

# Approximability of Stable Matching Problems

Kazuo Iwama\*

School of Informatics, Kyoto University, Kyoto 606-8501, Japan  
iwama@kuis.kyoto-u.ac.jp

The stable matching problem, also called the stable marriage problem, was first introduced by Gale and Shapley in 1962: In this problem, we are given two parties of men and women, and for each person in one party his/her preference list that orders the members of the other party. The question is to find a stable matching, i.e., a matching containing no pair (blocking pair) of man and woman who prefer each other to their current partners. The above paper gave a linear time algorithm, now called the Gale-Shapley algorithm, for this problem.

The problem has a wide range of real-world applications including assigning residents to hospitals, known as the hospital/residents problem, used in NRMP in US and in many other similar programs. For such large scale applications, however, the original setting that each preference list must be strict and complete is clearly unrealistic. Hence, the study of two natural relaxations, allowing ties and incomplete lists, has been started shortly after the Gale-Shapley paper, and before long it turned out that there are still linear time algorithms, namely the problem does not become inherently harder, if we allow either one of the two relaxations.

However, for the case that both are allowed at the same time, the complexity of the optimization problem (obtaining a matching of maximum size) was open for long time. In 1999, Iwama et al. proved that the problem is NP-hard, which immediately initiated the study of approximation algorithms for this relaxed version of the problem. In 2007 Iwama et al. were able to first obtain an approximation ratio strictly less than 2.0 (=1.875), which was quickly followed by Király (1.67) in 2008 and McDermid (1.5) in 2009.

This talk gives basic ideas of those approximation algorithms as well as some other recent extensions such as the case that ties exist only in one party's lists and introduction of minimum quota in the hospital/residents problem.

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\* Supported in part by KAKENHI, Ministry of Japan, 16092101, 1920000 and 2224001.