Using High Dimensions to Compare Drawings of Graphs[*]

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1 Introduction

We describe a simple visualization technique which allows a user to quickly assess
the overall similarity of drawings of similar graphs, and to easily find regions of
stability and of change. It can be used to simultaneously compare any number of
drawings, and does not require that the layouts be adjusted to minimize changes.

The underlying idea — grouping vertices whose relative positions undergo
little change — is shared by the animation technique of Friedrich and Houle [1],
but both our approach to identifying such groups (visually via planes in high
dimensions) and our application (comparing drawings) are quite different.

2 Algorithm

Let D_1 and D_2 be 2D drawings of some graph G, and let (x_{v_i}, y_{v_i}) be the co-
ordinates assigned to vertex v in drawing i. Create a 4D drawing D of G in
which each vertex is associated with the 4-tuple $(x_{v_1}, y_{v_1}, x_{v_2}, y_{v_2})$, then project
that 4D drawing into 3D for viewing. Groups of vertices whose relative positions
are maintained under translation, rotation, and/or scaling appear on the same
plane in both the 4D drawing D and the 3D projection, with different planes
indicating different combinations of transformations. See Figure 1.

The algorithm can be extended to include edges by evenly spacing a number
of points along each edge and creating 4D "edge points" in the same manner
as vertex points. Changes in the graph structure can be accommodated by as-
signing appropriate coordinates to the missing points. In addition, k drawings
can be compared simultaneously by building a $2k$-tuple for each point from the
coordinates in each drawing, then projecting to 3D for viewing.

3 Applications

In dynamic graph drawing and layout adjustment, the user must frequently
orient herself to a new drawing of the same or nearly the same graph. The "high-
dimensional comparison" technique of section 2 can be used to provide visual

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(a)   (b)

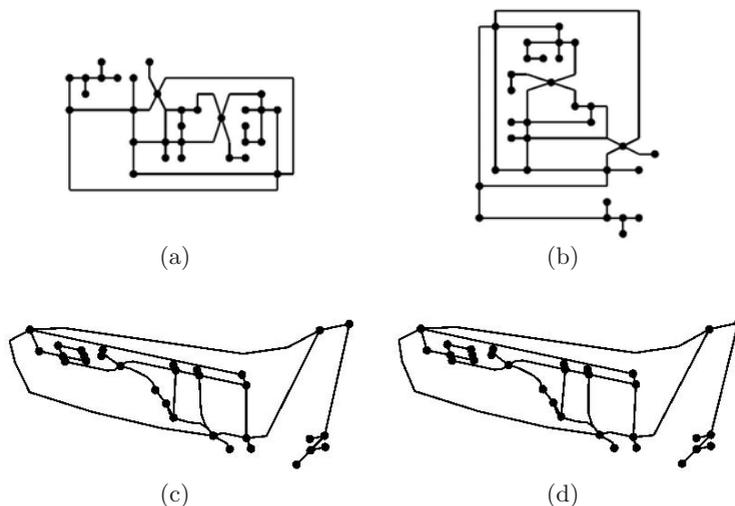(c)   (d)

Fig. 1. D_1 and D_2 are shown in (a) and (b). Finding similar regions in these drawings requires repeatedly looking back and forth between the drawings and a lot of mental gymnastics. Figures (c) and (d) are a stereo pair showing a 3D projection of D — defocus your eyes until (c) and (d) merge into a third picture. (The visualization is most effective when the user can also manipulate the viewing angle to enhance the 3D sense.) Flat areas indicating similar regions can be identified immediately. Brushing can be used in an interactive viewer to match flat regions to the original drawings.

cues to aid the reorientation process, and is particularly useful when different parts of the layout have undergone different transformations (see Figure 1).

Time-varying graphs are often drawn by creating a separate layout for each time step, and either animating the transitions between time steps or stacking the layouts to create a 2.5D drawing. The success of these techniques requires adjusting consecutive layouts to reduce changes between time steps, which precludes the use of drawing algorithms designed to display particular structural elements of the graph (such as clusters). High-dimensional comparison provides a new way to visualize the change and stability between layers and over a series of time steps, and does not require adjustment of consecutive layouts.

In addition, the fit of the high-dimensional points to a single plane provides a potential similarity measure for comparing drawings and has the novel feature of being suitable for comparing more than two drawings at once.

Finally, several other applications are based on automatically clustering high-dimensional points which lie on or near the same plane (e.g. using the techniques of Friedrich and Houle [1] or Tseng [2]). Clustering information can be used to color-code similar groups in the D_i drawings, to produce a meta-visualization of how cluster position and membership changes over a series of drawings, or to define similarity measures based on common features rather than individual nodes.

References

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2. Tseng, P.: Nearest q-flat to m points. J. Optim. Theory Appl. 105(1), 249–252 (2000)