Chapter 2
Background and Fundamentals

This chapter introduces the terminology and background that are necessary for the description of all the protocols discussed in this book. Precisely, it gives some key definitions and describes a percolation model to study coverage and connectivity in two-dimensional and three-dimensional wireless sensor networks. In addition, it presents an energy consumption model and defines the default network model.

2.1 Introduction

In this book, we use some terminology and different models, such as Voronoi diagram model, energy model, sensing model, and continuum percolation model, to describe our proposed approaches and protocols for connected $k$-coverage, duty-cycling, and geographic forwarding in wireless sensor networks. Furthermore, our work is based on a specific network model. The goal of this chapter is to present the different terms and models we use in this book.

The remainder of this chapter is organized as follows: Section 2.2 presents the key definitions and fundamental concepts that are used in this book. Section 2.3 presents deterministic and stochastic sensing models. Section 2.4 discusses different types of network connectivity. Section 2.5 describes the energy model while Sect. 2.6 presents the percolation model. Section 2.7 presents the network model that we use in the design of our energy-efficient framework for joint $k$-coverage, duty-cycling, and geographic forwarding in wireless sensor networks. Section 2.8 summarizes the chapter.

2.2 Terminology

In this section, we give the key definitions and describe some fundamental concepts used throughout this book.

Definition 2.1: The sensing range (or detection range) of a sensor $s_i$ is a region where every event that takes place in this region can be detected by $s_i$. The sensing neighbour set of a sensor $s_i$, denoted by $SN(s_i)$, is the set of all the sensors that are located in its sensing range.

Definition 2.2: The communication range of a sensor $s_i$ is a region such that $s_i$ can communicate with any sensor located in this region. The communication
neighbour set of a sensor $s_i$, denoted by $CN(s_i)$, is a set of all the sensors that are located in its communication range.

Throughout this book, we use communication range and transmission range interchangeably. To illustrate the following definitions, we assume that the sensing ranges and communication ranges of the sensors in a two-dimensional (respectively, three-dimensional) wireless sensor network are represented by disks (respectively, spheres). Furthermore, the sensors are assumed to have the same sensing range and the same communication range. Figure 2.1 shows the sensing and communication ranges of the sensors of radii $r$ and $R$, respectively.

![Fig. 2.1 Schematic of overlapping concentric disks (respectively, spheres)](image)

**Definition 2.3:** Two sensors $s_i$ and $s_j$ in a two-dimensional (respectively, three-dimensional) wireless sensor network are said to be collaborating if the Euclidean distance between the centres of their sensing disks (respectively, spheres) satisfies $|\xi_i - \xi_j| \leq 2r$, where $r$ is the radius of the sensing disks (respectively, spheres) of
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Fig. 2.3 (a) Covered, (b) connected, and (c) coordinated components

the sensors. Intuitively, the two sensing disks (respectively, spheres) centred at \( \xi_i \) and \( \xi_j \) are either tangential or overlapping (see Fig. 2.2a). The collaborating set of a sensor \( s_j \), denoted by \( \text{Col}(s_i) \), includes all the sensors it can collaborate with, i.e., \( \text{Col}(s_i) = \{ s_j : |\xi_i - \xi_j| \leq 2r \} \).

**Definition 2.4:** Two sensors \( s_i \) and \( s_j \) in a two-dimensional (respectively, three-dimensional) wireless sensor network are said to be communicating if the Euclidean distance between the centres of their communication disks (respectively, spheres) satisfies \( |\xi_i - \xi_j| \leq R \), where \( R \) is the radius of the communication disks (respectively, spheres) of the sensors (see Fig. 2.2b). The communicating set of a sensor \( s_i \) is the set of sensors it can communicate with, i.e., \( \text{Com}(s_i) = \{ s_j : |\xi_i - \xi_j| \leq R \} \).
Definition 2.5: Two sensors $s_i$ and $s_j$ in a two-dimensional (respectively, three-dimensional) wireless sensor network are said to be coordinating if and only if they both collaborate and communicate (Fig. 2.2c). The coordinating set of a sensor $s_i$ is the set of sensors it can collaborate and communicate with at the same time.

Definition 2.6: A covered component (or covered region) in a two-dimensional (respectively, three-dimensional) wireless sensor network is a maximal set of sensing disks (respectively, spheres), i.e., not included in any other subset except when it is equal to the original entire set of sensing disks (respectively, spheres) whose corresponding sensors are collaborating directly or indirectly (Fig. 2.3a shows three covered components). A covered $k$-component, denoted by $CC_k$, is a covered component having $k$ sensing disks (respectively, spheres).

Definition 2.7: A connected component in a two-dimensional (respectively, three-dimensional) wireless sensor network is a maximal set of communication disks (respectively, spheres) whose corresponding sensors are communicating directly or indirectly (Fig. 2.3b shows three connected components). A connected $k$-component, denoted by $CC_k$, is a connected component with $k$ communication disks (respectively, spheres).

Definition 2.8: A coordinated component in a two-dimensional (respectively, three-dimensional) wireless sensor network is a maximal set of concentric sensing and communication disks (respectively, spheres) whose corresponding sensors are coordinating directly or indirectly (Fig. 2.3c shows three coordinated components).

Definition 2.9: A wireless sensor network is said to be connected if there is a communication path between any pair of sensors. In other words, any pair of sensors can communicate with each other either directly or indirectly.

Definition 2.10: A two-dimensional (respectively, three-dimensional) wireless sensor network is said to be homogeneous if all of its sensors have the same storage, processing, battery power, sensing, and communication capabilities. In particular, all deployed sensors have the same radius $r$ of their sensing disks (respectively, spheres) and the same radius $R$ of their communication disks (respectively, spheres). Otherwise, the network is said to be heterogeneous.

Definition 2.11: The width of a closed convex area is the maximum distance between parallel lines that bound it.

Definition 2.12: The breadth of closed convex volume is the maximum distance between tangential planes on opposing faces or edges of the volume.

Definition 2.13: The largest enclosed disk (respectively, sphere) of closed convex area (respectively, volume) $A$ is a disk (respectively, sphere) that lies inside $A$ and whose diameter is equal to the minimum distance between any pair of points on the boundary of $A$. 
The Voronoi diagram [36], also known as Dirichlet tessellation, represents one of the most fundamental data structures in computational geometry. It has interesting mathematical and algorithmic properties and potential applications.

Definition 2.14: Let \( S = \{s_1, \ldots, s_n\} \) be a finite set of \( n \) sites (or points) in the plane. The Voronoi diagram of \( S \), denoted by \( \text{Vor}(S) \), is a subdivision of the plane containing \( S \) into \( n \) cells \( \text{VC}(s_i), 1 \leq i \leq n \), such that each cell \( \text{VC}(s_i) \) includes only one site \( s_i \) with the property that any point \( p \) located in \( \text{VC}(s_i) \) is closer to \( s_i \) than any other site in \( S \). The cell \( \text{VC}(s_i) \) corresponding to site \( s_i \) is called the Voronoi cell of \( s_i \), which is a (possibly unbounded) open convex polygonal region. The edges of a Voronoi cell are called Voronoi edges and its endpoints are called Voronoi vertices. The Voronoi diagram of \( S \) is the union of the Voronoi cells of all sites in \( S \). The Delaunay triangulation, denoted by \( \text{DT}(S) \), is the dual of the Voronoi diagram \( \text{Vor}(S) \) [36]. A \( \text{DT}(S) \) graph has an edge between two sites \( s_i \) and \( s_j \) if and only if their Voronoi cells \( \text{VC}(s_i) \) and \( \text{VC}(s_j) \), respectively, share a common edge.

Notice that \( \text{DT}(S) \) is a planar graph whose Delaunay edges are orthogonal to their corresponding Voronoi edges. Figure 2.4 shows a Voronoi diagram while Fig. 2.5 shows a Voronoi diagram and its dual, i.e., Delaunay triangulation.
Let $CN(s_i)$ be the communication neighbour set of a sensor $s_i$, which are located in its communication disk whose radius is equal to $R_i$. From $CN(s_i)$, the sensor $s_i$ considers only a subset of sensors, denoted by $SCN(s_i,s_m)$, located between $s_i$ and the sink $s_m$ to act as data forwarders to the sink.

**Definition 2.15:** Let $s_j \in SCN(s_i,s_m)$. The Voronoi diagram computed by $s_i$, denoted by $Vor([s_i,s_m] \cup SCN(s_i,s_m))$, is said to be localized. A Voronoi cell $VC(s_m)$ of the sink $s_m$ is said to be adjacent to the sensor $s_j$ if $VC(s_m)$ and $VC(s_j)$ of $s_m$ and $s_j$, respectively, have at least one common Voronoi edge.

**Definition 2.16:** A localized Delaunay triangulation of a sensor $s_i$, denoted by $LDT(s_i)$, is the Delaunay triangulation computed by $s_i$ with respect to $SCN(s_i) \cup \{s_i,s_m\}$.

**Definition 2.17:** The candidate checkpoints of a sensor $s_i$, denoted by $CCP(s_i,s_m)$, are the sensors that are adjacent to the sink $s_m$ in the localized Delaunay triangulation of $s_i$, $LDT(s_i)$.

**Definition 2.18:** Let $s_j \in SCN(s_i,s_m)$. A sensor $s_j$ is said to be a candidate forwarder of $s_i$ if $VC(s_m)$ is adjacent to $s_j$, where $VC(s_m) \in Vor([s_i,s_m] \cup SCN(s_i,s_m))$. The set of candidate forwarders of $s_i$ is denoted by $CF(s_i,s_m)$.

Assume that a source $s_0$ wishes to disseminate its data to the sink $s_m$. Fig. 2.6 shows the localized Voronoi diagram of $s_0$, where $SCN(s_0,s_m) = \{s_i | 1 \leq i \leq 18\}$. Notice that $CF(s_0,s_m) = \{s_9,s_{10},s_{15},s_{17},s_{18}\}$, where the Voronoi cell of each of

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Fig. 2.6 Localized Voronoi diagram
those sensors shares one Voronoi edge with that of $s_m$. These Voronoi cells are shaded in green.

**Definition 2.19:** Let $s_{ref} \in SCN(s_i, s_m)$. A sensor $s_{ref}$ is said to be a reference sensor of $s_i$ if $s_{ref}$ has the highest remaining energy amongst all sensors in $SCN(s_i, s_m)$.

**Definition 2.20:** A data forwarding path is a path that is traversed by a data packet originated from a source sensor (or simply source) and destined to the sink. It includes all the sensors (including the source) that forwarded the data packet to the sink on behalf of the source.

**Definition 2.21:** A forwarding scheme is said to be long range if each sensor in any data forwarding path can use at most one of its one-hop neighbours to forward a data packet towards its ultimate destination. A forwarding scheme is said to be short range if each sensor in any data forwarding path can use multiple one-hop neighbours to forward a data towards its destination.

### 2.3 Deterministic and Stochastic Sensing Models

In the deterministic sensing model (also known as binary), a point (or event) $p$ in a field is sensed/covered (or detected) by a sensor $s_i$ based on the Euclidean distance $\delta(p, s_i)$ between $p$ and $s_i$. Throughout this chapter, we use “coverage of a point” and “detection of an event” interchangeably. Formally, the coverage $Cov(p, s_i)$ of a point $p$ by a sensor $s_i$ under the deterministic sensing model is defined as follows:

\[
Cov(p, s_i) = \begin{cases} 
1 & \text{if } \delta(p, s_i) \leq r \\
0 & \text{otherwise}
\end{cases}
\] (2.1)

As can be seen, the deterministic sensing model for two-dimensional (respectively, three-dimensional) wireless sensor networks considers the sensing range of a sensor as a disk (respectively, sphere), and hence all sensor readings are precise and have no uncertainty. However, it has been found that the communication range of the radios is highly probabilistic and irregular [224]. Thus, the deterministic sensing model does not reflect the real behaviour of the sensing units of the sensors, which are irregular in nature. Hence, given the signal attenuation and the presence of noise associated with sensor readings, it is necessary to consider a more realistic sensing model by defining the coverage $Cov(p, s_i)$ using some probability function. In other words, the sensing capability of a sensor needs to be modelled as the probability of successful detection of an event. Specifically, the sensor’s sensing capability should depend on the distance between it and the event as well as the type of propagation model being used (free-space vs. multi-path). Indeed, it has been demonstrated that the probability that an event in a distributed detection application can be detected by an acoustic sensor depends on the distance between the event and sensor [77]. A realistic sensing model for passive infrared (PIR) sensors that reflects their non-isotropic range is presented in [54].
This sensing irregularity of PIR sensors has been verified by simulations [54]. Thus, in a stochastic sensing model, the coverage Cov(p,si) is defined as the probability of detection P(p,si) of an event occurring at point p by sensor si as follows:

\[
P(p,si) = \begin{cases} 
e^{-\beta \delta(p,si)^a} & \text{if } \delta(p,si) \leq r \\ 0 & \text{otherwise} \end{cases}
\]  

(2.2)

where \( \beta \) represents the physical characteristic of the sensing units of the sensors and \( 2 \leq a \leq 4 \) is the path-loss exponent. Precisely, \( \beta \) measures the uncertainty introduced by the sensing unit of the sensors. Also, for the free-space model, we have \( a = 2 \) and for the multi-path model, \( 2 < a \leq 4 \). Our stochastic sensing model is motivated by the one introduced by Elfes [80], where the sensing capability of a sonar sensor is modeled by a Gaussian probability density function. Moreover, a probabilistic sensing model for coverage and target localization in wireless sensor networks is proposed in [229]. This sensing model is similar to ours except that it considers \( \delta(p,si) - (r - r_e) \) instead of \( \delta(p,si) \), where \( r \) is the detection range of the sensors and \( r_e < r \) is a measure of uncertainty in the sensor detection capability. Our stochastic sensing model is also similar to the one in [228], except that ours uses \( a \), and reduces to the deterministic sensing model if we set \( \beta = 0 \).

**Definition 2.22:** Under the deterministic sensing model, a point \( p \) in a field is said to be \( k \)-covered if it belongs to the intersection of the sensing ranges of at least \( k \) sensors.

**Definition 2.23:** Under the stochastic sensing model, a point \( p \) in a field is said to be probabilistically \( k \)-covered if the detection probability of an event occurring at \( p \) by at least \( k \) sensors is at least equal to some threshold probability \( 0 < p_{th} < 1 \).

**Definition 2.24:** For both sensing models, a region \( A \) is said to be \( k \)-covered if every point \( p \in A \) is \( k \)-covered. A \( k \)-covered wireless sensor network is a network that fully \( k \)-covers a field.

**Definition 2.25:** We call degree of coverage provided by a wireless sensor network the maximum value of \( k \) such that a field is fully \( k \)-covered.

**Definition 2.26:** A given \( k \)-coverage is said to be connected if the network induced by all the sensors that are selected to achieve \( k \)-coverage is connected.

**Definition 2.27:** A region of interest \( A \) is said to be energy-efficiently \( k \)-covered if a small number of sensors is selected to \( k \)-cover \( A \).

### 2.4 Network Connectivity and Fault Tolerance

**Definition 2.28:** A communication graph of a homogeneous (respectively, heterogeneous) wireless sensor network is an undirected (respectively, directed) graph, \( G = (S,E) \), where \( S \) is a set of sensors and \( E \) is a set of edges (respectively, arcs) between them such that for all \( s_i, s_j \in S \), \( (s_i,s_j) \in E \) if \( \delta(p_i,p_j) \leq R_i \), where \( p_i \) and \( R_i \) stand for the location and radius of the communication disk of sensor \( s_i \).
respectively. The vertex connectivity (or simply, connectivity) of $G$ is equal to $K$ if $G$ can be disconnected by the removal (or failure) of at least $K$ nodes. The fault tolerance of the underlying network is equal to $K - 1$.

**Definition 2.29:** A forbidden faulty set of a graph $G = (S, E)$ of a wireless sensor network is a set of sensors $F \subseteq S$ that cannot fail at the same time.

Let $P$ be “The faulty set cannot include the neighbour set of any sensor”, $F_{P} \subseteq S$ a faulty set satisfying property $P$, and $G = (S, E)$ a communication graph representing a wireless sensor network.

According to our conditional fault-tolerance model, a faulty sensor set $F_{P}$ is given by $F_{P} = \{ U \subseteq S | \forall s_{i} \in S: CN(s_{i}) \not\subseteq U \}$, where $CN(s_{i})$ is the communication neighbour set of sensor $s_{i}$. Thus, the communication neighbour set of a sensor cannot fail simultaneously, and hence it is a forbidden faulty set. More precisely, any faulty sensor set that includes the communication neighbour set of at least one sensor $s_{i}$ is considered as a forbidden faulty set.

**Definition 2.30:** The conditional connectivity of $G = (S, E)$ with respect to the property $P$, denoted by $\kappa(G : P)$, is the minimum size of $F_{P}$ such that the resulting graph $G_{\text{dis}} = (S - F_{P}, E_{\text{dis}})$ is disconnected into components each satisfying $P$.

Another generalization of connectivity, called restricted connectivity, was proposed by Esfahanian [81] in which the restriction is on the faulty set (i.e., set of nodes that can fail). Restricted connectivity uses the concept of forbidden faulty set in which the entire neighbour set of a node cannot be faulty at the same time.

**Definition 2.31:** The conditional fault tolerance of $G = (S, E)$ with respect to the property $P$ is given by $\eta(G : P) = \kappa(G : P) - 1$.

### 2.5 Energy Consumption Model

The energy consumption of the sensors is dominated by data transmission and reception. Let $s_{i}$ and $s_{j}$ be two neighbouring sensors. According to the energy consumption model specified by Heinzelman et al. [102], when data is sent from a transmitter to a receiver, there is energy consumption incurred at both ends (i.e., transmitter and receiver). While at the receiver end, the energy consumption is due to only one component, called the transceiver, the energy consumption at the transmitter end depends on two components, namely the transceiver and the transmitter amplifier. The energy consumed by the latter component depends on the size of the data packet that has been sent, the distance between the transmitter and the receiver, and another constant, called transmitter amplifier and denoted by $\varepsilon$. The value of this constant depends on whether the free-space or the multipath model is being considered. Formally, according to [102], the energy spent in data transmission and reception are given by $E_{\text{tx}}(s_{i}, s_{j}) = a(E_{\text{elec}} + \varepsilon \delta^{2}(s_{i}, s_{j}))$ and $E_{\text{rx}}(s_{i}) = aE_{\text{elec}}$, respectively, where $\delta$ is the Euclidean distance function, $a$ is data size in bits, $E_{\text{elec}}$ is the electronics energy and is equal to 50 nJ/bit,
\( \varepsilon \in \{ \varepsilon_{fs}, \varepsilon_{mp} \} \) is the transmitter amplifier in the free-space \( (\varepsilon_{fs} = 10pJ/bit/m^2) \) or the multi-path \( (\varepsilon_{mp} = 0.013pJ/bit/m^2) \) model, and \( 2 \leq \alpha \leq 4 \) is the path-loss exponent. When the identities of sender and receiver are not important, we simply write \( E_{tx} = a (E_{elec} + \varepsilon d^\alpha) \) and \( E_{rx} = a E_{elec} \), where \( d \) stands for the transmission distance used by a sender.

**Definition 32:** The delay is defined as the time elapsed between the departure of the data from a source \( s_0 \) and its arrival to the sink \( s_m \). This delay is given by

\[
D(s_0, s_m) = (qd + td + pd) N_f(s_0, s_m)
\]

where \( qd \) is the average queuing delay per intermediate forwarder, \( td \) is the average transmission delay, \( pd \) is the average propagation delay, and \( N_f(s_0, s_m) \) is the number of intermediate forwarders between \( s_0 \) and \( s_m \).

Given that the size of the field is in the order of a few miles, the average propagation delay is negligible. Thus, the delay \( D(s_0, s_m) \) is proportional to \( N_f(s_0, s_m) \), i.e.,

\[
D(s_0, s_m) = c N_f(s_0, s_m) \approx N_f(s_0, s_m)
\]

where \( c = qd + td \). A similar result can be found in [120].

### 2.6 Percolation Model

Assume that the sensing and communication ranges of the sensors in a two-dimensional (respectively, three-dimensional) wireless sensor network are modelled by disks (respectively, spheres).

Let \( X = \{ \xi_i : i \geq 1 \} \) be a two-dimensional (respectively, three-dimensional) homogeneous Poisson point process of density \( \lambda \), where \( \xi_i \) represents the location of a sensor \( s_i \). Let \( X_\lambda(A) \) be a random variable representing the number of points in an area (respectively, volume) \( A \). The probability that there are \( k \) points inside \( A \) is computed as

\[
P(X_\lambda(A) = k) = \frac{\lambda^k |A|^k}{k!} e^{-\lambda |A|}
\]  

for all \( k \geq 0 \), where \( |A| \) is the size of the area (respectively, volume) of \( A \).

**Definition 2.33:** In a two-dimensional (respectively, three-dimensional) wireless sensor network, the covered area (respectively, volume) fraction of a Poisson Boolean model \( (X_\lambda, \{B_i(r) : i \geq 1 \}) \) given by \( A(r) = 1 - e^{-b\lambda} \) [100] is the mean fraction of area (respectively, volume) covered by the sensing disks (respectively, spheres) \( B_i(r) \), for \( i \geq 1 \), in a region of unit area (respectively, volume), where
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\[ b = \pi r^2 \] (respectively, \( b = 4\pi r^3 / 3 \)) is the area (respectively, volume) of the sensing disks (respectively, spheres) of the sensors and \( \lambda \) is the density of the Poisson point process \( X_\lambda \).

Assume that \( \lambda \) is not a constant as the sensors could appear and disappear independently of one another. We want to compute the density \( \lambda_c \), called critical percolation density (or critical density) such that there exists an infinite covered component when \( \lambda > \lambda_c \), and hence the Boolean model \( (X_\lambda, \{D_i(r) : i \geq 1\}) \) is said to be percolating. Otherwise, there is no infinite covered component and hence the Boolean model \( (X_\lambda, \{D_i(r) : i \geq 1\}) \) does not percolate.

**Definition 2.34: The critical covered area (respectively, volume) fraction of \( (X_\lambda, \{D_i(r) : i \geq 1\}) \), computed as \( A_c(r) = 1 - e^{-a \lambda_c} \), is the fraction of area (respectively, volume) covered at critical percolation, where \( \lambda_c \) is the associated density of \( X_\lambda \).**

Percolation processes were introduced by Broadbent and Hammersley [51] to model the random flow of a fluid through a medium. Because of their simplicity of description and display of critical behaviour, where the behaviour of a model changes abruptly (phenomenon known as phase transition) as the value of a parameter crosses a threshold, percolation models are attractive in several areas of mathematics, physical science, and engineering. A percolation model can be viewed as an ensemble of points distributed in space, where some pairs are adjacent (or connected) [83]. We consider a Boolean model [155] which is defined as follows:

**Definition 2.35: A Boolean model consists of two components, namely point process \( X_\lambda \) and connection function \( h \). The set \( X_\lambda = \{\xi_i : i \geq 1\} \) is a homogeneous Poisson point process of density \( \lambda \) in a two-dimensional Euclidean plane \( \mathbb{R}^2 \) (respectively, three-dimensional Euclidean space \( \mathbb{R}^3 \)), where the elements of \( X_\lambda \) are the locations of the sensors used to cover a field. The connection function, \( h \), is defined such that two points \( \xi_i \) and \( \xi_j \) are adjacent with probability \( h(\|\xi_i - \xi_j\|) = 1 \) independent of all other points if \( \|\xi_i - \xi_j\| \leq d \) and \( h(\|\xi_i - \xi_j\|) = 0 \) if \( \|\xi_i - \xi_j\| > d \), where \( d \geq 0 \) and \( \|\xi_i - \xi_j\| \) is the Euclidean distance between \( \xi_i \) and \( \xi_j \). In other words, \( h(\|\xi_i - \xi_j\|) \) given by

\[
h(\|\xi_i - \xi_j\|) = \begin{cases} 
1 & \text{if } \|\xi_i - \xi_j\| \leq d \\
0 & \text{otherwise}
\end{cases}
\]

2.6.1 Why a Continuum Percolation Model?

We consider a continuum percolation model rather than a discrete percolation model for the following reason. In discrete percolation [95], also known as lattice
model, the sites, which are randomly occupied in a discrete lattice, may have different configurations, namely square, triangle, honeycomb, etc. In continuum percolation [155], the positions of the sites are randomly distributed and thus there is no need to have different analysis for each of these regular lattices. Precisely, we consider a continuum percolation model, which consists of homogeneous disks (respectively, spheres) whose centres representing the locations of the sensors are randomly distributed in two-dimensional (respectively, three-dimensional), according to a spatial Poisson point process of density \( \lambda \). In percolation theory, we are interested in the critical density \( \lambda_c \) above which an infinite cluster of overlapping disks (respectively, spheres) first appears. The density \( \lambda_c \) is the critical value for the density \( \lambda \) such that there exists no infinite cluster of overlapping disks (respectively, spheres) almost surely when \( \lambda < \lambda_c \), but there is an infinite cluster of overlapping disks (respectively, spheres) almost surely when \( \lambda > \lambda_c \) and we say that percolation occurs.

### 2.7 Network Model

In this section, we specify the default network model used in this book unless stated otherwise.

We assume that all the sensors in a two-dimensional (respectively, three-dimensional) wireless sensor network are static and isotropic. In other words, all the sensors do not move and have the same sensing and communication ranges. Furthermore, the latter follow the unit disk model (respectively, sphere model). That is, the sensing range of a sensor \( s_i \) is modelled by a disk (respectively, sphere) whose radius is \( r \), and its communication range is represented by a disk (respectively, sphere) with radius equal to \( R \). Moreover, both of the sensing and communication ranges of the sensor \( s_i \) are centred at \( p_i \), i.e., the location of the sensor \( s_i \). Therefore, each sensor \( s_i \) is characterized by two concentric disks (respectively, spheres) associated with its sensing and communication ranges, respectively, as shown in Fig. 2.1.

Moreover, all the sensors are always-on, meaning that they constantly report their sensed data to a single static sink. Hence, the sensors cannot be turned off while monitoring a physical phenomenon. Also, each sensor has a unique id (an integer, for instance) and is aware of its own location information through Global Positioning System (GPS) or some localization technique [114]. The sensors advertise their location information only once when they start their sensing task. In addition, each sensor advertises its remaining energy by piggybacking it on the data sent to the sink. The sensors are randomly, uniformly deployed in a field whose size is much larger than that of the sensing and communication ranges of the sensors. Moreover, the sensors are supposed to be densely deployed. As indicated in Chap. 1, the limited battery power of the sensors and the difficulty of replacing and/or recharging batteries on the sensors in hostile environments require that the sensors be deployed with high density to extend the network lifetime. Also, we assume that each sensor has transmit-power control and hence can adjust its transmission distance so it can transmit its data over a distance that is
less than or equal to the radius of its communication range. The communication links between the sensors are perfectly reliable while the sensors can fail or die independently due to low battery power.

We should mention that some of these assumptions will be further relaxed so as to promote the use of our proposed protocols in real-world sensing applications. These relaxations will be discussed later.

2.8 Summary

In this chapter, we defined useful terms that are used throughout this book. Also, we discussed the notion of unconditional (or traditional) connectivity as well as conditional connectivity, and derived the network connectivity and fault tolerance. Moreover, we described the energy consumption and percolation models. Furthermore, we specified our default network model, which is used in the design and description of all the protocols proposed in this book.