Adaptive Dynamic Surface Control of a Class of Nonlinear Systems with Unknown Duhem Hysteresis

Yong-Hua Liu[1], Ying Feng[1,⋆], Juan Du[1], and Chun-Yi Su[1,2]

[1] College of Automation Science and Engineering,
South China University of Technology, Guangzhou, Guangdong, 510641, China
[2] Department of Mechanical and Industrial Engineering,
Concordia University, Montreal, Quebec, H3G 1M8, Canada
{scutliux,chunyi.su}@gmail.com, {yfeng,dujuan}@scut.edu.cn

Abstract. In this paper, the tracking problem for a class of uncertain perturbed strict-feedback nonlinear systems with unknown Duhem hysteresis input is investigated. Different with the conventional nonlinear systems, the existence of the unknown preceded hysteresis will affect the system performance and bring a challenge for the controller design. To overcome the difficulties caused by the unknown hysteresis, the Duhem model is used to describe the hysteresis in this paper. The properties of the Duhem model are utilized to get the explicit expression of the hysteresis output, which makes it possible to deal with the unknown hysteresis input. Following the conventional backstepping design procedure, a dynamic surface control method in each step is used to avoid "the explosion complexity" in the backstepping design, and the Nussbaum function method is used to solve the time-varying coefficient problem in the explicit expression of the Duhem model. Under the proposed control approach, the semiglobal uniform ultimate boundedness of all the signals in the close-loop system is guaranteed. The effectiveness of the proposed design scheme is validated through a simulation example.

Keywords: Nonlinear systems, hysteresis, dynamic surface control.

1 Introduction

As a class of strongly nonlinear phenomena, hysteresis nonlinearities occur in numerous physical systems and industrial elements, such as electromagnetic fields, mechanical systems, and smart materials-based actuators [1–3]. Compared with the conventional nonlinearities, non-smooth and multi-values properties of hysteresis limit the system performance, and the available traditional control approaches may not be effective for these systems. Therefore, the modeling and control problems for the controlled systems with hysteresis have attracted more attention, and the unknown hysteresis as the systems input becomes a new challenge for the control system design.

⋆ Corresponding author.

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Addressing this challenge, the hysteresis modeling methods become the primary step for the control design. So far, hysteresis models can be simple classified as operator-based hysteresis models, such as Preisach model, Krasnosel'skii-Pokrovskii (KP) model, and Prandtl-Ishlinskii (PI) model etc. [4–6], and differential equation-based hysteresis models, such as backlash-like model, Bouc-Wen model and Duhem model etc. [7–9].

Recently, lots of new control strategies based on the above various hysteresis model are developed to suppress the detrimental effects caused by hysteresis nonlinearities [10–12]. These control approaches can also be classified as constructing the hysteresis inverse and without constructing the hysteresis inverse. Constructing the hysteresis inverse approach is pioneered by Tao and Kokotovic [13]. The main advantage of this approach is to cancel the negative effects caused by hysteresis directly, when the inverse model matches the hysteresis exactly. However, this method is very sensitive to the model parameters and may cause a new difficulty for the stability analysis. To avoid these difficulties, another alternative control approach without constructing the hysteresis inverse is developed in [7]. As an illustration, a robust adaptive control law was investigated for a class of nonlinear systems with unknown backlash-like hysteresis [7]. Adaptive variable structure control was proposed for a class of nonlinear systems preceded by PI hysteresis in [14]. The common feature of this scheme is the hysteresis model can be decomposed as linear component and nonlinear bounded component[15–17], and this property can be utilized in the control design.

In this paper, the Duhem model is used to represent the hysteresis nonlinearities. The Duhem model can describe a class of general hysteresis shapes by choosing different input functions. However, due to the existence of the nonlinear input functions, it generates the difficulty for the controller design, which needs a special new treatment. By exploring the characteristics of the Duhem model, the explicit expression of the Duhem model can be transferred as a linear input with time-varying coefficient, which facilitates the control design. Different with the classical backstepping method, the dynamic surface control (DSC) method without the inverse of the Duhem hysteresis is discussed in this paper. This method can mitigate the effects caused by the Duhem hysteresis effectively and avoid "the explosion complexity" coming from the backstepping by applying the low-pass filters in the design of dynamic control laws [18]. Under the proposed control approach, semiglobal uniform ultimate boundedness of all the signals in the close-loop system is guaranteed. Finally, the effectiveness of the proposed design scheme is validated through a simulation.

2 Problem Statement

Consider a class of perturbed strict-feedback nonlinear systems with unknown hysteresis nonlinearities described as

$$\begin{cases} \dot{x}_i = \theta_i f_i(\bar{x}_i(t)) + g_i x_{i+1}(t) + d_i(x(t), t) \\ \dot{x}_n = \theta_n f_n(x(t)) + g_n w(t) + d_n(x(t), t) \\ y = x_1 \end{cases} \tag{1}$$

where $\bar{x}_i(t) = [x_i(t), ..., x_i(t)]^T \in R^i, i = 1, ..., n$ are the system states, $x(t) := \bar{x}_n(t) = [x_1(t), ..., x_n(t)]^T \in R^n$. $y \in R$ is the system output. $g_i, \theta_i, i = 1, \ldots, n$ are unknown system parameters. $d_i(x(t), t), i = 1, \ldots, n$ denote the unknown uncertain disturbances. $f_i(\cdot), i = 1, \ldots, n$ are known smooth functions. $w(t) \in R$ is the system input, which is also the output of the preceded hysteresis. In this paper, the hysteresis is represented by the Duhem model in [9] as follows:

$$\frac{dw}{dt} = \alpha \left| \frac{du}{dt} \right| (\lambda(u) - w) + \frac{du}{dt} \psi(u) \tag{2}$$

where u is the input of the hysteresis, α is a constant, and the Duhem hysteresis model discussed in this paper satisfies three conditions[6].

Condition 1. $\lambda(u)$ is piecewise smooth, monotone increasing, odd, with $\lim_{u \to \infty} \dot{\lambda}(u)$ finite.

Condition 2. $\psi(u)$ is piecewise continuous, even, with

$$\lim_{u \to \infty} \psi(u) = \lim_{u \to \infty} \dot{\lambda}(u) \tag{3}$$

Condition 3. $\dot{\lambda}(u) > \psi(u) > \alpha e^{\alpha u} \int_u^\infty |\dot{\lambda}(\zeta) - \psi(\zeta)| e^{-\alpha\zeta} d\zeta$ for all $u > 0$.

Satisfying the above conditions, the Duhem model defined in (2) can be solved explicitly as [6]

$$w = \lambda(u) + \varphi(u) \tag{4}$$

where

$$\begin{aligned} \varphi(u) = {} & [w_0 - \lambda(u_0)] e^{-\alpha(u-u_0)sgn(\dot{u})} \\ & + e^{-\alpha u sgn(\dot{u})} \int_{u_0}^{u} [\psi(\zeta) - \dot{\lambda}(\zeta)] e^{\alpha\zeta sgn(\dot{u})} d\zeta \end{aligned}$$

In [6], it has been proven that $\varphi(u)$ is bounded.

Since $\lambda(u)$ in Duhem model satisfies Condition 1, it is obvious that the mean value theorem can be used for $\lambda(u)$. By choosing $\lambda(\theta) = 0$, $\lambda(u)$ in (4) can be expressed as follows

$$\lambda(u) = \lambda(u) - \lambda(\theta) = \dot{\lambda}(\vartheta(u))(u - \theta) \tag{5}$$

where $\vartheta(u) = \varsigma u + (1 - \varsigma)\theta$ with $0 \leq \varsigma \leq 1$.

Utilizing this transform based on mean value theorem, the Duhem hysteresis output w can re-presented as

$$w(t) = \dot{\lambda}(\vartheta(u))u - \dot{\lambda}(\vartheta(u))\theta + \varphi(u(t)) \tag{6}$$

For the convenience of expression, we define the function $L(t)$ as

$$L(t) = \dot{\lambda}(\vartheta(u(t))) \tag{7}$$

then w can be expressed as

$$w(t) = L(t)u(t) + S(t) \tag{8}$$

where $S(t) = -L(t)\theta + \varphi(u(t))$.

Substituting (8) into the controlled systems defined in (1), it has

$$\begin{cases} \dot{x}_i = \theta_i f_i(\bar{x}_i(t)) + g_i x_{i+1}(t) + d_i(x(t), t) \\ \dot{x}_n = \theta_n f_n(x(t)) + g_n[L(t)u(t) + S(t)] + d_n(x(t), t) \\ y = x_1 \end{cases} \tag{9}$$

3 Adaptive DSC Design and Stability Analysis

In this section, the adaptive dynamic surface control design method and the stability of the closed-loop system are presented.

In order to present the developed control laws, the following assumptions regarding the systems (9) and a lemma are required.

Assumption 1. The desired trajectory vectors are continuous and available, and $[y_d, \dot{y}_d, \ddot{y}_d]^T \in \Omega_d$ with known compact set $\Omega_d = \{[y_d, \dot{y}_d, \ddot{y}_d]^T : y_d^2 + \dot{y}_d^2 + \ddot{y}_d^2 \leq B_0\} \subset R^3$, whose size B_0 is a known positive constant.

Assumption 2. The signs of g_i are known, and there exist unknown positive constants g_{i0} and g_{i1} such that $0 < g_{i0} \leq |g_i| \leq g_{i1} < \infty$. Without loss of generality, it is assumed that $0 < g_{i0} \leq g_i$, $i = 1, ..., n$.

Assumption 3. The disturbances terms $d_i(x(t), t), i = 1, \ldots, n$ satisfy

$$|d_i(x(t), t)| \leq b_i \rho_i(\bar{x}_i(t)) \tag{10}$$

where $\rho_i(\bar{x}_i(t))$ are known positive smooth functions and b_i are unknown positive constants.

Assumption 4. There exist unknown positive constants h_0 and h_1, such that

$$0 < h_0 \leq \dot{\lambda}(u) \leq h_1 \tag{11}$$

Remark 1. Assumption 1 is a basic requirement of dynamic surface control method. For Assumptions 2, it is reasonable to assume the bound of the disturbances terms $d_i(x(t), t), i = 1, \ldots, n$. Assumption 3 is a basic condition for control system (9) to avoid the controller singularity. It should be noted that the values of g_{i0} and g_{i1} are not needed to be known. Assumption 4 implies that $L(t)$ and $\varphi(u(t))$ are bounded, and further means the term $S(t)$ is bounded, we denote it as D, where D is an unknown positive constant.

Lemma 1. Let $V(.), \zeta(.)$ be the smooth functions defined on $[0, t_f]$ with $V(t) \geq 0, \forall t \in [0, t_f]$, and let $N(.)$ be an ever smooth Nussbaum-type function [19]. If the following inequalities holds:

$$V(t) \leq c_0 + e^{-c_1 t} \int_0^t [G(.)N(\zeta) + 1]\dot{\zeta} e^{c_1 \tau} d\tau \tag{12}$$

where c_0 represents some suitable constant, c_1 is a positive constant, and G(.) is a time-varying parameter which takes values in the unknown closed intervals $I = [l^-, l^+]$, with $0 \notin I$, and then $V(t), \zeta(t)$, and $\int_0^t G(.)N(\zeta)\dot{\zeta} d\tau$ must be bounded on $[0, t_f]$.

3.1 Adaptive DSC Design

Due to the strong nonsmooth and multi-values properties of the hysteresis, the conventional control approaches may not be effective for the systems preceded by hysteresis. Besides, the system input is not obtained since the output of the hysteresis is unknown, which brings a new challenge for the controller design. In this section, one adaptive dynamic surface controller is investigated for a class of nonlinear systems to explore the way handling the unknown hysteresis input.

Firstly, the following coordinate transformation are used: $z_1 = x_1 - y_d$ and $z_i = x_i - s_{i-1}, i = 2, ..., n$, where s_{i-1} are the output of a first order filters with the input α_{i-1} as

$$\mu_i \dot{s}_i + s_i = \alpha_i, \quad s_i(0) = \alpha_i(0), \quad i = 1, ..., n-1. \tag{13}$$

where μ_i are the filter parameters, α_i are the intermediate control for the ith subsystems and their definitions will be given thereinafter.

For the dynamic surface control design, the boundary filter errors e_i are defined as

$$e_i = s_i - \alpha_i, \quad i = 1, ..., n-1. \tag{14}$$

Step i $(1 \leq i \leq n-1)$. For convenience, we denote $\frac{e_0}{\mu_0} = -\dot{y}_d$, $g_0 = 0$. Utilizing $z_i = x_i - s_{i-1}$ and the definitions for s_i and e_i in (13) and (14), it has

$$s_i = e_i + \alpha_i, \dot{s}_i = -\frac{e_i}{\tau_i}, i = 1, ..., n-2. \tag{15}$$

$$\begin{aligned} \dot{z}_i &= \theta_i f_i(\bar{x}_i(t)) + g_i x_{i+1}(t) + d_i(x(t), t) + \frac{e_{i-1}}{\tau_{i-1}} \\ &= \theta_i f_i(\bar{x}_i(t)) + g_i[z_{i+1} + \alpha_i + e_i] + d_i(x(t), t) + \frac{e_{i-1}}{\tau_{i-1}} \end{aligned} \tag{16}$$

Choose the following Lyapunov functions as

$$V_i = V_{i-1} + \frac{1}{2}\left(\frac{1}{g_i} z_i^2 + \frac{1}{\gamma_{\theta_i}} \tilde{\theta}_{g_i}^2 + \frac{1}{\gamma_{b_i}} \tilde{b}_{g_i}^2 + \frac{1}{\gamma_{\bar{g}_i}} \tilde{\bar{g}}_{g_i}^2\right) \tag{17}$$

where $\tilde{\theta}_{g_i} = \theta_{g_i} - \hat{\theta}_{g_i}, \tilde{b}_{g_i} = b_{g_i} - \hat{b}_{g_i}$ and $\tilde{\bar{g}}_{g_i} = \bar{g}_{g_i} - \hat{\bar{g}}_{g_i}$ with $\hat{\theta}_{g_i}, \hat{b}_{g_i}$ and $\hat{\bar{g}}_{g_i}$ as the estimation of $\theta_{g_i} = \theta_i/g_i, b_{g_i} = b_i/g_i$ and $\bar{g}_{g_i} = 1/g_i$, respectively. $\gamma_{\theta_i}, \gamma_{b_i}$ and $\gamma_{\bar{g}_i}$ are positive design parameters. Then we have

$$\begin{aligned}
\dot{V}_i &= \dot{V}_{i-1} + \frac{z_i}{g_i}(\theta_i f_i(\bar{x}_i(t)) + g_i[z_{i+1} + \alpha_i + e_i] + d_i(x(t),t) + \frac{e_{i-1}}{\tau_{i-1}}) \\
&\quad + \frac{1}{\gamma_{\theta_i}}\tilde{\theta}_{g_i}\dot{\tilde{\theta}}_{g_i} + \frac{1}{\gamma_{b_i}}\tilde{b}_{g_i}\dot{\tilde{b}}_{g_i} + \frac{1}{\gamma_{\bar{g}_i}}\tilde{\bar{g}}_{g_i}\dot{\tilde{\bar{g}}}_{g_i} \\
&\le \dot{V}_{i-1} + z_i(z_{i-1} + \theta_{g_i} f_i(\bar{x}_i(t)) + \alpha_i + b_{g_i}\rho_i(\bar{x}_i(t))\tanh(\frac{z_i\rho_i(\bar{x}_i(t))}{\omega}) + \bar{g}_{g_i}\frac{e_{i-1}}{\tau_{i-1}}) \\
&\quad + z_i z_{i+1} - z_i z_{i-1} + z_i e_i + 0.2785\omega b_{g_i} + \frac{1}{\gamma_{\theta_i}}\tilde{\theta}_{g_i}\dot{\tilde{\theta}}_{g_i} + \frac{1}{\gamma_{b_i}}\tilde{b}_{g_i}\dot{\tilde{b}}_{g_i} + \frac{1}{\gamma_{\bar{g}_i}}\tilde{\bar{g}}_{g_i}\dot{\tilde{\bar{g}}}_{g_i} \\
&= \dot{V}_{i-1} + z_i(z_{i-1} + \hat{\theta}_{g_i} f_i(\bar{x}_i(t)) + \alpha_i + \hat{b}_{g_i}\rho_i(\bar{x}_i(t))\tanh(\frac{z_i\rho_i(\bar{x}_i(t))}{\omega}) + \hat{\bar{g}}_{g_i}\frac{e_{i-1}}{\tau_{i-1}}) \\
&\quad + z_i z_{i+1} - z_i z_{i-1} + z_i e_i + 0.2785\omega b_{g_i} + \tilde{\theta}_{g_i}(z_i f_i(\bar{x}_i(t)) - \frac{1}{\gamma_{\theta_i}}\dot{\hat{\theta}}_{g_i}) \\
&\quad + \tilde{b}_{g_i}(z_i\rho_i(\bar{x}_i(t))\tanh(\frac{z_i\rho_i(\bar{x}_i(t))}{\omega}) - \frac{1}{\gamma_{b_i}}\dot{\hat{b}}_{g_i}) + \tilde{\bar{g}}_{g_i}(z_i\frac{e_{i-1}}{\tau_{i-1}} - \frac{1}{\gamma_{\bar{g}_i}}\dot{\hat{\bar{g}}}_{g_i}) \qquad (18)
\end{aligned}$$

By choosing the adaptive virtual control α_i and adaptive laws for $\hat{\theta}_{g_i}, \hat{b}_{g_i}$ and $\hat{\bar{g}}_{g_i}$ for the ith subsystem as

$$\begin{aligned}
\alpha_i &= -k_i z_i - z_{i-1} - \hat{\theta}_{g_i} f_i(\bar{x}_i(t)) \\
&\quad - \hat{b}_{g_i}\rho_i(\bar{x}_i(t))\tanh(\frac{z_i\rho_i(\bar{x}_i(t))}{\omega}) - \hat{\bar{g}}_{g_i}\frac{e_{i-1}}{\tau_{i-1}}
\end{aligned} \qquad (19)$$

$$\dot{\hat{\theta}}_{g_i} = \gamma_{\theta_i}(z_i f_i(\bar{x}_i(t)) - \varpi_i\hat{\theta}_{g_i}) \qquad (20)$$

$$\dot{\hat{b}}_{g_i} = \gamma_{b_i}(z_i\rho_i(\bar{x}_i(t))\tanh(\frac{z_i\rho_i(\bar{x}_i(t))}{\omega}) - \mu_i\hat{b}_{g_i}) \qquad (21)$$

$$\dot{\hat{\bar{g}}}_{g_i} = \gamma_{\bar{g}_i}(z_i\frac{e_{i-1}}{\tau_{i-1}} - \nu_i\hat{\bar{g}}_{g_i}) \qquad (22)$$

where $k_i, \varpi_i, \mu_i, \nu_i$ are positive design parameters, and using the following inequalities

$$\varpi_i\tilde{\theta}_{g_i}\hat{\theta}_{g_i} \le \frac{\varpi_i}{2}(-\tilde{\theta}_{g_i}^2 + \theta_{g_i}^2) \qquad (23)$$

$$\mu_i\tilde{b}_{g_i}\hat{b}_{g_i} \le \frac{\mu_i}{2}(-\tilde{b}_{g_i}^2 + b_{g_i}^2) \qquad (24)$$

$$\nu_i\tilde{\bar{g}}_{g_i}\hat{\bar{g}}_{g_i} \le \frac{\nu_i}{2}(-\tilde{\bar{g}}_{g_i}^2 + \bar{g}_{g_i}^2) \qquad (25)$$

it can be obtained

$$\dot{V}_i \leq -\sum_{j=1}^{i} k_j z_j^2 - \sum_{j=1}^{i}(\frac{\varpi_1}{2}\tilde{\theta}_{g_1}^2 + \frac{\mu_1}{2}\tilde{b}_{g_1}^2 + \frac{\nu_i}{2}\tilde{\bar{g}}_{g_1}^2)$$
$$+z_i z_{i+1} + \sum_{j=1}^{i}(z_j e_j + 0.2785\omega b_{g_j} + \frac{\varpi_j}{2}\theta_{g_j}^2 + \frac{\mu_i}{2}b_{g_j}^2 + \frac{\nu_i}{2}\bar{g}_{g_j}^2) \quad (26)$$

Step n. In the last step, the control law $u(t)$ will be designed to ensure the performance of the closed-loop system. Similarly, Considering $z_n = x_n - s_{n-1}$ and $\dot{s}_{n-1} = -e_{n-1}/\mu_{n-1}$, it has

$$\dot{z}_n = \theta_n f_n(x(t)) + g_n[L(t)u(t) + S(t)] + d_n(x(t), t) + \frac{e_{n-1}}{\tau_{n-1}} \quad (27)$$

and the Lyapunov-Krasovskii function for the system can be chosen as

$$V_n = V_{n-1} + \frac{1}{2}\left(z_n^2 + \frac{1}{\gamma_{\theta_n}}\tilde{\theta}_{g_n}^2 + \frac{1}{\gamma_{b_n}}\tilde{b}_{g_n}^2 + \frac{1}{\gamma_{\bar{g}_n}}\tilde{\bar{g}}_{g_n}^2\right) \quad (28)$$

where $\tilde{\theta}_{g_n} = \theta_{g_n} - \hat{\theta}_{g_n}, \tilde{b}_{g_n} = b_{g_n} - \hat{b}_{g_n}$ and $\tilde{\bar{g}}_{g_n} = \bar{g}_{g_n} - \hat{\bar{g}}_{g_n}$ with $\hat{\theta}_{g_n}, \hat{b}_{g_n}$ and $\hat{\bar{g}}_{g_n}$ as the estimation of $\theta_{g_n} = \theta_n, b_{g_n} = b_n, \bar{g}_{g_n} = g_n D$, respectively. $\gamma_{\theta_n}, \gamma_{b_n}$ and $\gamma_{\bar{g}_n}$ are positive design parameters.

Based on the expression for z_n in (27), we have

$$\dot{V}_n = \dot{V}_{n-1} + z_n(\theta_n f_n(x(t)) + g_n[L(t)u(t) + S(t)] + d_n(x(t), t) + \frac{e_{n-1}}{\tau_{n-1}})$$
$$+\frac{1}{\gamma_{\theta_i}}\tilde{\theta}_{g_n}\dot{\tilde{\theta}}_{g_n} + \frac{1}{\gamma_{b_n}}\tilde{b}_{g_n}\dot{\tilde{b}}_{g_n} + \frac{1}{\gamma_{\bar{g}_n}}\tilde{\bar{g}}_{g_n}\dot{\tilde{\bar{g}}}_{g_n}$$
$$= \dot{V}_{n-1} + z_n\theta_n f_n(x(t)) + z_n g_n L(t)u(t) + z_n g_n S(t) + z_n d_n(x(t), t)$$
$$+z_n\frac{e_{n-1}}{\tau_{n-1}} + \frac{1}{\gamma_{\theta_n}}\tilde{\theta}_{g_n}\dot{\tilde{\theta}}_{g_n} + \frac{1}{\gamma_{b_n}}\tilde{b}_{g_n}\dot{\tilde{b}}_{g_n} + \frac{1}{\gamma_{\bar{g}_n}}\tilde{\bar{g}}_{g_n}\dot{\tilde{\bar{g}}}_{g_n} \quad (29)$$

By using the following inequalities in [20]

$$z_n g_n S(t) \leq \bar{g}_{g_n}|z_n| \leq \bar{g}_{g_n} z_n \tanh(\frac{z_n}{\omega}) + 0.2785\omega\bar{g}_{g_n}$$

$$z_n d_n(x(t), t) \leq b_n|z_n|\rho_n(x(t))$$
$$\leq b_n z_n\rho_n(x(t))\tanh(\frac{z_n\rho_n(x(t))}{\omega}) + 0.2785\omega b_n \quad (30)$$

we have

$$\dot{V}_n \leq \dot{V}_{n-1} + z_n\theta_n f_n(x(t)) + z_n g_n L(t)u(t) + \bar{g}_{g_n} z_n\tanh(\frac{z_n}{\omega})$$
$$+0.2785\omega\bar{g}_{g_n} + 0.2785\omega b_n + b_n z_n\rho_n(x(t))\tanh(\frac{z_n\rho_n(x(t))}{\omega})$$
$$+z_n\frac{e_{n-1}}{\tau_{n-1}} + \frac{1}{\gamma_{\theta_n}}\tilde{\theta}_{g_n}\dot{\tilde{\theta}}_{g_n} + \frac{1}{\gamma_{b_n}}\tilde{b}_{g_n}\dot{\tilde{b}}_{g_n} + \frac{1}{\gamma_{\bar{g}_n}}\tilde{\bar{g}}_{g_n}\dot{\tilde{\bar{g}}}_{g_n}$$

$$\leq \dot{V}_{n-1} + z_n z_{n-1} + z_n \hat{\theta}_n f_n(x(t)) + z_n g_n L(t) u(t) + \hat{\bar{g}} z_n \tanh(\frac{z_n}{\omega})$$
$$+0.2785\omega \bar{g}_{g_n} + 0.2785\omega b_n + \hat{b}_n z_n \rho_n(x(t)) \tanh(\frac{z_n \rho_n(x(t))}{\omega}) + z_n \frac{e_{n-1}}{\tau_{n-1}}$$
$$+\tilde{\theta}_{g_n}(z_n f_n(x(t)) - \frac{1}{\gamma_{\theta_n}}\dot{\hat{\theta}}_{g_n}) + \tilde{b}_{g_n}(z_n \rho_n(x(t)) \tanh(\frac{z_n \rho_n(x(t))}{\omega}) - \frac{1}{\gamma_{b_n}}\dot{\hat{b}}_{g_n})$$
$$+\tilde{\bar{g}}_{g_n}(z_n \tanh(\frac{z_n}{\omega}) - \frac{1}{\gamma_{\bar{g}_n}}\dot{\hat{\bar{g}}}_{g_n}) - z_n z_{n-1} \tag{31}$$

In the last step, the adaptive virtual control u and adaptive laws for $\zeta, \hat{\theta}_{g_n}, \hat{b}_{g_n}$ and $\hat{\bar{g}}_{g_n}$ for the nth subsystem can be chosen as

$$u = N(\zeta)[k_n z_n + z_{n-1} + \hat{\theta}_{g_n} f_n(x_(t)) + \hat{\bar{g}}_{g_n} \tanh(\frac{z_n}{\omega})$$
$$+\hat{b}_{g_n} \rho_n(x(t)) \tanh(\frac{z_n \rho_n(x(t))}{\omega}) + \frac{e_{n-1}}{\tau_{n-1}}] \tag{32}$$

$$\dot{\zeta} = k_n z_n^2 + z_n z_{n-1} + z_n \hat{\theta}_{g_n} f_n(x(t)) + \hat{\bar{g}}_{g_n} z_n \tanh(\frac{z_n}{\omega})$$
$$+z_n \hat{b}_{g_n} \rho_n(x(t)) \tanh(\frac{z_n \rho_n(x(t))}{\omega}) + z_n \frac{e_{n-1}}{\tau_{n-1}} \tag{33}$$

$$\dot{\hat{\theta}}_{g_n} = \gamma_{\theta_n}(z_n f_n(x(t)) - \varpi_n \hat{\theta}_{g_n}) \tag{34}$$

$$\dot{\hat{b}}_{g_n} = \gamma_{b_n}(z_n \rho_n(x(t)) \tanh(\frac{z_n \rho_n(x(t))}{\omega}) - \mu_n \hat{b}_{g_n}) \tag{35}$$

$$\dot{\hat{\bar{g}}}_{g_n} = \gamma_{\bar{g}_n}(z_n \tanh(\frac{z_n}{\omega}) - \nu_i \hat{\bar{g}}_{g_i}) \tag{36}$$

where $k_i, \varpi_i, \mu_i, \nu_i$ are positive design parameters.

By using the following inequalities

$$\varpi_n \tilde{\theta}_{g_n} \hat{\theta}_{g_n} \leq \frac{\varpi_n}{2}(-\tilde{\theta}_{g_n}^2 + \theta_{g_n}^2) \tag{37}$$

$$\mu_n \tilde{b}_{g_n} \hat{b}_{g_n} \leq \frac{\mu_n}{2}(-\tilde{b}_{g_n}^2 + b_{g_n}^2) \tag{38}$$

$$\nu_n \tilde{\bar{g}}_{g_n} \hat{\bar{g}}_{g_n} \leq \frac{\nu_n}{2}(-\tilde{\bar{g}}_{g_n}^2 + \bar{g}_{g_n}^2) \tag{39}$$

it can be obtained

$$\dot{V}_n \leq -\sum_{j=1}^{n} k_j z_j^2 - \sum_{j=1}^{n}(\frac{\varpi_j}{2}\tilde{\theta}_{g_j}^2 + \frac{\mu_j}{2}\tilde{b}_{g_j}^2 + \frac{\nu_j}{2}\tilde{\bar{g}}_{g_j}^2)$$

$$+[g_n L(t)N(\zeta)+1]\dot{\zeta}+0.2785\omega\bar{g}_{g_n}+\sum_{j=1}^{n-1}(z_j e_j)$$
$$+\sum_{j=1}^{n}(0.2785\omega b_{g_j}+\frac{\varpi_j}{2}\theta_{g_j}^2+\frac{\mu_i}{2}b_{g_j}^2+\frac{\nu_i}{2}\bar{g}_{g_j}^2) \quad (40)$$

3.2 Stability Analysis

The semiglobal boundedness of all of the signals in the closed-loop system will be given.

Based on (15) and (19), it can be obtained that

$$\begin{aligned}\dot{e}_i &= \dot{s}_i-\dot{\alpha}_i\\ &= -\frac{e_i}{\tau_i}+(\frac{\partial\alpha_i}{\partial z_i}\dot{z}_i+\frac{\partial\alpha_i}{\partial\hat{\theta}_{g_i}}\dot{\hat{\theta}}_{g_i}+\frac{\partial\alpha_i}{\partial\hat{b}_{g_i}}\dot{\hat{b}}_{g_i}+\frac{\partial\alpha_i}{\partial\hat{\bar{g}}_{g_i}}\dot{\hat{\bar{g}}}_{g_i})\\ &= -\frac{e_i}{\tau_i}+B_i(z_1,...,z_i,\hat{\theta}_{g_1},...,\hat{\theta}_{g_i},\hat{b}_{g_1},...,\hat{b}_{g_i},\hat{\bar{g}}_{g_1},...,\hat{g}_{g_i},y_d,\dot{y}_d,\ddot{y}_d)\end{aligned} \quad (41)$$

where $B_i(z_1,...,z_i,\hat{\theta}_{g_1},...,\hat{\theta}_{g_i},\hat{b}_{g_1},...,\hat{b}_{g_i},\hat{\bar{g}}_{g_1},...,\hat{g}_{g_i},y_d,\dot{y}_d,\ddot{y}_d)=\frac{\partial\alpha_i}{\partial z_i}\dot{z}_i+\frac{\partial\alpha_i}{\partial\hat{\theta}_{g_i}}\dot{\hat{\theta}}_{g_i}+\frac{\partial\alpha_i}{\partial\hat{b}_{g_i}}\dot{\hat{b}}_{g_i}+\frac{\partial\alpha_i}{\partial\hat{\bar{g}}_{g_i}}\dot{\hat{\bar{g}}}_{g_i}$, which are continuous functions, $i=1,...,n-1$.

Thus, it follows

$$e_i\dot{e}\le-\frac{e_i^2}{\tau_i}+\left|e_iB_i(z_1,...,z_i,\hat{\theta}_{g_1},...,\hat{\theta}_{g_i},\hat{b}_{g_1},...,\hat{b}_{g_i},\hat{\bar{g}}_{g_1},...,\hat{g}_{g_i},y_d,\dot{y}_d,\ddot{y}_d)\right| \quad (42)$$

Denote $\Omega_i:=\{[z_1,...,z_i,\hat{\theta}_{g_1},...,\hat{\theta}_{g_i},\hat{b}_{g_1},...,\hat{b}_{g_i},\hat{\bar{g}}_{g_1},...,\hat{g}_{g_i}]:V_n+\sum_{i=1}^{n-1}e_i^2\le 2P_0\}\subset R^{4i}$ as the compact set of the initial conditions with P_0 a positive constant. Combining Assumption 1, for any $B_0>0,P_0>0$, the set Ω_d and Ω_i are compact in R^4 and R^{4i}. Thus, $B_i(z_1,...,z_i,\hat{\theta}_{g_1},...,\hat{\theta}_{g_i},\hat{b}_{g_1},...,\hat{b}_{g_i},\hat{\bar{g}}_{g_1},...,\hat{g}_{g_i},y_d,\dot{y}_d,\ddot{y}_d)$ has a maximum value $M_i, i=1,...,n-1$ on $\Omega_d\times\Omega_i$.

Theorem 1. Under Assumptions 1-4, considering the closed-loop system (1) with unknown Duhem hysteresis (2), the designed controller and adaptive control laws are given in (32)-(36), then for any initial conditions Ω_i, there exist control feedback gains k_i and filter parameters μ_i, such that the closed-loop control system is semiglobally stable in the sense that all signals in the closed-loop remain ultimately bounded.

The proof is omitted due to the space limit.

4 Simulation Studies

In this section,a nonlinear system (43) with Duhem hysteresis is used to illustrate the effectiveness of the proposed scheme in Section III.

$$\begin{cases}\dot{x}=\theta f(x(t))+gw(t)+d(x(t),t)\\ y=x\end{cases} \quad (43)$$

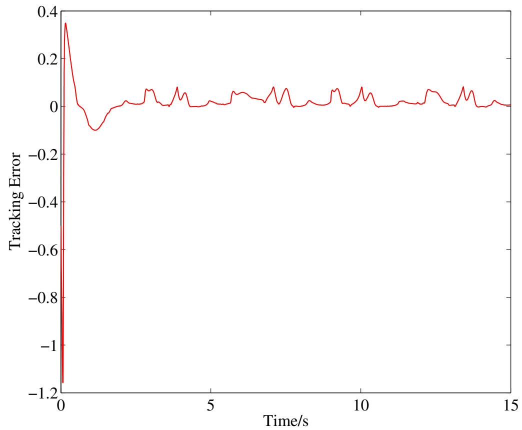

Fig. 1. Tracking error of the closed-loop system

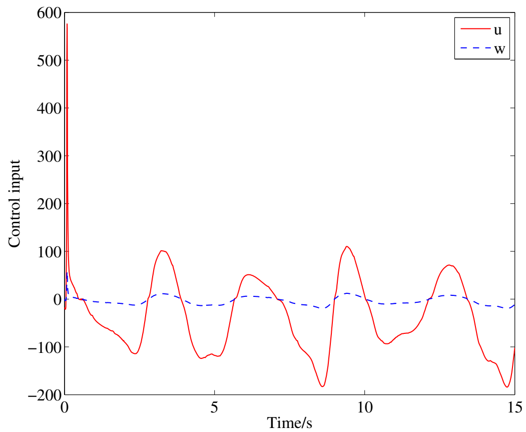

Fig. 2. Control signal u and the Duhem hysteresis output w

where $\theta = 1, g = 1, f(x(t)) = \frac{1-e^{-x}}{1+e^{-x}}, d(x(t), t) = e^{-0.5x}$. Correspondingly, $b = 1, \rho(x) = e^{-0.5x}$. For the Duhem model, $\lambda(u) = \tanh(u) + 0.1u$ and $\psi(u) = \dot{\lambda}(u)(1 - 0.58e^{-|u|})$. The objective is make the output y of system (43) to track the desired trajectory $x_d(t) = 5\sin(2t) + \cos(3.2t)$.

In this simulation, the Nussbaum function is chosen as $N(\zeta) = e^{\zeta^2}\cos(\frac{\pi\zeta}{2}), \omega = 0.01$.The initial parameters for update laws are chosen as $\hat{\theta}_{g_1}(0) = 0, \hat{b}_{g_1}(0) = 0, \hat{\bar{g}}_{g_1}(0) = 0, \zeta(0) = 0$, and the initial condition of system is chosen as $x(0) = 0.5$. The control parameters are chosen as $k = 15, \gamma_{\theta_1} = 5, \gamma_{b_1} = 8, \gamma_{\bar{g}_1} = 10$. The filter parameter and σ-modification parameters is defined as $\tau_1 = 0.01$ and $\varpi_1 = \mu_1 = \nu_1 = 0.1$ respectively.

The simulation results are shown in Figs. 1-2. In Fig. 1, the tracking error is shown and Fig. 2 shows the control input u. From the results, the proposed control scheme can overcome the effects of the hysteresis and ensure the boundedness of the closed-loop system.

5 Conclusion

In this paper, an adaptive dynamic surface controller for a class of uncertain perturbed strict-feedback nonlinear systems with unknown Duhem hysteresis input is developed. By utilizing DSC technique, “the explosion complexity” in the classical backstepping design method is avoided. To avoid the difficulties of the last recursive step caused by the unknown Duhem hysteresis, the unknown Duhem model is decomposed as nonlinear smooth component and nonlinear bounded component. By using mean value theorem, the nonlinear smooth component can be transformed as an unknown time-varying coefficient form, which makes it possible to solve the control design difficulty. Semiglobal uniform ultimate boundedness of all the signals in the close-loop system is guaranteed under the proposed control approach. Finally, simulation studies are given to demonstrate the effectiveness of the proposed design scheme.

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