A Tutorial on Analog Computation: Computing Functions over the Reals

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The best known programmable analog computing device is the differential analyser. The concept for the device dates back to Lord Kelvin and his brother James Thomson in 1876, and was constructed in 1932 at MIT under the supervision of Vannevar Bush. The MIT differential analyser used wheel-and-disk mechanical integrators and was able to solve sixth-order differential equations. During the 1930's, more powerful differential analysers were built. In 1941 Claude Shannon showed that given a sufficient numbers of integrators the machines could, in theory, precisely generate the solutions of all differentially algebraic equations. Shannon's mathematical model of the differential analyser is known as the GPAC.

Graça and Costa improved the GPAC model and showed that all of the interesting functions it can define are solutions of polynomial differential equations. From the point of view of computability two natural questions arise: Are all those functions computable? Are all computable functions definable by a GPAC? To answer those questions one has to agree upon a notion of computability of real functions. We will use Computable Analysis, which is a model of computation based on type-2 Turing machines. Roughly, a function f is considered computable if from a sequence that converges rapidly to x the machine can compute in discrete steps a sequence that converges rapidly to $f(x)$. Computable analysis is an effective model of computation over the real numbers and it cannot use real numbers with infinite precision.

In this tutorial, we consider that analog models of computation handle real numbers represented exactly rather than by strings of digits. We also consider that the state of the model evolves in a continuum. The GPAC, other continuous dynamical systems, and more general real recursive functions fit into that framework and will also be addressed.

A series of papers from Bournez, Campagnolo, Hainry, Graça and Ojakian establish equivalences between several analog models and Computable Analysis, showing that in that sense digital and analog computation are ultimately not too far apart. In the tutorial a unified view of those results will be provided, using a technique called "approximation". If two classes approximate each other sufficiently, we can derive an equality using the notion of limit which is implicit in Computable Analysis.

In short, the goal of this tutorial is to relate various computational models over the reals, using the notion of approximation as a unifying tool and language.

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