, M., Deriche, R.: A review of statistical approaches to level set segmentation: Integrating color, texture, motion and shape. *Int. J. Comput. Vision* 72(2), 195–215 (2007)
5. Cuzzolin, F.: A geometric approach to the theory of evidence. *IEEE Trans. on Syst., Man, and Cyber., Part C* 38(4), 522–534 (2008)
6. De Vylder, J., Rooms, F., Philips, W.: Convex formulation and global optimization for multimodal active contour segmentation. In: 2011 7th International Symposium on Image and Signal Processing and Analysis (ISPA), pp. 165–170 (2011)
7. Dempster, A.P., Chiu, W.F.: Dempster-shafer models for object recognition and classification. *Int. J. Intell. Syst.* 21(3), 283–297 (2006)
8. Denoeux, T.: Maximum likelihood estimation from uncertain data in the belief function framework. *IEEE Trans. Knowl. Data Eng.* 25(1), 119–130 (2013)
9. Derraz, F., Taleb-Ahmed, A., Pinti, A., Peyrodie, L., Betrouni, N., Chikh, A., Bereksi-Reguig, F.: Fast unsupervised texture segmentation using active contours model driven by bhattacharyya gradient flow. In: Bayro-Corrochano, E., Eklundh, J.-O. (eds.) *CIARP 2009. LNCS*, vol. 5856, pp. 193–200. Springer, Heidelberg (2009)

10. Goldstein, T., Bresson, X., Osher, S.: Geometric applications of the split bregman method: Segmentation and surface reconstruction. *J. Sci. Comput.* 45(1-3), 272–293 (2010)
11. Goldstein, T., Osher, S.: The split bregman method for  $l_1$ -regularized problems. *SIAM J. Img. Sci.* 2(2), 323–343 (2009)
12. Lelandais, B., Gardin, I., Mouchard, L., Vera, P., Ruan, S.: Using belief function theory to deal with uncertainties and imprecisions in image processing. In: Denœux, T., Masson, M.-H. (eds.) *Belief Functions: Theory & Appl. AISC*, vol. 164, pp. 197–204. Springer, Heidelberg (2012)
13. Martin, D.R., Fowlkes, C.C., Malik, J.: Learning to detect natural image boundaries using local brightness, color, and texture cues. *IEEE Trans. Pattern Anal. Mach. Intell.* 26(5), 530–549 (2004)
14. Masson, M.-H., Denœux, T.: Ecm: An evidential version of the fuzzy c. *Pattern Recognition* 41(4), 1384–1397 (2008)
15. Michailovich, O., Rathi, Y., Tannenbaum, A.: Image segmentation using active contours driven by the bhattacharyya gradient flow. *IEEE Transactions on Image Processing* 16(11), 2787–2801 (2007)
16. Niemeijer, M., van Ginneken, B., Cree, M., Mizutani, A., Quellec, G., Sanchez, C., Zhang, B., Hornero, R., Lamard, M., Muramatsu, C., Wu, X., Cazuguel, G., You, J., Mayo, A., Li, Q., Hatanaka, Y., Cochener, B., Roux, C., Karray, F., Garcia, M., Fujita, H., Abramoff, M.: Retinopathy online challenge: Automatic detection of microaneurysms in digital color fundus photographs. *IEEE Transactions on Medical Imaging* 29(1), 185–195 (2010)
17. Parzen, E.: On estimation of a probability density function and mode. *The Annals of Mathematical Statistics* 33(3), 1065–1076 (1962)
18. Quellec, G., Lamard, M., Josselin, P., Cazuguel, G., Cochener, B., Roux, C.: Optimal wavelet transform for the detection of microaneurysms in retina photographs. *IEEE Transactions on Medical Imaging* 27(9), 1230–1241 (2008)
19. Rombaut, M., Zhu, Y.M.: Study of dempster–shafer theory for image segmentation applications. *Image and Vision Computing* 20(1), 15–23 (2002)
20. Rousson, M., Paragios, N.: Prior knowledge, level set representations & visual grouping. *Int. J. Comput. Vision* 76(3), 231–243 (2008)
21. Scheuermann, B., Rosenhahn, B.: Feature quarrels: The dempster-shafer evidence theory for image segmentation using a variational framework. In: Kimmel, R., Klette, R., Sugimoto, A. (eds.) *ACCV 2010, Part II. LNCS*, vol. 6493, pp. 426–439. Springer, Heidelberg (2011)
22. Tschumperle, D., Deriche, R.: Vector-valued image regularization with pdes: a common framework for different applications. *IEEE Trans. Pattern Anal. Mach. Intell.* 27(4), 506–517 (April)