Generalized Predictive Control for a Flexible Single-Link Manipulator

Rahma Boucetta

Control and **E**nergy **M**anagement **Lab**oratory (CEM Lab)
University of Sfax, Sfax Engineering School, BP W, 3038, Sfax, Tunisia
rboucetta@yahoo.fr

Abstract. This paper presents the development of a generalized predictive controller applied to a flexible single-link manipulator robot to compare to a fuzzy supervisory controller in input tracking and end-point vibration suppression. A dynamic model of the flexible manipulator is derived using finite elements method and Lagrange's equations to determine dynamics behavior. A generalized predictive controller is then developed and introduced in the system closed-loop to minimize end-point residual vibrations. A fuzzy supervisory controller is also synthesized to compare simulation results between the two methods of control in terms of input tracking and disturbance rejection.

Keywords: Flexible manipulator robot, dynamic model, generalized predictive control, fuzzy supervisory control.

1 Introduction

Robotic manipulators are generally built using heavy material to maximize stiffness, in an attempt to minimize system vibration and achieve good positional accuracy. As a consequence, such robots are usually heavy with respect to the operating payload. The operation speed of the robot manipulation is limited, so the actuators size is increased boosting energy consumption and increasing the overall cost. Moreover, the robot has a low payload to robot weight ratio. In order to solve these problems, robotic manipulators are designed to be lightweight.

Conversely, flexible manipulators exhibit many advantages over their rigid counterparts: they require less material, are lighter in weight, have higher manipulation speed, lower power consumption, require smaller actuators, are more maneuverable and transportable, are safer to operate due to reduced inertia, have enhanced back-drive ability due to elimination of gearing, have less overall cost and higher payload to robot weight ratio.

However, the control of flexible robotic manipulators to maintain accurate positioning is an extremely challenging problem. Due to the flexible nature and distributed characteristics of the system, the dynamics are highly non-linear and complex. Problems arise due to precise positioning requirement, vibration due to system

K. Saeed et al. (Eds.): CISIM 2013, LNCS 8104, pp. 499–510, 2013.

flexibility, difficulty in obtaining an accurate model and non-minimum phase characteristics of the system. Therefore, flexible manipulators have not favored in production industries, due to un-attained end-point positional accuracy requirements in response to input commands. Thus, the design of control algorithms for flexible systems possessing nonlinear time-varying and ill-modeled dynamics presents great challenges for all conventional methodologies.

M.A. Arteaga and B. Siciliano collected a number of recent results on modeling, nonlinear control and observer for flexible-link manipulators [8]. S.S. Ge proposed energy-based robust control strategies for the control of flexible link robots without using the dynamics of the systems explicitly [8]. O. Al Jarrah, Y.F. Zheng and K.-Y. Yi presented three approximation methods of the optimal trajectories and a compliant control scheme [8]. R.N. Banavar and P. Dominic applied an LQG/H_∞ controller for a flexible manipulator [11]. S.B. Choi and J.W. Cheon proposed a vibration control of a single-link flexible arm subjected to disturbances [12]. A neural network control is developed by C.-F.J. Kuo and C.-J. Lee for a rotating elastic manipulator [13]. L. Tian and C. Collins proposed firstly a dynamic recurrent neural network-based controller for a rigid-flexible manipulator system [14] and secondly an adaptive neuro-fuzzy control for a flexible manipulator [15]. A self-organizing fuzzy logic controller is used by G.L.C.M. de Abreu and J.F. Ribeiro for the active control of flexible structures using piezoelectric actuators [9]. Soft computing methods are applied to the control of a flexible robot manipulator by B. Subudhi and A.S. Morris [10]. It is in the last area that our contribution can be located.

The main purpose of this work is to determine the adequate controller to minimize human intervention and to increase performance responses. So, this paper is organized as follows: Section 2 describes the flexible manipulator system and how to derive its dynamic model using Lagrange's equations and finite elements method. The open-loop analysis is given in section 3. Section 4 presents the GPC controller applied to the flexible single-link manipulator. Section 5 developed the supervisory fuzzy controller given to the system. Finally, a comparative assessment of performances between the different strategies in terms of vibration suppression and input tracking is presented and discussed.

2 The Flexible Manipulator System

A schematic representation of the single-link flexible manipulator system is shown in figure 1, where a control torque $\tau(t)$ is applied at the hub of a motor with E, I, ρ, L and I_H represent Young's modulus, second moment of area, mass density per unit volume, length, and hub inertia moment respectively [1].

The angular displacement of the link in the X_0OY_0 coordinates is denoted by $\theta(t)$. w(x,t) represents the elastic deflection of the manipulator at a distance x from the hub, measured along the OX axis. X_0OY_0 and XOY represent the stationary and moving frames respectively.

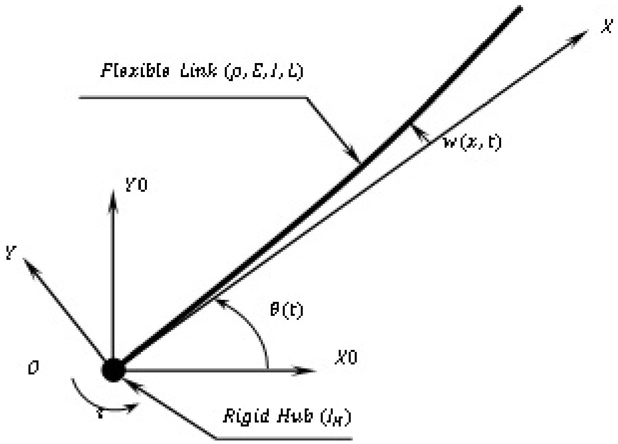

Fig. 1. Flexible manipulator scheme

The height (width) of the link is assumed to be much greater than its depth, thus allowing the manipulator to vibrate dominantly in the horizontal direction (X_0OY_0 plane). To avoid difficulties arising from time varying lengths, the length of the manipulator is assumed to be constant. Moreover, the shear deformation, the rotary inertia and the effect of axial force are ignored. For an angular displacement θ and an elastic deflection w, the total displacement y(x,t) of a point along the manipulator at a distance x from the hub can be described as a function of both the rigid body motion θ(t) and the elastic deflection w(x,t), i.e. [3, 7]

$$y(x,t) = x.\theta(t) + w(x,t) \tag{1}$$

Thus, by allowing the manipulator to be dominantly flexible in the horizontal direction, the elastic deflection of the manipulator can be assumed to be confined to the horizontal plane only.

Kinetic energy of the flexible manipulator, depending of hub rotation, modes rotation in the X0OY0 and XOY frames, has the following expression

$$T = \frac{1}{2} I_H \dot{\theta}^2 + \frac{1}{2} (\dot{q} + L\dot{\theta})^T M (\dot{q} + L\dot{\theta}) \tag{2}$$

Potential energy just depending of link flexibility has the form

$$V = \frac{1}{2} q^T K q \tag{3}$$

After applying Lagrange's equations, the dynamic model can be written as

$$(I_H + L^T M L)\ddot{\theta} + L^T M \ddot{q} = \tau \tag{4}$$

$$M\ddot{q} + L^T M \ddot{\theta} + Kq = 0 \tag{5}$$

where M, K and L are the mass matrix, the stiffness matrix and the length array respectively, and q is the elastic modes vector.

3 Dynamic Behavior

The dynamic equations can be presented in a state-space form as

$$\dot{v} = Av + Bu$$

$$y = Cv + Du$$

where the state-space matrices are

$$A = \begin{pmatrix} 0 & I \\ -M^{-1}K & 0 \end{pmatrix}, B = \begin{pmatrix} 0 \\ M^{-1} \end{pmatrix}, C = (I \quad 0), D = (0)$$

The state and control vectors are given by

$$v^T = (\theta \quad q_1 \quad q_2 \quad \dots \quad \dot{\theta} \quad \dot{q}_1 \quad \dot{q}_2 \quad \dots)$$

$$u^T = (\tau \quad 0 \quad \dots)$$

In order to simulate the flexible manipulator system, an aluminum link of dimensions L=0.61m and S=3×10^{-5}m², with E=200×10^{9}N/m², I=2.5×10-^{12}m^4, I_H=4.3×10^{-3}Kg.m² and ρ=7.8×103Kg/m^3 is considered [16]. The link is discretized into two elements.

Solving the state-space matrices gives the vector of states v, that is, the hub angle, the elastic modes and their velocities. The derived dynamic model is a nonminimum phase system, not strictly proper, and unstable. Also, the model has zeros very close to the imaginary axis; this deteriorates the time domain performance of the closed-loop system.

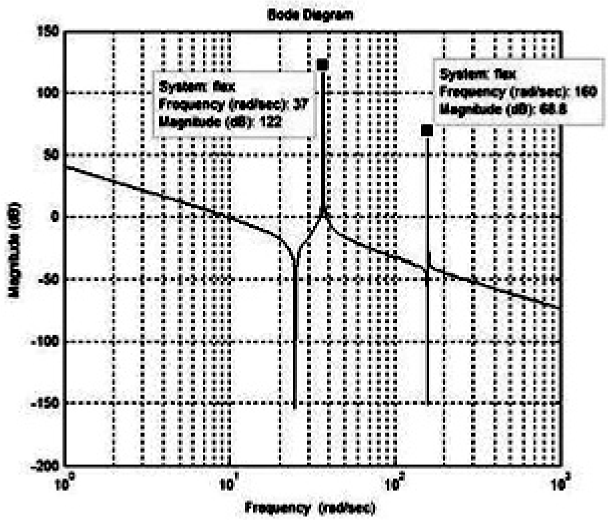

Fig. 2. Open-loop frequency response

Generally, linear models of flexible structures used in design of controllers are derived under restrictive assumptions which are often not valid for large motions that occur during slewing maneuvers. Hence, considerable uncertainty in the linear model exists. Another feature characteristic of lightly damped systems is the occurrence of poles (±159.67j, ±37j, 0, 0) and zeros (±158j, ±25j) very close to the imaginary axis that gives rise to ill-conditioned systems. The state-space matrices arising out of such systems have largely separated singular values, posing considerable computational difficulty in controller design. In the spectral density given by figure 2, the vibration frequencies of the system are obtained as 37rad/s and 160rad/s, i.e. 5.9Hz and 25.46Hz, and the magnitude of frequency response for the two resonance modes are 122dB and 68.8dB.

4 Generalized Predictive Control

4.1 ARIMAX Model

The Generalized Predictive Controller was introduced by Clarke and al. in 1987 [22]. The computation of the output predictions supposes the knowledge of a model of the system that is in the ARIMAX form:

$$A(q^{-1})y(t) = B(q^{-1})u(t - T_e) + \frac{C(q^{-1})}{\Delta(q^{-1})}\xi(t) \tag{6}$$

where:

- q^{-1} is the backward operator
- $y(t)$ is the output signal
- $u(t)$ is the input signal
- $\xi(t)$ is the disturbance (white noise) process with $E\big(\xi(t)\big) = 0$
- A and C are $p \times p$ monic polynomial matrices where C can be used to model a colored noise. To simplify the problem, we consider that $C = c(q^{-1})I_{p\times p}$ where c is a monic polynomial
- B is a $p \times m$ polynomial matrix
- The operator Δ is defined as $\Delta = 1 - q^{-1}$ used to make noise be non-stationary, which suitable to model any perturbation in a control loop.

The polynomial matrices A, B, C are respectively of order n_a, n_b, and n_c:

$$A(q^{-1}) = I_{p\times p} + A_1 q^{-1} + \cdots + A_{na} q^{-na}$$

$$B(q^{-1}) = B_0 q^{-d} + B_1 q^{-d-1} + \cdots + A_{nb} q^{-nb-d}$$

$$C(q^{-1}) = I_{p\times p} + C_1 q^{-1} + \cdots + C_{nc} q^{-nc}$$

where d is a positive integer representing the delay of the system. The operator $\Delta(q^{-1})$ allows the rejection of constant perturbations and is equivalent to the introduction of an integral action in the controller.

4.2 Cost Function

The GPC is based on the minimization of a cost function J over a finite receding horizon [18, 19, 21]:

$$J = E\left\{\sum_{j=N_1}^{N_2} \|y(t + jT_e) - r(t + jT_e)\|^2 + \lambda \sum_{j=0}^{N_u-1} \|\Delta u(t + jT_e)\|^2\right\} \tag{7}$$

with $N_u < N_2$ and $\Delta u(t + jT_e) = u(t + jT_e) - u(t + (j - 1)T_e) = 0\ \forall j \geq N_u$ where λ is a positive scalar and N_1, N_2 and N_u are positive integers defined as follows:

- N_1 is the minimum costing horizon,
- N_2 is the maximum costing horizon,
- N_u is the length of the control cost horizon,
- λ weights the relative importance of the control energy,
- T_e is the sampling time
- $r(t)$ is the reference trajectory

The aim is to compute the N_u future control increments, so as to drive the actual system outputs towards the theoretical ones, or, equivalently, to compensate for the measurement disturbances. This controller is predictive because it takes into account the future references. Indeed, the minimization of (6) requires the computation of N_2 predictions of the output using the future reference signals. The arguments of the minimization are the N_u future steps of the control input.

4.3 Control Computing

The minimization of the cost function J on a finite horizon allows us to compute the predictor on j optimal number of sampling periods to predict real output in $(k - j)$ sample periods. The vector $\underline{Y_c}$ is setting as

$$\underline{Y_c} = [y_c(k + HI), \ldots, y_c(k + HP)]^T \tag{8}$$

The prediction depending of past and present measures $\underline{E\hat{Y}_a}$ is defined as

$$\underline{E\hat{Y}_a} = \underline{\hat{Y}_a} - \underline{Y_c} = R^*\underline{\Delta U^*} + G^*Y^* - \underline{Y_c} \tag{9}$$

Similarly, the prediction depending of the sequence of future control increments $\underline{E\hat{Y}_p}$ minimizing the cost function is given by

$$\underline{E\hat{Y}_p} = Q^*\underline{\Delta U_p} \tag{10}$$

The prediction margin ranged from HI to HP, and the system delay is known a priori, so, the $(d - 1)$ first lines of matrices G^*, R^* and Q^* are not taken into account, and eq. 9 and eq. 10 are rewritten as

$$\underline{E\hat{Y}_a} = R^{**}\underline{\Delta U^*} + G^{**}\underline{Y^*} - \underline{Y_c}$$

$$\underline{E\hat{Y}_p} = Q^{**}\underline{\Delta U_p}$$

The aim of control is to compute optimal control sequence $\underline{\Delta U_p}$ minimizing the cost function. Length of this vector is reduced to $(HC, 1)$, and $(HP - HC)$ last columns of Q^{**} matrix are not taken into account. Consequently, a new matrix Q_* and a new expression of $\underline{E\hat{Y}_p}$

$$\underline{E\hat{Y}_p} = Q_*\underline{\Delta U_p}$$

$$\underline{\hat{Y}} = \underline{\hat{Y}_p} + \underline{\hat{Y}_a} = Q_*\underline{\Delta U_p} + R^{**}\underline{\Delta U}^* + G^{**}\underline{Y}^*$$

The cost function has the following matrix form

$$J = [Q_*\underline{\Delta U_p} + \underline{E\hat{Y}_a}]^T\left[Q_*\underline{\Delta U_p} + \underline{E\hat{Y}_a}\right] + \underline{\Delta U_p^T}\lambda I_{HC}\underline{\Delta U_p}$$

The minimal value of the cost function is

$$\frac{\partial J}{\partial \underline{\Delta U_p}} = 2Q_*^T\left[Q_*\underline{\Delta U_p} + \underline{E\hat{Y}_a}\right] + 2\lambda\underline{\Delta U_p} = 0$$

The optimal control for the minimal cost function has the following form

$$\underline{\Delta U_p} = -[Q_*^T Q_* + \lambda I_{HC}]^{-1} Q_*^T [R^{**}\underline{\Delta U}^* + G^{**}\underline{Y}^* - \underline{Y_c}$$

$$= -M[R^{**}\underline{\Delta U}^* + G^{**}\underline{Y}^* - \underline{Y_c}]$$

with

$$M = [Q_*^T Q_* + \lambda I_{HC}]^{-1} Q_*^T = \begin{bmatrix} m_1^T \\ m_2^T \\ \vdots \\ m_{HC}^T \end{bmatrix}$$

Finally, the resulted control signal is the first element of the control vector $\underline{\Delta U_p}$, given by the following expression

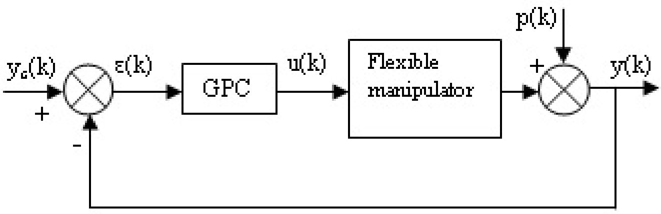

Fig. 3. GPC control scheme

$$u(k) = \Delta u(k) + u(k-1)$$

with

$$\Delta u(k) = -m_1^T[R^{**}\underline{\Delta U}^* + G^{**}\underline{Y}^* - \underline{Y_c}]$$

5 Fuzzy Supervisory Control

A highest level supervisor uses any available data from the control system to characterize the system's current behavior so that it knows how to change the controller and ultimately achieve the desired specifications. In addition, the supervisor can be used to integrate other information into the control decision-making process. It can incorporate certain user inputs, or inputs from other subsystems [2]. Conceptually, the design of the supervisory controller can then proceed in the same manner as it did for direct fuzzy controllers [18]: either via the gathering of heuristic control knowledge or via training data that we gather from an experiment. The type of heuristic knowledge that is used in a supervisor may take one of the following two forms

- Information from a human control system operator who observes the behavior of an existing control system and knows how this controller should be tuned under various operating conditions.
- Information gathered by a control engineer who knows that under different operating conditions controller parameters should be tuned according to certain rules.

A higher level of control can be achieved for monitoring and adjusting the direct fuzzy controller. The expert controller expands or compresses the universes of discourse by simply changing the scaling gains. When the universe is expanded, a coarse control given by the table 1 is achieved, and when it is compressed, a fine control given by the table 2 is achieved. The supervisor would be a fuzzy system that can gradually rather than abruptly switch between the two conditions using a Sugeno fuzzy system based on the following rule-base:

- If error is negative (positive) Then control action is coarse
- If error is zero Then control action is fine

Table 1. Rule-base for the coarse control

u		ε'						
		-3	-2	-1	0	+1	+2	+3
	-3	-4	-3	-3	-2	-2	-1	0
	-2	-3	-3	-2	-2	-1	0	+1
	-1	-3	-2	-2	-1	0	+1	+2
ε	0	-2	-2	-1	0	+1	+2	+2
	+1	-2	-1	0	+1	+2	+2	+3
	+2	-1	0	+1	+2	+2	+3	+3
	+3	0	+1	+2	+2	+3	+3	+4

Table 2. Rule-base for the fine control

u		ε'						
		-3	-2	-1	0	+1	+2	+3
ε	-3	-4	-4	-3	-3	-2	-1	0
	-2	-4	-3	-3	-2	-1	0	+1
	-1	-3	-3	-2	-1	0	+1	+2
	0	-2	-1	0	0	0	+1	+2
	+1	-2	-1	0	+1	+2	+3	+3
	+2	-1	0	+1	+2	+3	+3	+4
	+3	0	+1	+2	+3	+3	+4	+4

6 Simulation Results

To study the GPC and supervisory control performances and to compare simulation results in terms of input tracking, vibration suppression and disturbance rejection, the GPC controller is introduced in the closed-loop position control of the flexible single-link manipulator. Sampling time is chosen 0.01s.

Figure 5 depicts the time evolutions of both consign and hub angle responses. A transient appears at t=10s, corresponding to the start of the step consign chosen $pi/4$.

The control signal can compensate the fast change of consign at t=10s. So, the hub angle output varies before the consign signal and has very fast rise explaining the appearance of an overshoot peak over the consign value. After that, the plant output oscillates around the desired value with very low vibration magnitude.

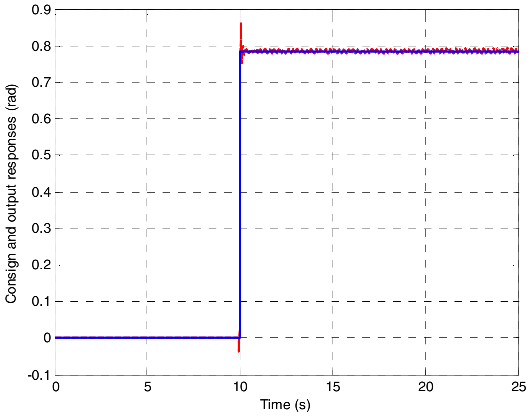

Fig. 4. Consign and output responses without disturbances

In figure 6, an impulse disturbance with amplitude of $pi/3$ is applied at t=15s, the plant output receives a small variation and continues with the same specifications.

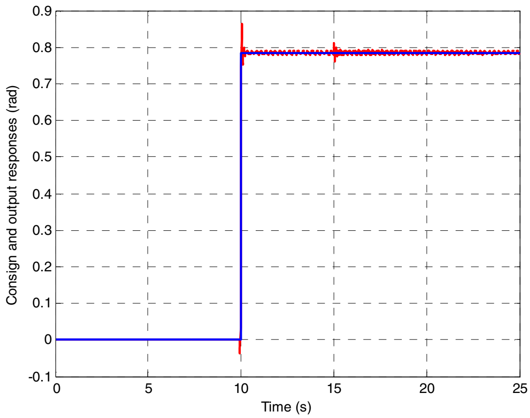

Fig. 5. Consign and output responses with disturbance at t=15s

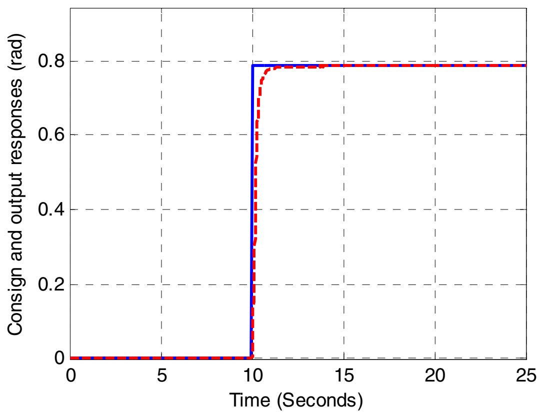

Fig. 6. Consign and output responses with supervisory controller

Figure 7 gives the hub angle response deduced from the fuzzy supervisory control, the output response has a fast transient phase to reach consign without overshoot, with rise time and settling time about 0.5s.

The introduction of a disturbance at t=15s in the fuzzy supervisor control case yields the result of figure 8, we can notice that the hub angle receives a slight peak and returns to its stable state.

For the two controllers, we can easily noticed that the GPC controller gives a fast hub angle response with a good input tracking but it has an overshoot in the rise, contrary to the fuzzy supervisory controller that has a slower rise time compared to the GPC, without vibrations or overshoot. The GPC controller has a good disturbance rejection against the fuzzy supervisor.

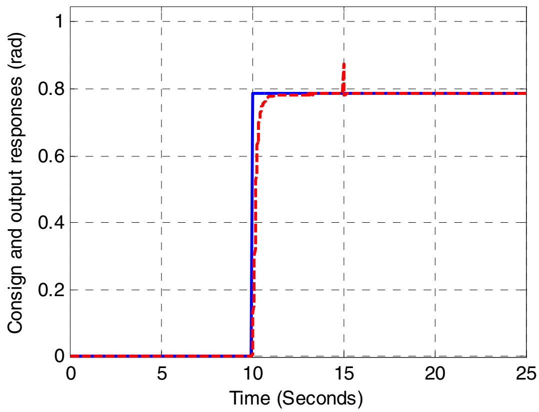

Fig. 7. Consign and output responses with supervisory controller and disturbance

7 Conclusion

A comparative assessment between the GPC and the fuzzy supervisory control strategies applied to a flexible single-link manipulator has been presented in this paper. A dynamic model of the flexible manipulator system is first derived using Lagrange's equations and finite element method. A GPC controller is then introduced in the closed-loop of the flexible system. Next, a fuzzy supervisory controller is developed to be incorporated in the system closed loop to increase level of desired performances. Simulation results are compared in terms of input tracking, disturbance rejection and vibration reduction.

References

1. Meirovitch, L.: Elements of Vibration Analysis. McGraw-Hill International Book Company (1982)
2. Passino, K.M., Yurkovich, S.: Fuzzy Control. Addison-Wesley Longman (1998)
3. Wolovich, W.A.: Automatic Control Systems, Basic Analysis and Design, International Edition (1994)
4. Md Zain, M.Z., Tokhi, M.O., Mailah, M., Mohamed, Z.: Improving Performance in Single-link Flexible Manipulator using Hybrid Learning Control. Jurnal Mekanikal, Bil. 18, 13–18 (2004)
5. Knani, J.: Contribution à la Commande Adaptative des Robots Manipulateurs Rigides et Flexibles. Habilitation Universitaire, ENIT (2003)
6. Azad, A.K.M., Tokhi, M.O., Anand, N.: Teaching of Control for Complex Systems through Simulation. In: Proceedings of the 2003 ASEE/WFEO International Colloquium. American Society for Engineering Education (2003)
7. Moberg, S.: On Modeling and Control of Flexible Manipulators, Thesis, Division of Automatic Control, Department of Electrical Engineering, Linköping Studies in Science and Technology, Sweden (2007)

8. Wang, F.-Y., Gao, Y.: Advanced Studies of Flexible Robotic Manipulators, Modeling, Design, Control and Applications. World Scientific Edition (2003)
9. de Abreu, G.L.C.M., Ribeiro, J.F.: A self-organizing fuzzy logic controller for the active control of flexible structures using piezoelectric actuators. Applied Soft. Computing, 271–283 (2002)
10. Subudhi, B., Morris, A.S.: Soft computing methods applied to the control of a flexible robot manipulator. Applied Soft. Computing, 149–158 (2009)
11. Banavar, R.N., Dominic, P.: An LQG/H Controller for a Flexible Manipulator. IEEE Transactions on Control Systems Technology 3(4) (December 1995)
12. Choi, S.B., Cheon, J.W.: Vibration control of a single-link flexible arm subjected to disturbances. Journal of Sound and Vibration 271, 1147–1156 (2004)
13. Kuo, C.-F.J., Lee, C.-J.: Neural Network Control of a Rotating Elastic Manipulator. Computers and Mathematics with Applications 42, 1009–1023 (2001)
14. Tian, L., Collins, C.: A dynamic recurrent neural network-based controller for a rigid-flexible manipulator system. Mechatronics 14, 471–490 (2004)
15. Tian, L., Collins, C.: Adaptive neuro-fuzzy control of a flexible manipulator. Mechatronics 15, 1305–1320 (2005)
16. Boucetta, R., Chabir, A., Chelly, N., Abdelkrim, M.N.: Building and Control of a Flexible One-link Manipulator. In: The 1st IAA Mediterranean Astronautical Conference, Tunis Science City (November 17-19, 2008)
17. Boucetta, R., Bel Hadj Ali, S., Abdelkrim, M.N.: On the Fuzzy Control of a Flexible Single-link Manipulator. In: SSD 2011, Sousse, Tunisia (March 2011)
18. Abdelkrim, R., Boucetta, R.: Compensation des mouvements respiratoires par la commande prédictive généralisée à modèle répétitif d'un robot médical. In: CRATT 2013 (2013)
19. Abdelkrim, R.: commande par retour visuel d'un robot médical, projet de fin d'étude, Université de Gabès, ENIG (2012)
20. Ott, L., Zanne, P., Nageotte, F., de Mathelin, M., Gangloff, J.: Physiological Motion Rejection in Flexible Endoscopy Using Visual Servoing. In: IEEE International Conference on Robotics and Automation, Pasadena, Californie USA (2008)
21. Ginhoux, R.: Compensation des mouvements physiologiques en chirurgie robotisée par commande prédictive, Thèse, Université de Louis Pasteur Strasbourg (2003)
22. Clarke, D.W., Mohtadi, C.: Properties of Generalized Predictive Control, vol. 25, pp. 859–875. Pergamon Press, International Federation of Automatic Control (1989)