The First Simple Symmetric 11-Venn Diagram

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An *n-Venn diagram* is a collection of n simple closed curves in the plane with the following properties: (a) Each of the 2^n different intersections of the open interiors or exteriors of the curves is a non-empty connected region; (b) there are only finitely many points where the curves intersect. If each of the intersections is of only two curves, then the diagram is said to be *simple*. The purpose of this poster is to highlight how we discovered the first simple symmetric 11-Venn diagram.

A n-Venn diagram is *symmetric* if it is left fixed (up to a relabeling of the curves) by a rotation of the plane by $2\pi/n$ radians. Interest in symmetric Venn diagrams was initiated by Henderson in a 1963 paper in which he showed that symmetric n-Venn diagrams do not exist when n is a composite number [6]. Of course, it is easy to draw symmetric 2- and 3-Venn diagrams using circles as the curves, but it was not until 1975 that Grünbaum [3] published a simple symmetric 5-Venn diagram, one that could be drawn using ellipses. Some 20 years later, in 1992, simple symmetric 7-Venn diagrams were discovered independently by Grünbaum [4] and by Edwards [1]. Hamburger [5] was the first to discover a (non-simple) symmetric 11-Venn diagram in 2002 and later Griggs, Killian and Savage (GKS) showed how to construct symmetric, but highly non-simple, n-Venn diagrams whenever n is prime [2].

Part of the interest in Venn diagrams is due to the fact that their geometric dual graphs are planar spanning subgraphs of the hypercube; furthermore, if the Venn diagram is simple then the subgraph is maximum in the sense that every face is a quadrilateral. Symmetric drawings of Venn diagrams imply symmetric drawings of certain spanning subgraphs of the hypercube.

We define a *crosscut* of a Venn diagram as a segment of a curve which sequentially "cuts" every other curve without repetition. A symmetric n-Venn diagram is *crosscut-symmetric* if aside from the innermost face and the outermost face, it can be partitioned into n congruent connected collection of regions, each of size $(2^n - 2)/n$ which we call it a *cluster*; such that for any curve C not containing the crosscut, the sequence of curves crossing C in the cluster is palindromic. Figure 1(a) is a simple crosscut-symmetric 7-Venn diagram with a cluster has been shaded. The cluster is redrawn in Figure 1(b) such that a central shaded section has a reflective symmetry about the crosscut. The sequence of curves intersecting C_5 for example, is $[C_4, C_6, C_3, C_6, C_4, C_1, C_4, C_6, C_3, C_6, C_4]$.

A Venn diagram is *monotone* if it is drawable in the plane with all curves convex. Given π as the permutation of curve labels along a ray emanating from the centre of a monotone simple symmetric n-Venn diagram, we use a sequence of length $2^n - 2$ of integers which we call it *crossing sequence* to represent the diagram . An entry of value i, $1 \leq i < n$, of the crossing sequence indicates the intersection of

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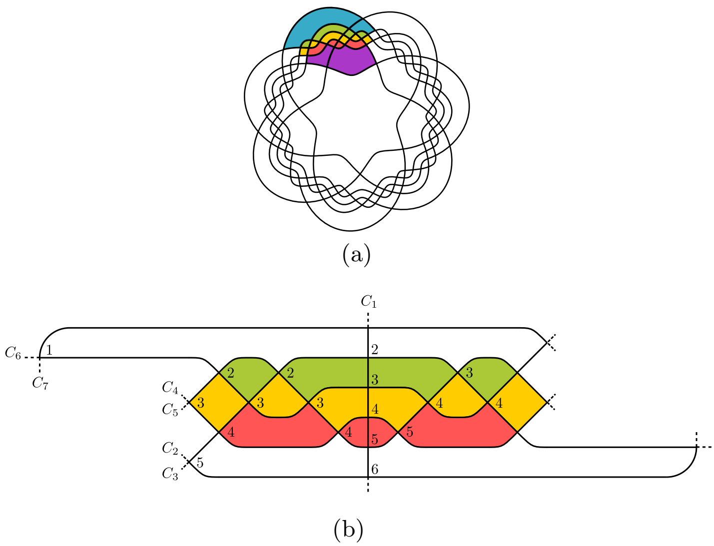

Fig. 1. (a) A simple symmetric monotone 7-Venn diagram. (b) A cluster of the diagram, showing the reflective aspect of crosscut symmetry.

curves $\pi(i)$ and $\pi(i+1)$. For example, the 7-Venn diagram of Figure 1(a) could be represented by the crossing sequence $\underbrace{1,3,2,5,4}_{\rho}, \underbrace{3,2,3,4}_{\alpha}, \underbrace{6,5,4,3,2}_{\delta}, \underbrace{5,4,3,4}_{\alpha^{r+}}$.

Theorem 1. *A simple monotone rotationally symmetric n-Venn diagram is crosscut symmetric if and only if it can be represented by a crossing sequence of the form $\rho, \alpha, \delta, \alpha^{r+}$ where*

- *ρ is $1,3,2,5,4,\ldots,n-2,n-3$ and δ is $n-1,n-2,\ldots,3,2$.*
- *α and α^{r+} are two sequences of length $(2^{n-1}-(n-1)^2)/n$ such that α^{r+} is obtained by reversing α and adding 1 to each element; that is, $\alpha^{r+}[i] = \alpha[|\alpha|-i+1]$.*

Using an exhaustive search of α sequences, we found more than 200,000 simple monotone symmetric Venn diagrams which settles a long-standing open problem in this area. The search algorithm is of the backtracking variety; for each possible case of α, we construct the crossing sequence $S = \rho, \alpha, \delta, \alpha^{r+}$ checking along the way whether it currently satisfies the Venn diagram constraints, and then doing a final check of whether S represents a valid symmetric Venn diagram. Some illustrations of our first 11-Venn diagram may be found at `http://webhome.cs.uvic.ca/~ruskey/Publications/Venn11/Venn11.html`. Following Anthony Edwards' tradition of naming symmetric diagrams [1], we named it Newroz which means "the new day" or "the new sun" in Kurdish; for English speakers, Newroz sounds also like "new rose", perhaps also an apt description. Below is the α sequence of Newroz.

323434543234345434545654565676543254346545676787656543457654658765457656876546576567

References

1. Edwards, A.W.F.: Seven-set Venn diagrams with rotational and polar symmetry. Combin. Probab. Comput. 7(2), 149–152 (1998)
2. Griggs, J., Killian, C.E., Savage, C.D.: Venn diagrams and symmetric chain decompositions in the Boolean lattice. Electron. J. Combin. 11(1), Research Paper 2, 30 (electronic) (2004)
3. Grünbaum, B.: Venn diagrams and independent families of sets. Mathematics Magazine 48, 12–23 (1975)
4. Grünbaum, B.: Venn diagrams. II. Geombinatorics 2(2), 25–32 (1992)
5. Hamburger, P.: Doodles and doilies, non-simple symmetric Venn diagrams. Discrete Math. 257(2-3), 423–439 (2002), kleitman and combinatorics: a celebration (Cambridge, MA, 1999)
6. Henderson, D.W.: Venn Diagrams for More than Four Classes. Amer. Math. Monthly 70(4), 424–426 (1963)