Physical-Layer Network Coding in Wireless Multi Access Relay Channel

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Abstract. Network Coding recognized as an efficient solution on improving network throughput is a hot spot in the area of wireless network. However, interference affects quality of communication seriously, which is a harmful factor in wireless fading environment. One protocol named physical-layer network coding not only increase network throughput but also takes advantage of interference between signals. How to separate the mixed signal resulting from the broadcast nature of wireless channel is an area with little discussion. This paper firstly proposes a PNC decoding solution for mixed signal separation in Multi Access Relay Channel(MARC) model. Then we use our estimation result about the channel coefficients to give a discussion on the capacity theorem of MARC model using PNC. In conclusion, the simulation result tells that the proposed method ARD PNC performs well in PNC with higher bandwidth utility and lower error rate. At the same time, the capacity using PNC is also improved comparing to traditional direct transmission.

Keywords: Physical-layer Network Coding, Wireless Fading Channel, Capacity Theorem, Multi Access Relay Channel.

1 Introduction

Wireless medium is significantly different from wired network with wireless channel fading and multipath effect which set a big trouble for wireless communication. Meanwhile, the broadcast nature of wireless links incurring interference into wireless networks. Accordingly, wireless networks suffer from high bit error rate and low throughput. Recent research has proved that wireless medium has set fertile ground for network coding which is effective in improving the multicast throughput. One should also be shocked by the evolutional development of network coding in communication field during these years. In standard network coding (Fig.1 Alice and Bob scenario) [1,2,8,9], two source nodes each use one time slot to transmit one packet to the relay node in a conventional time-division multiple access(TDMA). Then relay node manipulates the exclusive-or of these two packets in digital form and broadcast the result during the third time slot. It is easy to discover that network coding needs some prior information in its decoding.

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Physical-layer network coding(PNC)[3], one novel protocol of network coding, employs broadcast nature of wireless communication and the idea of reaching down to physical-layer[3,8]. PNC is much more effective than network coding for it does not need any prior information and can release relay nodes from coding and save the energy of the node. However, due to the prominent scale path loss and multi-path fad ing in general wireless environment, there are plenty of problems to overcome.

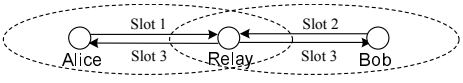

Fig. 1. Alice-Bob Topology

The purpose of this paper is twofold. First, we desire to describe the PNC scheme in perspective angle. Then we explore a joint diagonalization separation method named APNC to decode the mixed signal. Second, we use the channel coefficients estimated by APNC algorithm to deduce the throughput of MARC model using PNC, and compare it with conventional direct transmission scheme.

This paper is organized as follows. The MARC system model is described in Section 2. For better illustration and comparison, we mention two types of generic schemes for MARC before PNC scheme. In Section 3, there is an algorithm about the PNC decoding with joint diagonalization separation. The capacity of PNC is developed in 6, and the numerical results are shown in Section 5 for performance comparison. Finally, concluding remarks are given in Section 4.

2 System Model

MARC system, shown in Fig.1, is a model for network topology where multiple sources communicate with one single destination with the help of a relay node. The MARC model in our paper has two source nodes (S_1, S_2), one relay node (R) and one destination node (D). The meaning of existence of R is to assist source nodes in transmitting information to destination node. It has a symmetric positioning of the sources with respect to relay and destination. The geometric configuration of source nodes, relay node and destination node is a collinear setup and the destination with a unit distance away from the source, as in Fig 2, and distance between source node and relay node is set to d(0<d<1). The most important step before designing ARD PNC is to decide the modulation scheme, and we chose BPSK for its high utilization and valid antinoise. Attenuation and phase shift can be abstracted from the effect of the wireless fading channel[4,5]. Consequently, we set signals in wireless medium as a discrete function over complex domain. After colliding in wireless channel, original signals $s[m]$ will incur attenuation, multi-path, time shift and phase shift. Therefore, signals arriving at the receivers are:

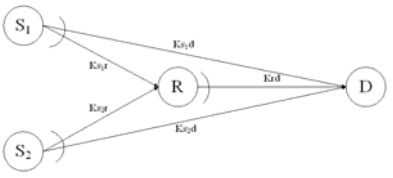

Fig. 2. MARC Model

$$y[m] = k_{ij} s[m] + n[m] \tag{1}$$

where $k_{i,j} \in C(i, j \in \{s_1, s_2, r, d\})$ stands for the gain of signals received by the j^{th} node and send by the i^{th} node, which is composed of two parts - large fading part and small fading part. The large part (path lost factor) is parameterized to $1/d^2$, and the small part (fading effect factor) is proportional by Rayleigh distribution. $n[m]$ depicts the additive white Gaussian noise variable at the receiver with zero-mean and σ^2 variance. Here, we assume that $s_1[m]$ and $s_2[m]$ have equal power P, and the additive Gaussian noise with power P_ω.

The transmission over wireless fading channel will lead to collision and interference. R receives a signal y_r mixed with white additive Gaussian noise variable $n_r[m]$:

$y_r[m] = k_{s_1 r} s_1[m] + k_{s_2 r} s_2[m] + n_r[m]$ D receives a signal y_{d_1}, one sample of S_1 and S_2, mixed with white additive Gaussian noise variable $n_1[m]$:

$y_{d_1}[m] = k_{s_1 d} s_1[m] + k_{s_2 d} s_2[m] + n_1[m]$ Before focusing on PNC, we should pay attention to two generic schemes for MARC:

2.1 Traditional Scheme

The key idea of traditional scheme[8] is to avoid the effect of interference. Accordingly, this scheme, a 4 steps scheme, chooses to transmit only one packet from one single node in one time slot. In the first time slot, S_1 sends a packet for R. In the second time slot, R transmits the packet to D. Similarly, when S_2 wants to transmit a packet for D, it also needs two time slots.

2.2 Network Coding Scheme

Network coding scheme[1] takes advantage of the idea of cooperative communication which is helpful to increase wireless network throughput because coding permits the relay node to compress the transmitted information given what is known at various nodes(as in a traditional 3 nodes Alice and Bob scenario). In our model, it is a 3 steps scheme. In the first time slot, S_1 sends one packet s_1 for R. In the second time slot, S_2 sends one packets s_2 for R. Then R sends a computed signal $s_1 \oplus s_2$ ($\oplus$ delegates exclusive or manipulation) to D.

2.3 Physical-Layer Networking Scheme

1) In the 1st time slot:
Firstly, S_1 and S_2 send out signals simultaneously, both R and D receive mixed signals of S_1 and S_2 through wireless free space, for the broadcast nature character of communication in wireless channel.
2) In the 2nd time slot:
R retransmits and forwards the mixed signals received in the former time slot to D. Therefore, D gets another sample of the mixed signals of S_1 and S_2. R retransmits signal y_r to D, and then D receives a signal y_{d_2} as:

$$\begin{aligned} y_{d_2}[m] &= \frac{k_{rd}}{\beta} y_r[m] + n_2[m] \\ &= \frac{k_{rd}k_{s_1r}}{\beta} s_1[m] + \frac{k_{rd}k_{s_2r}}{\beta} s_2[m] + \frac{k_{rd}}{\beta} n_1[m] + n_2[m] \end{aligned}$$

where $n_2[m]$ is the additive white Gaussian noise variable of wireless fading channel and $\beta = \sqrt{\left|k_{s_1r}\right|^2 + \left|k_{s_2r}\right|^2 + \frac{1}{\rho}}$ ($\rho = P / P_{\omega}$) is the factor for the power constraint. This equation provides another sample about S_1 and S_2 at D.

We rearrange (3)-(4) into matrix form and define K as channel coefficients, which expresses the relationship between source signal matrix S at S_1 and S_2, and observed signal matrix Y at D. Accordingly, we can easily summarize an equation $Y = K \cdot S + N$,

$$\begin{bmatrix} y_{d_1}[m] \\ y_{d_2}[m] \end{bmatrix} = K \begin{bmatrix} s_1[m] \\ s_2[m] \end{bmatrix} + \begin{bmatrix} n_1[m] \\ \frac{k_{rd}}{\beta} n_1[m] + n_2[m] \end{bmatrix}$$

where the channel coefficients are as matrix form,

$$K = \begin{bmatrix} k_{s_1d} & k_{s_2d} \\ \frac{k_{rd}k_{s_1r}}{\beta} & \frac{k_{rd}k_{s_2r}}{\beta} \end{bmatrix}$$

3 Physical-Layer Network Decoding

Channel coefficient estimation is essential for our latter discussion about the channel throughput. However, the whitening process of channel state information estimated by joint diagonalization method is often limited by channel noise. We implement ARD[6] in the decoding operation which employs the observed signals directly without whitening process. The process of decoding algorithm is reference to [7] in detail. In this paper, we briefly introduce the main process of the decoding algorithm.

With "inverse matrix" V, the original signals can be estimated by $\hat{S} = V \cdot Y$ and channel state coefficients are $\hat{K} = V^+$, where V^+ represents the pseu-doinverse of V. To computing out V is the essential of APNC algorithm. To approximate V, we define a cost function C to evaluate the independence between signals. C is relevant to fourth-order cumulants of the observed signals, which form a series of target matrix G. In

short, APNC algorithm is to update V by minimizing the cost function C and jointing diagonalization of a set of target matrix G.

As each element in channel coefficients can tell the channel factor, we can obtain CSI by the eqution $K = V^{+}$

Table 1. Channel Coefficient Estimation

Decoding with ARD algorithm
if signal detected then
Initialize
$b = (1,1,\ldots,1)^T \in R^{b \times n}$

$$V_0 = \begin{bmatrix} 1 & \cdots & 1 \\ \vdots & \ddots & \vdots \\ 1 & \cdots & 1 \end{bmatrix}_{n \times n}$$

Optimize with iteration:
for $i = 1:n$

$$B = (V_1, V_2, \cdots, V_{i-1}, V_{i+1}, \cdots, V_n)$$

$$R = \sum_{k=1}^{n^2} G_k B B^H G_k^H$$

$$G = \begin{bmatrix} \begin{bmatrix} Q_1 & Q_2 \\ Q_3 & Q_4 \end{bmatrix} & \begin{bmatrix} Q_9 & Q_{10} \\ Q_{11} & Q_{12} \end{bmatrix} \\ \begin{bmatrix} Q_5 & Q_6 \\ Q_7 & Q_8 \end{bmatrix} & \begin{bmatrix} Q_{13} & Q_{14} \\ Q_{15} & Q_{16} \end{bmatrix} \end{bmatrix}$$

$$Q_y = \sum_{i=1}^{2}\sum_{j=1}^{2}\sum_{k=1}^{2}\sum_{l=1}^{2} Cum(y_i, y_j^*, y_k, y_l^*), 1 \le i, j, k, l \le 2$$
$$= [Q_1, Q_2, \ldots, Q_{16}]$$

Compute the unit eigenvector h associated with the minimum eigenvalue of **R**.

update $V_i = b_i \cdot h$

$V = (V_1, V_2, \cdots, V_{i-1}, b_i h, V_{i+1}, \cdots, V_n)^T$, equal to update cost function **C**

$$C(G,V) = \sum_{k=1}^{4} 2 \sum_{j=1, i \ne j}^{2} \left(V_i^H G_k V_j\right)^2$$
$$= 2V_i^T \left[\sum_{k=1}^{4} G_k \left(\sum_{j=1, i \ne j}^{2} V_j V_j^H\right) G_k\right] V_i$$
$$= 2(b_i)^2 \left(\frac{V_\perp}{b_i}\right)^H \left[\sum_{k=1}^{4} G_k \left(\sum_{j=1, i \ne j}^{2} V_j V_j^H\right) G_k\right] \left(\frac{V_\perp}{b_i}\right)$$

end for
end if

4 MARC Model Capacity

We apply the channel coefficient estimated in matrix V to help with approaching to the channel capacity in MARC model.

4.1 The Outer Bound

In PNC, during the transmitting of N bits, we assume that half of the time is spending on S_1 and S_2 transmit signal to both R and D. And the other half time is using for R transmits the mixed signal to D.

In the first half time, from (3), the achievable rate R can be determined by

$$R_{sd,\ time1_{st}} = \frac{1}{2}\log[1+\rho(\left|k_{s_1d}\right|^2+\left|k_{s_2d}\right|^2)] \tag{6}$$

In the second half time, from (3), the achievable rate R is

$$R_{sd,\ time2_{nd}} = \frac{1}{2}\log\left[1+\frac{\rho(\left|k_{rd}\right|^2\left|k_{s_1r}\right|^2+\left|k_{rd}\right|^2\left|k_{s_2r}\right|^2)}{\left|k_{rd}\right|^2+1}\right] \tag{7}$$

An outer bound of the PNC MARC is

$$R = R_{sd,\ time1_{st}} + R_{sd,\ time2_{nd}} \tag{8}$$

4.2 The Inner Bound

The capacity region is

$$\begin{aligned} C=\{&R: R\geq 0,\\ &R\leq \textstyle\int_0^{1/2}\log(1+S_1(f)T_1(f))df+\\ &\textstyle\int_0^{1/2}\log(1+S_2(f)T_2(f))df\} \end{aligned} \tag{9}$$

where

$$T_1(f)=\frac{\beta^2k_{s_1d}^2+k_{rd}^2k_{s_1r}^2+2\beta k_{rd}k_{s_1d}k_{s_2r}\cos 2\pi f}{k_{rd}^2+\beta^2}$$

$$T_2(f)=\frac{\beta^2k_{s_2d}^2+k_{rd}^2k_{s_2r}^2+2\beta k_{rd}k_{s_2d}k_{s_2r}\cos 2\pi f}{k_{rd}^2+\beta^2}$$

then the capacity region is easily be manipulated by

$$\begin{aligned} &C=\{R: R\geq 0,\\ &R\leq\frac{1}{2}\log(1+\frac{\beta^2k_{s_1d}^2+k_{rd}^2k_{s_1r}^2}{k_{rd}^2+\beta^2}+\frac{\beta^2k_{s_2d}^2+k_{rd}^2k_{s_2r}^2}{k_{rd}^2+\beta^2})\} \end{aligned} \tag{10}$$

However, in the direct transmission scheme the channel coefficient matrix is

$$K=\begin{bmatrix} k_{s_1d} & 0\\ 0 & k_{s_2d}\end{bmatrix},$$

$$R_{DT}=\frac{1}{2}\log\det\left[\mathrm{I}_2+K^HC_n^{-1}K\right] \tag{11}$$

where $C_n^{-1} = E\{nn^H\} = \sigma^2 \mathrm{I}_2$, and the mutual information is:

$$R_{DT} \geq \frac{1}{2}\log[1+\rho(|k_{s_1d}|^2 + |k_{s_2d}|^2) + 2\rho^2 |k_{s_1d}|^2 |k_{s_2d}|^2] \tag{12}$$

In our model, we can easily find that all channel coefficients are determined by the variable d. With plenty of experiment,

5 Numerical Illustration

In this section, some simulations and numerical results are provided to support the conclusion that PNC will increase the throughput considerably. All the following results are done with the Rayleigh factor 2. Fig.3 depicts the law of bit error rate change affected by the distance between source and relay. We can observe that the nearer the distance between relay and destination the better the quality of communication. In Fig.4, the result shows that PNC provide lower bit error rate than direct transmission. Fig 5 depicts the ergodic capacity about PNC comparing with direct transmission. While there are some areas that the throughput of PNC is lower than DT, the upper ergodic capacity bound of PNC is higher than that of DT. It indicates that there should be some rules in relay node selection which is a good point for further research. As the distance d between source and relay is a key factor in channel coefficient decision, we can also learn how ergodic capacity changed with different d in Fig. 6

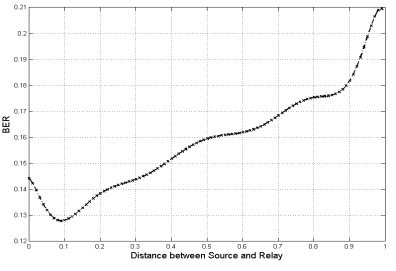

Fig. 3. Relationship between BER and Distance. Here, the distance of abscissa depicts the space from Sourse Node to Relay Node.

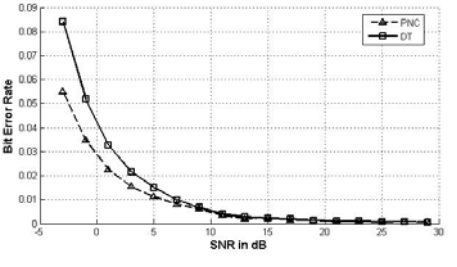

Fig. 4. Bit Eerror Rate vs SNR

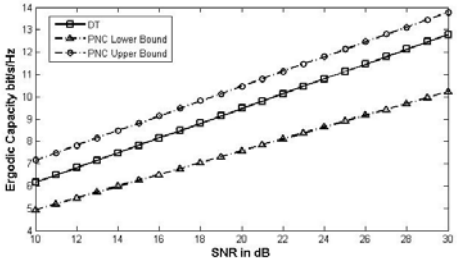

Fig. 5. Ergodic Cpacity vs. SNR. Solid line represents the conventional direct transimmion, dash-dash line reprensents physical-layer network coding.

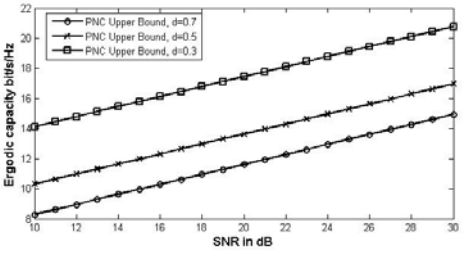

Fig. 6. Ergodic Capacity vs. SNR with different d.

6 Conclusion

Traditional wireless communication protocols strive to avoid data stream at the same time for the broadcast nature of wireless medium incurring interference with each other. However, PNC is considered to be suitable for more efficient, reliable and scalable wireless network taking advantage of the harmful interference. This paper briefly presents a decoding algorithm for PNC which will also helps with estimating the channel coefficients which is the foundation for latter discussion about the capacity of the MARC channel with PNC. The analytical and confirming simulation results show that the proposed protocol can achieve better performance than comparable schemes, which demonstrate that PNC can achieve higher system throughput than a direct transmission scheme, which also show the tendency on relay selection for PNC to achieve better capacity than direct transmission.

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