

Research on Identifying Method of Freezing-Thawing Soil Hydraulic Properties

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Abstract. The identification of freezing-thawing soil hydraulic properties is the basis and key problem of studying soil water movement. Current identifying methods of the properties have many limitations, such as big error, high uncertainty and low operability. Thus, the identifying method of freezing-thawing soil hydraulic properties was researched in the research. The estimating parameters of soil freezing characteristic curve were calculated by classical statistical method. Targeting at the maximum absolute difference between measured and simulated value of unsaturated soil hydraulic properties, the particle swarm optimization algorithm was used to estimate the optimal values of soil water characteristic curve, unsaturated hydraulic conductivity and specific water capacity. The numerical simulation of freezing-thawing soil water movement proved that the measured and simulated value of soil unfrozen water content had high degree of agreement. The identifying method proposed in the research is effective and feasible.

Keywords: Freezing-thawing soil, Soil hydraulic properties, Identifying method, Particle swarm optimization.

1 Introduction

Mathematical simulation is the method of researches on seasonal freezing-thawing soil water movement. The freezing-thawing soil hydraulic properties are the necessary data and the basis of quantification study precision whether in the analytic or numerical solutions. The hydraulic properties include soil freezing characteristic curve, soil water characteristic curve, unsaturated hydraulic conductivity (K), unsaturated diffusivity (D) and specific water capacity (C). For the later three properties can be related by the relational expression $D = K/C$, generally, two of them have to be identified.

The identifying method of freezing-thawing soil hydraulic properties includes direct manner and indirect manner [1]. The direct manner involves laboratory test and field experiment [2-3]. The indirect manner is the priority research area and can be found in many interrelated studies, such as pedo-transfer functions (PTFs), fractal,

soil morphology, numerical inversion and empirical formulae [4]. The current means have great error due to the complexity of freezing-thawing soil water movement [5-11]. After comprehensive analysis of current means, a new set of identifying methods of freezing-thawing soil hydraulic properties was proposed and its validity was verified in the study.

2 Basic Theories

2.1 Soil Freezing Characteristic Curve

Soil liquid water is not completely frozen when soil temperature is lower than 0°C and a certain amount of soil unfrozen water exists all along. The content of unfrozen water in frozen soil depends on earthiness, environmental condition and freezing-thawing history. Soil freezing characteristic curve reflects a dynamic balance relation between unfrozen water content and negative temperature in frozen soil and can be expressed as follows:

$$\theta_u = AT_{abs}^{-B} \quad (1)$$

where θ_u is the soil unfrozen water content, T_{abs} is the absolute value of negative temperature, A and B are the empirical coefficients related with earthiness.

Soil freezing characteristic curve is a normal straight line and can be obtained by regression analysis of experimental data.

2.2 Van Genuchten-Mualem Model

van Genuchten-Mualem model is an empirical model of calculating unsaturated soil hydraulic properties. It is widely used in soil water movement research for it can properly fit filed experimental data, although its mathematical form is complex [12-13]. The formulae of van Genuchten-Mualem model used in the study are as follows:

$$|\psi| = \{[(\theta - \theta_r)/(\theta_s - \theta_r)]^{\frac{n}{1-n}} - 1\}^{\frac{1}{n}} / a \quad (2)$$

$$K = K_s \{1 - (a|\psi|)^{n-1} [1 + (a|\psi|)^n]^{\frac{1-n}{n}}\}^2 / [1 + (a|\psi|)^n]^{\frac{n-1}{2n}} \quad (3)$$

$$C = (n-1)(\theta - \theta_r) \{1 - [(\theta - \theta_r)/(\theta_s - \theta_r)]^{\frac{n}{n-1}}\} / |\psi| \quad (4)$$

where θ_s is the saturated soil water content, θ_r is the soil residual water content, ψ is the soil matric potential, a and n are the empirical parameters.

The two empirical parameters in van Genuchten-Mualem model can be obtained by reversely solving the relation curve of the measured soil unfrozen water content data

and the corresponding soil matric potential data. However, the curve is complicated and its parameter estimation is a nonlinear fitting problem, therefore the particle swarm optimization algorithm is performed in the study to solve the problem.

Particle swarm optimization (PSO) algorithm was proposed by Kennedy and Eberhart in 1995 [14-15]. It is a kind of self-adapting evolutionary computation technology and swarm intelligence algorithm on the basis of population search. Each solution of optimization problem is considered as a particle among the searching space in PSO algorithm. Each particle has its own position, speed and adaptive value (*fitness*) decided by a certain optimization problem. The best position that a particle has experienced during the flight is the best solution found by the particle (called the personal extremum, *pbest*). The best position that the whole group has experienced is the best solution found by the whole group (called the global extremum, *gbest*). Every particle constantly updates itself by using *pbest* and *gbest* and then creates a new population. At last, the whole population can comprehensively search the solution region and find the optimal values [16].

Let N be the particle population size, x_i be the position of the i th particle ($i=1, 2, \dots, N$), v_i be the speed and f_i be the adaptive value of the particle. After the initial position and speed generate randomly, a particle updates itself by following the personal extremum $pbest_i(t)$ and the global extremum $gbest(t)$ in each iteration. At the moment of $t+1$, any particle i can update its own position and speed by the following formulae:

$$v_i(t+1) = wv_i(t) + c_1r_1(t)(pbest_i(t) - x_i(t)) + c_2r_2(t)(gbest(t) - x_i(t)) \tag{5}$$

$$x_i(t+1) = x_i(t) + v_i(t+1) \tag{6}$$

where c_1 and c_2 are the learning factors, r_1 and r_2 are the random numbers evenly distributing in the interval of (0, 1), w is the inertia weight.

Then the personal extremum of each particle and the global extremum of all particles can update by the following formulae:

$$pbest_i(t+1) = \begin{cases} x_i(t+1) & f_i(t+1) \geq f(pbest_i(t)) \\ pbest_i(t) & f_i(t+1) < f(pbest_i(t)) \end{cases} \tag{7}$$

$$gbest(t+1) = x_{\max}(t+1) \tag{8}$$

where $f_i(t+1)$ is the adaptive value of the i th particle at the moment of $t+1$, $f(pbest_i(t))$ is the best adaptive value of the i th particle in its searching history, $x_{\max}(t+1)$ is the position of the particle that corresponds with the biggest $f(pbest_i(t))$ among all the particles at the moment of $t+1$.

The two parameters (a and n) are considered as the particle of PSO. The objective function of PSO can be expressed by the following formula:

$$\min Q = \max(|Y_1 - y_1|, |Y_2 - y_2|, \dots, |Y_m - y_m|) \tag{9}$$

where Y_i and y_i are the calculated and measured value respectively, m is the number of the measuring point in soil profile.

The algorithm will stop running when the difference between the former and later iteration value of the global optimal particle is less than error threshold. Thus, the convergence criterion of PSO can be expressed as follows:

$$\left| a_{gbest}^p - a_{gbest}^{p-1} \right| \leq e_1 \ \& \ \left| n_{gbest}^p - n_{gbest}^{p-1} \right| \leq e_2 \tag{10}$$

where a_{gbest} and n_{best} are the global optimal values of the model parameters (a and n) in each iteration, p and $p-1$ are the iteration number, e_1 and e_2 are the convergence thresholds of the model parameters (a and n).

3 Results and Analysis

3.1 Fixing Soil Freezing Characteristic Curve

The soil profile in the study was divided into two layers, one extended from 0 to 100cm and the other one is under 100cm. The functional relations of soil unfrozen water content and negative temperature were built separately on the basis of the measured data from two different soil layers. The curve fitting situations were showed in Fig. 1.

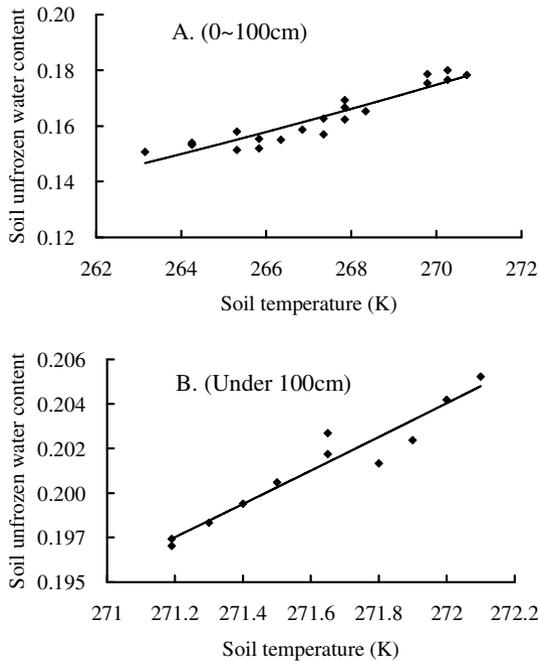


Fig. 1. Soil freezing characteristic curve

The expressions of soil freezing characteristic curves of the two soil layers are as follows:

$$\theta_u = 2E - 18(T + 273.16)^{6.935} \quad (0\sim 100\text{cm}) \tag{11}$$

$$\theta_u = 6E - 32(T + 273.16)^{12.535} \quad (\text{Under } 100\text{cm}) \tag{12}$$

where θ_u is the soil unfrozen water content, T is the negative temperature.

3.2 Estimating Unsaturated Soil Hydraulic Properties

The PSO algorithm was programmed in Matlab 7.0. Set population size of particle swarm is 200; the maximum iteration time is 10000; the convergence thresholds as 1×10^{-8} and 1×10^{-4} ; the learning factors as 2 and the inertia weight changing from 0.9 to 0.1. After running the PSO algorithm program, the optimal solution of the objective function was found when the program ran to the convergence criterion and the model parameters of the two soil layers were listed in Table 1.

Table 1. Results of estimating model parameters

Depth (cm)	θ_s	θ_r	$a \text{ (cm}^{-1}\text{)}$	n
0~100	0.46735	0.02	0.000086	1.6939
Under 100	0.458	0.03	0.0684	1.1271

The soil water characteristic curves of the two layers were fixed by putting the parameters from Table 1 back in the van Genuchten-Mualem model, and showed in Fig. 2.

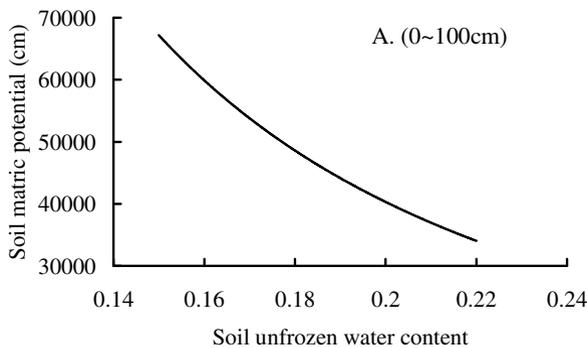


Fig. 2. Soil water characteristic curve

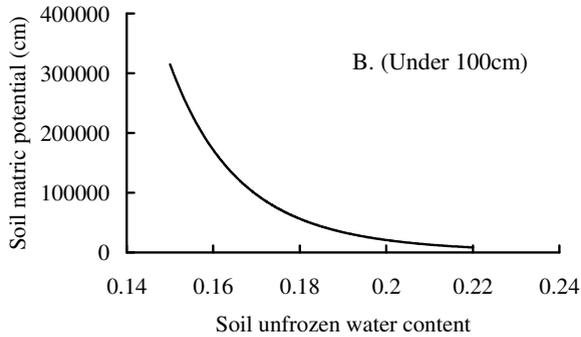


Fig. 2. (Continued)

The expressions of unsaturated hydraulic conductivity and specific water capacity in each soil layer were as follows:

$$\begin{cases} K = \frac{K_s \left\{ 1 - (8.6 \times 10^{-5} |\psi|)^{0.6939} \left[1 + (8.6 \times 10^{-5} |\psi|)^{1.6939} \right]^{-0.4096} \right\}^2}{\left[1 + (8.6 \times 10^{-5} |\psi|)^{1.6939} \right]^{0.2048}} \\ C = \frac{0.6939(\theta - 0.02) \left\{ 1 - [(\theta - 0.02) / 0.44735]^{2.4411} \right\}}{|\psi|} \end{cases} \quad (0 \sim 100\text{cm}) \quad (13)$$

$$\begin{cases} K = \frac{K_s \left\{ 1 - (0.0684 |\psi|)^{0.1271} \left[1 + (0.0684 |\psi|)^{1.1271} \right]^{-0.1128} \right\}^2}{\left[1 + (0.0684 |\psi|)^{1.1271} \right]^{0.0564}} \\ C = \frac{0.1271(\theta - 0.03) \left\{ 1 - [(\theta - 0.03) / 0.428]^{8.8678} \right\}}{|\psi|} \end{cases} \quad (\text{Under } 100\text{cm}) \quad (14)$$

3.3 Verifying Validity of Identifying Method

In order to verify the validity of the identifying method of freezing-thawing soil hydraulic properties, the comparison between numerical simulated value and filed measured value is necessary. Thus, the numerical simulation of freezing-thawing soil water movement was conducted on a period in 2008.

The soil unfrozen water content distribution in December 19, 2008 was simulated and the freezing status was that the frozen depth had developed to 55cm. The comparison between simulated and measured values was showed in Fig. 3.

According to Fig. 3, the simulated and measured soil unfrozen water content in December 19, 2008 coincided with each other very well and had the same trend. The average relative error of the numerical simulation of soil unfrozen water content was 20.18%, therefore, the simulation was successful.

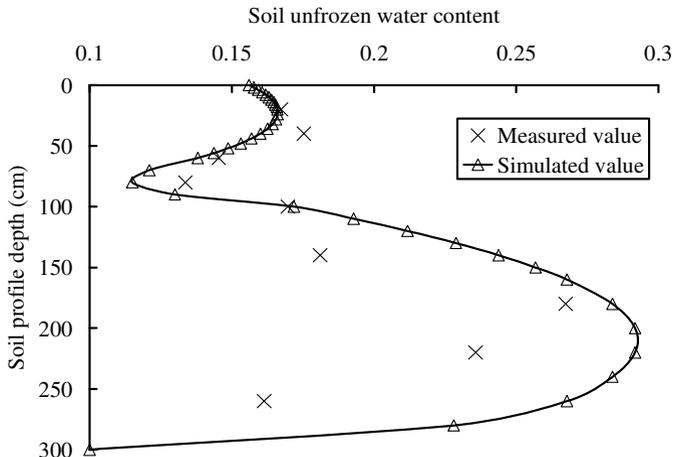


Fig. 2. Comparison between the measured and simulated soil unfrozen water content (2008-12-19)

4 Conclusions

A new set of identifying methods of freezing-thawing soil hydraulic properties was proposed in the study. It includes two part, one part is fixing soil freezing characteristic curve by a regression and fitting analysis of measured data, the other part is using particle swarm optimization algorithm to estimate the empirical parameters of van Genuchten-Mualem model and then to obtain soil water characteristic curve, unsaturated hydraulic conductivity and specific water capacity. At last, a comparison and error analysis was performed on the simulated and measured data of freezing-thawing soil water movement for the purpose of verifying method validity. The results showed that the identifying method of freezing-thawing soil hydraulic properties proposed in the study had good effectiveness and suitability.

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