

# Cryptographic Hardness of Random Local Functions – Survey

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Constant parallel-time cryptography allows performing complex cryptographic tasks at an ultimate level of parallelism, namely, by local functions that each of their output bits depend on a constant number of input bits. The feasibility of such highly efficient cryptographic constructions was widely studied in the last decade via two main research threads.

The first is an encoding-based approach, developed in [1,2], in which standard cryptographic computations are transformed into local computations via the use of special encoding schemes called *randomized encoding* of functions. The second approach, initiated by Goldreich [3], is more direct and it conjectures that almost all non-trivial local functions have some cryptographic properties.

In this survey we focus on the latter approach. We consider *random local functions* in which each output bit is computed by applying some fixed  $d$ -local predicate  $P$  to a randomly chosen  $d$ -size subset of the input bits. Formally, this can be viewed as selecting a random member from a collection  $\mathcal{F}_{P,n,m}$  of  $d$ -local functions where each member  $f_{G,P} : \{0,1\}^n \rightarrow \{0,1\}^m$  is specified by a  $d$ -uniform hypergraph  $G$  with  $n$  nodes and  $m$  hyperedges, and the  $i$ -th output of  $f_{G,P}$  is computed by applying the predicate  $P$  to the  $d$  inputs that are indexed by the  $i$ -th hyperedge.

In this talk, we will investigate the cryptographic hardness of random local functions. In particular, we will survey known attacks and hardness results, discuss different flavors of hardness (one-wayness, pseudorandomness, collision resistance, public-key encryption), and mention applications to other problems in cryptography and computational complexity. We also present some open questions with the hope to develop a systematic study of the cryptographic hardness of local functions.

## References

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