

# A Neuro Fuzzy Model for Image Compression in Wavelet Domain

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**Abstract.** Image compression forms the backbone for several applications such as storage of images in a database, picture archiving, TV and facsimile transmission, and video conferencing. Compression of images involves taking advantage of the redundancy in data present within an image. This work evaluates the performance of an image compression system based on fuzzy vector quantization, wavelet based sub band decomposition and neural network. Vector quantization is often used when high compression ratios are required. The implementation consists of three steps. First, image is decomposed into a set of sub bands with different resolution corresponding to different frequency bands. Different quantization and coding schemes are used for different sub bands based on their statistical properties. At the second step, the wavelet coefficients corresponding to lowest frequency band are compressed by differential pulse code modulation (DPCM) and the coefficients corresponding to higher frequency bands are compressed using neural network. The result of the second step is used as input to fuzzy vector quantizer. Image quality is compared objectively using mean squared error and PSNR along with the visual appearance. The simulation results show clear performance improvement with respect to decoded picture quality as compared to other image compression techniques.

## 1 Introduction

Interest in digital image compression techniques, dates back to a few decades ago. Here we are concerned with minimizing the number of bits required to represent an image primarily for achieving information transmission and storage efficiency. All image compression algorithms strive to remove statistical redundancy and exploit perceptual irrelevancy while reducing the amount of data as much as possible. Over the last few decades researches have proposed many competing techniques such as prediction coders, transform coders, vector quantizers, trellis-coded quantizers and fractal image representation. However, due to the nature of each scheme every algorithm has its own advantages and disadvantages. Among all these schemes, however, the discrete cosine transform used in the JPEG standard has the advantage that it is well understood and it is quite mature. On the other hand vector quantization is a more recently developed technique. VQ performance is directly proportional to the codebook size and the vector size According to Shannon's rate distortion theory

larger vectors would result in better VQ performance. However, with increased vector size the required codebook size also increases, and that in-turn results in an exponential increase in encoding complexity. Consequently, for practical purposes one should limit the vector size despite the fact that better VQ performance is theoretically possible. To date, VQ techniques used in image compression may be classified into two broad categories, based on either a hard or soft decision in a membership function, namely, K-means and Fuzzy K-mean. The K-means algorithm is the most popular minimization technique using a descent algorithm. It is simple to implement but it depends strongly on the selection of the initial codebook. It is based on hard decisions and ignores the possibility that the training vector may belong to a different cluster. Artificial neural networks are receiving renewed attention in many fields where high computation rates are required. In the field of image processing applications can be found in concurrent parallel processing structures, in the recognition of characters or 2-D patterns, in some 2-D signal processing tasks and more recently in data compression for image coding systems.

Recent advances in signal processing tools such as wavelets opened up a new horizon in sub band image coding. Studies in wavelets showed that the wavelet transform exhibits the orientation and frequency selectivity of images [1-3]. Image coding using wavelets has been attempted by Antonini et al. [1], who used bi-orthogonal wavelets to obtain a set of bi-orthogonal sub bands. Woods and O'Neil [4] extended sub band decomposition to two-dimensional (2-D) signals and proposed a method for QMF design that eliminates possible aliasing error due to non ideal sub band filters. The original image was decomposed at different scales or sub bands of frequency using Mallat's pyramidal architecture [2], in which horizontal and vertical orientations are considered preferential. After the image was decomposed, the wavelet coefficients were vector quantized producing a set of multi-resolution codebooks. Averbuch et al. [5] proposed a similar image compression approach that combines pyramidal wavelet decomposition and vector quantization. In order to improve the quality of the reconstructed images, they used higher bit rate per pixel for shapes found to be more important. Codebook design is an essential process in lossy image compression techniques based on vector quantization [6-8]. Paliwal et al [9] modified the K mean algorithm for vector quantization for fast convergence without affecting the optimality of the codebook. Kadono et al [10] combined wavelet transform and vector quantization on color images.

Based on the concept of fuzzy sets, Ruspini [25] developed the first fuzzy clustering algorithm. Dunn [26] considered an alternative formulation of the clustering process and proposed the fuzzy k-means algorithm. Bezdek [27] extended Dunn's formulation and produced a family of fuzzy k-means algorithms, which includes Dunn's original algorithm as a special case. Sasazaki et al [28] suggested to use local fractal dimension to design a codebook and used fuzzy k means algorithm to group the training vectors into clusters.

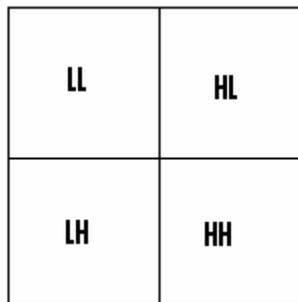
Recently, artificial neural networks [11] are increasingly being examined and considered as possible solutions to problems and for application in many fields where high computation rates are required [12]. People have proposed several kinds of image compression methods [13]. The typical image compression methods are based on back propagation network [14-15]. The number of neurons in the hidden layer should be smaller than the number of units in the input and output layers. The image

is compressed in the hidden layers. However, the compression results are generally worse than traditional approaches [ 16-17]. Improved results probably cannot be achieved unless neural networks are incorporated directly into a state-of-the-art framework. Most successful modern coding techniques are built around filter banks [18-19]. There is evidence that the use of neural networks can improve the performance of sub band coders. Neural networks have been used to find vector quantization maps for wavelet coefficients [20], and recently Burges et al. [21] showed that preprocessing the detail coefficients using neural network DPCM reduced both the entropy of these bands and the length of the coded stream.

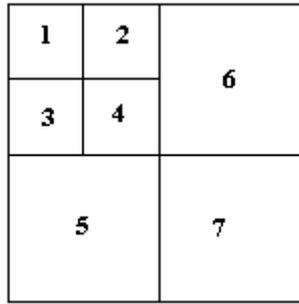
The main objective of this paper is to develop a Neuro-Fuzzy network based on wavelet transform and report its application to Image compression. The paper is organized as follows: Section 2 briefly reviews filter bank concepts for image compression and section 3 introduces the background necessary to define the desired input-output relations that are required to train the neural network with back propagation. Section 4 describes the fuzzy k means algorithm for vector quantization, and Section 5 presents an implementation of the image compression algorithm. Section 6 reports sample simulation results and section 7 provides concluding remarks.

## 2 Wavelet Transform

Wavelet transforms are multi resolution decompositions that can be used to analyze signals and images. They describe a signal by the power at each scale and position. Edges can be located very effectively in the wavelet transform domain. For image processing applications, one can use the hierarchical wavelet decomposition by Mallat [2]. The L and H filters are applied to the image in both the horizontal and vertical directions, and the filter outputs sub sampled by a factor of two, generating three orientation selective high-pass sub bands, HH, LH, HL, and a low-pass sub band LL. The process is then repeated on the LL band to generate the next level of the decomposition, and so on. As shown in Fig. 1, one octave of decomposition leads to four sub bands. Therefore, seven sub bands are obtained by iterating two times of such decomposition as shown in fig 2. The low-resolution version of the image is fed



**Fig. 1.** First stage of a discrete wavelet transform; The image is divided into 4 sub bands using separable filters

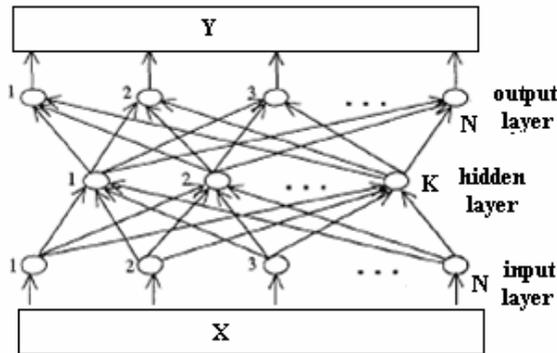


**Fig. 2.** Two stages of pyramidal decomposition leading to 7 octave sub bands

back to the input to produce one low-resolution version of the image and six sub bands which contain image details. The wavelet decomposition is an alternative representation of images. To compress an input image data using transform, we have to decide which coefficients to send and how many bits to encode them. Our compression algorithm consists of taking the low pass subband in full, and then deciding which coefficients within the remaining subbands to keep. Hence, the basic idea is this: the smaller the support of the wavelet, the less nonzerowavelet coefficients will correspond to an edge, so the more efficient will be our compression scheme.

### 3 Neural Network for Image Compression

In this section, we briefly review the basic idea of using a back propagation network to achieve image compression. A number of researchers [14-15] have shown that multilayer perceptron networks are able to learn a transform for reducing signal redundancy, and are capable of learning a reverse transform to recover the information (with some degradation) from a more compact (compressed) form of representation. In a multilayer configuration, the outputs of the units in one layer form the inputs to the next layer. The inputs to the first layer are considered as the network



**Fig. 3.** A multi layered neural network

inputs, and outputs of the last layer are the network outputs. The weights of the network are usually computed by training the network using the backpropagation algorithm. The backpropagation algorithm is a supervised learning algorithm which performs a surface to arrive at a gradient descent on a squared error energy minimum. The key to the use of this method on a multilayer perceptron is the calculation of error values for the hidden units by propagating the network. The network shown in Fig. 3 has  $N$  input nodes,  $K$  hidden layer nodes ( $K < N$ ) and  $N$  output nodes. The image compression in such a neural network proceeds in two stages; training and encoding. At the training stage, a set of input images go through the network one block by one block to allow the neural network to learn the best possible coupling weights for representation of the code-book.

After training, the network is ready for operational use. The wavelet coefficients are presented to the network one at a time. Following that, encoding begins by using the closest neuron weight to represent each individual block of the input image. The outputs of the hidden layer nodes constitute the compressed features of an input block. To achieve compression in a practical sense, the outputs of the hidden layer nodes are quantized.

## 4 Quantization

The process of quantization maps a signal into a series of  $K$  discrete messages. For the  $k^{\text{th}}$  message, there exists a pair of thresholds  $t_k$  and  $t_{k+1}$  and an output value  $q_k$  such that  $t_k \leq q_k \leq t_{k+1}$ . For a given set of quantization values, the optimal thresholds are equidistant from the values. The concept of quantizing data can be extended from scalar or one-dimensional data to vector data of arbitrary dimension. Instead of output levels, vector quantization (VQ) employs a set of representation vectors (for the one-dimensional case) or matrices (for the two dimensional case) [6-8] The set is referred to as the “codebook” and the entries as “codewords. Hard k-means algorithm executes a sharp classification, in which each object is either assigned to a class or not. The application of fuzzy sets in a classification function causes the class membership to become a relative one and an object can belong to several classes at the same time but with different degrees [24].

### 4.1 K-Means Algorithm

K-means clustering, is also known as c-means clustering. The k-means algorithm partitions a collection of  $N$  vector into  $c$  groups (clusters  $G_i$ ,  $i=1, \dots, c$ ). The aim of that algorithm finding cluster centers (centroids) for each group. The algorithm minimizes a dissimilarity (or distance) function which is given in Equation 1.

$c_i$  is the centroid of cluster  $i$ ;  $d(x_k - c_i)$  is the distance between  $i^{\text{th}}$  centroid ( $c_i$ ) and  $k^{\text{th}}$  data point; For simplicity, the Euclidian distance is used as dissimilarity measure and overall dissimilarity function is expressed as in Equation 2.

$$J = \sum_{i=1}^c J_i = \sum_{i=1}^c \sum_{k, x_k \in G_i} d(x_k - c_i) \quad (1)$$

$$J = \sum_{i=1}^c J_i = \sum_{i=1}^c \left( \sum_{k, x_k \in G_i} \|x_k - c_i\|^2 \right) \quad (2)$$

Partitioned groups can be defined by an  $c \times n$  binary membership matrix (U), where the element  $u_{ij}$  is 1 if the  $j$ th data point  $x_j$  belongs to group  $i$ , and 0 otherwise. This explanation is formulated in Equation 3.

$$u_{ij} = \begin{cases} 1 & \text{if } \|x_j - c_i\|^2 \leq \|x_j - c_k\|^2, \text{ for each } k \neq i, \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

Since a data point can only be in a group, the membership matrix (U) has two properties which are given equation 4 and equation 5.

$$\sum_{i=1}^c u_{ij} = 1, \forall j = 1, \dots, n \quad (4)$$

$$\sum_{i=1}^c \sum_{j=1}^n u_{ij} = n \quad (5)$$

Centroids are computed as the mean of all vectors in group  $i$ :

$$c_i = \frac{1}{|G_i|} \sum_{k, x_k \in G_i} x_k \quad (6)$$

$|G_i|$  is the size of  $G_i$ . The k-means algorithm [23] determines the following steps with a data set  $x_j, j=1, \dots, n$ ;

**Step 1.** Initialize the centroids  $c_i, i=1, \dots, c$ . This is typically achieved by randomly selecting  $c$  points from among all of the data points.

**Step 2.** Determine the membership matrix U by Equation 3.

**Step 3.** Compute the dissimilarity function by using Equation 2. Stop if its improvement over previous iteration is below a threshold.

**Step 4.** Compute new centroids using by Equation 6. Go to step 2.

While the k-means algorithm converges to a local minimum, it is not guaranteed to reach the global minimum. In addition, the algorithm is very sensitive to the initial codebook. Furthermore, the algorithm is slow since it requires an exhaustive search through the entire codebook on each iteration.

## 4.2 Fuzzy C-Means Clustering

Fuzzy C-means Clustering(FCM), is also known as Fuzzy ISODATA, is an clustering technique which is separated from hard k-means that employs hard partitioning. The

FCM employs fuzzy partitioning such that a data point can belong to all groups with different membership grades between 0 and 1. FCM is an iterative algorithm. The aim of FCM is to find cluster centers (centroids) that minimize a dissimilarity function.

To accommodate the introduction of fuzzy partitioning, the membership matrix (U) is randomly initialized according to Equation 7. The dissimilarity function which is used in FCM is given Equation 8

$$\sum_{i=1}^c u_{ij} = 1, \forall j = 1, \dots, n \quad (7)$$

$$J(U, c_1, c_2, \dots, c_c) = \sum_{i=1}^c J_i = \sum_{i=1}^c \sum_{j=1}^n u_{ij}^m d_{ij}^2 \quad (8)$$

$u_{ij}$  is between 0 and 1;  $c_i$  is the centroid of cluster  $i$ ;  $d_{ij}$  is the Euclidian distance between  $i_{th}$  centroid( $c_i$ ) and  $j_{th}$  data point;  $m \in [1, \infty)$  is a weighting exponent. To reach a minimum of dissimilarity function there are two conditions. These are given in Equation 9 and Equation 10. Detailed algorithm of fuzzy c-means proposed by Bezdek[24]. This algorithm determines the following steps [23].

$$c_i = \frac{\sum_{j=1}^n u_{ij}^m x_j}{\sum_{j=1}^n u_{ij}^m} \quad (9)$$

$$u_{ij} = \frac{1}{\sum_{k=1}^c \left( \frac{d_{ij}}{d_{kj}} \right)^{2/(m-1)}} \quad (10)$$

**Step 1.** Randomly initialize the membership matrix (U) that has constraints in Equation 7.

**Step 2.** Calculate centroids ( $c_i$ ) by using Equation 9.

**Step 3.** Compute dissimilarity between centroids and data points using equation 8. Stop if its improvement over previous iteration is below a threshold.

**Step 4.** Compute a new U using Equation 7 Go to Step 2.

By iteratively updating the cluster centers and the membership grades for each data point, FCM iteratively moves the cluster centers to the "right" location within a data set. FCM does not ensure that it converges to an optimal solution. Because of cluster centers (centroids) are initialize using U that randomly initialized.(Equation 14). Performance depends on initial centroids. For a robust approach there are two ways which is described below.

1) Using an algorithm to determine all of the centroids. (for example: arithmetic means of all data points)

2) Run FCM several times each starting with different initial centroids.

## 5 Proposed Method

Fig 4 shows the block diagram of complete image compression system. First the image is decomposed using wavelet transform. Because the human visual system has different sensitivity to different frequency components, the following scheme is adapted. The lowest frequency band, band-1 (fig 2) is encoded with DPCM. After that these coefficients are scalar quantized. The remaining frequency bands are coded using neural network. Band-2 and 3 contain the same frequency contents for different orientation. So same neural network is used to compress the data in these bands and different neural network is used for band 5 and 6. Band-4 coefficients are coded using separate neural networks as frequency characteristics of these bands does not match with other bands. Band-7 information is discarded as it contains little information to contribute to the image from the stand this band can be assumed to be zero with little effect on the quality of reconstructed image. The output of the hidden layer of neural network is then quantized. Finally these quantized values are entropy encoded. Huffman encoding is used here. This scheme was originally proposed in [20].

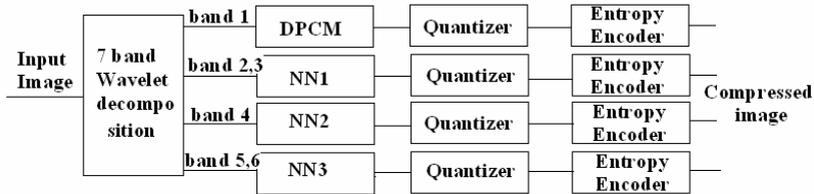


Fig. 4. Complete Image compression

## 6 Experimental Results and Discussion

This section presents the experimental evaluation of the proposed algorithm based on several experiments involving real image data. The set of experiments evaluate the effect of different wavelet filters on the quality of the reconstructed image. Experiments were conducted using the image 'lena', of size  $256 \times 256$ , with  $2^8 = 256$  gray levels. Images were decomposed using Daubechies' 1- coefficient filter (DAUB 1), 2- coefficient filter (DAUB 2), 4- coefficient filter (DAUB 4), 6- coefficient filter (DAUB 6) and 18 coefficient filter (DAUB 18), coiflet 5 coefficient filter and biorthogonal 9 coefficient filter. Band -1 is coded using DPCM, Band -2 and 3 is coded using a neural network with eight units in the input and the output and 6 hidden units i.e 8-6-8 neural network. Band -4 is coded using a 8-4-8 neural network and band -5 and 6 using 16-1-16 network. In different sets of experiments, scalar quantization (SQ) and vector quantization (VQ). was used on the coefficients of hidden layer. In the end, the coefficients are Huffman encoded.

### 6.1 Image Quality Evaluation

The image quality can be evaluated objectively and subjectively. A standard objective measure of image quality is reconstruction error. Two of the error metrics used to

compare the various image compression techniques are the mean square error (MSE) and the Peak Signal to Noise Ratio (PSNR). The MSE is the cumulative squared error between the compressed and the original image, whereas PSNR is a measure of the peak error. The mathematical formulae are

$$MSE = \frac{1}{MN} \sum_{i=1}^M \sum_{j=1}^N (I_{ij} - I'_{ij})^2 \tag{11}$$

$$PSNR = 20 * \log_{10} (255 / \text{sqrt}(MSE)) \tag{12}$$

where  $I(x,y)$  is the original image,  $I'(x,y)$  is the approximated version (which is actually the decompressed image) and  $M,N$  are the dimensions of the images, 255 is the peak signal value. A lower value for MSE means lesser error, and as seen from the inverse relation between the MSE and PSNR, this translates to a high value of PSNR. Logically, a higher value of PSNR is good because it means that the ratio of Signal to Noise is higher. Subjective quality is measured by psychophysical tests or questionnaires with numerical ratings.

**Table 1.** MSE and PSNR of training image ‘lena’ using fuzzy k-mean algorithm with  $\lambda =5$  with different wavelet filters

Filter Type	Filter Length	MSE with FVQ	PSNR with FVQ	Compression	
				Huffman Coded	Without Huff
Daubechies	1	160.77	26.06	0.1385	0.1879
Daubechies	2	101.73	28.056	0.1393	0.1879
Daubechies	4	87.95	28.689	0.1395	0.1879
Daubechies	6	83.62	28.903	0.1382	0.188
Daubechies	18	76.15	29.314	0.1380	0.2
Coiflet	5	71.86	29.566	0.1386	0.2
Biorhogonal	9	83.99	28.97	0.1395	0.198

**Table 2.** PSNR of training image ‘lena’ using VQ and FVQ  $\lambda =5$  with different wavelet filters

Filter Type	Filter Length	PSNR With VQ	Compression with VQ		PSNR With FVQ	Compression with FVQ	
			Huffman Coded	Without Huff		Huffman Coded	Without Huff
Daubechies	1	26.144	0.1378	0.1879	26.06	0.1385	0.1879
Daubechies	2	28.149	0.1390	0.1879	28.056	0.1393	0.1879
Daubechies	4	28.713	0.1382	0.1879	28.689	0.1395	0.1879
Daubechies	6	29.001	0.1378	0.188	28.903	0.1382	0.188
Daubechies	18	29.402	0.1379	0.200	29.314	0.1380	0.200
Coiflet	5	29.608	0.1378	0.200	29.566	0.1386	0.200
Biorhogonal	9	29.080	0.1382	0.198	28.97	0.1395	0.198

**Table 3.** PSNR of training image ‘lena’ reconstructed by db18 and fuzzy k-mean algorithm with different integer values of  $\lambda$ 

Filter Type	Filter Length	PSNR With FVQ					Compression with FVQ	
		$\lambda=1$	$\lambda=2$	$\lambda=3$	$\lambda=4$	$\lambda=5$	Huffman Coded	Without Huff
Daubechies	1	25.384	25.862	26.066	26.097	26.060	0.1378	0.1879
Daubechies	2	27.017	27.728	28.059	28.059	28.056	0.1390	0.1879
Daubechies	4	27.614	28.378	28.706	28.592	28.689	0.1382	0.1879
Daubechies	6	27.356	28.495	28.87	28.823	28.903	0.1378	0.188
Daubechies	18	27.599	28.498	29.182	29.275	29.314	0.1379	0.200
Coiflet	5	27.904	29.001	29.492	29.576	29.566	0.1378	0.200
Biorthogonal	9	27.737	28.480	28.875	28.962	28.97	0.1382	0.198



(a) Original



(b) PSNR= 26.06 db



(c) PSNR= 28.056 db



(d) PSNR= 28.689 db



(e) PSNR= 28.903 db



(f) PSNR= 29.314 db



(g) PSNR= 29.566 db



(h) PSNR= 28.97 db

**Fig. 5.** Results of ‘lena’ using fuzzy k-mean algorithm with  $\lambda=5$  with different wavelet filters (a) Original ‘Lena’ image (b) Reconstructed image with db1 (c) Reconstructed image with db2 (d) Reconstructed image with db4 (e) Reconstructed image with db6 (f) Reconstructed image with db18 (g) Reconstructed image with coiflet 5 (h) Reconstructed image with biorthogonal 9(a)  $\lambda=1$ (b)  $\lambda=2$ (c)  $\lambda=3$ (d)  $\lambda=4$ (e)  $\lambda=5$ **Fig. 6.** ‘lena’ reconstructed by db18 and fuzzy k-mean algorithm with different integer values of  $\lambda$  of  $\lambda$

The first set of experiments evaluated the performance the proposed algorithm with distinct QMF filters for the decomposition of the image, which raises the question of which filter is the best for our compression method. Table I presents the level of compression achieved by applying different types of filters on the training image 'lena' and fuzzy vector quantization with  $\lambda=5$ . The compression achieved is described in two ways: the compression achieved after applying Huffman coding on the resulted Vector Tables and without using the Huffman coding. Huffman coding, which is a lossless compression, increases the compression by 30%. This work uses Daubechies' wavelet of length 18 which yields the best PSNR as compared to other filters and its visual quality is better. The results are shown in Fig 9.

The second set of experiments compared the codebook design with VQ and FVQ. The wavelet coefficients in the hidden layer of the neural network can be quantized, among other methods, by VQ or FVQ. Comparing VQ versus FVQ should take into consideration two aspects : compression ratio achieved and its quality. Table II compares VQ and FVQ with the length of each vector as 16 and  $\lambda$  as 5 for FVQ on the training image 'lena'. It is observed that by using FVQ, though the SNR remains, the visual quality of the reconstructed image is better in comparison with VQ. The results are shown in Fig 10. The last set of experiments evaluated the quality of the reconstructed images with m. FVQ parameter m as evaluated as  $m = 1 + 1/\lambda$ , where  $\lambda = 1, 2, 3, 4, 5$ . Since the compression of images based on vector quantization is eventually based on crisp decisions, the quality of the codebook provided by fuzzy k mean algorithm depends strongly on the proximity of this parameter to unity. According to table IV &V, PSNR & the visual quality of the image increases as the parameter m approaches unity, with maximum SNR corresponding to  $m=1.2$  in most cases. The results of this set of experiments are summarized in Fig 11.

## 7 Conclusion

This paper presented a neuro-wavelet based approach using fuzzy vector quantization for image compression. Compared to the neural network applied on the original image wavelet based decomposition improved the quality of reconstructed images. Among the various wavelet filters tested in the experiments, DAUB 18 resulted in slightly better results. Fuzzy vector quantization makes use of uncertainty to the benefit of the clustering process. Use of fuzzy vector quantization on the hidden layer coefficients further improves the visual quality of the reconstructed image, but it is computationally more demanding than the vector quantization algorithm.

## References

- [1] Antonini, M., Barlaud, M., Mathieu, P., Daubechies, I.: Image coding using wavelet transform. IEEE Trans. Image Processing 1, 205–220 (1992)
- [2] Mallat, S.G.: A theory for multi resolution signal decomposition: The wavelet representation. IEEE Trans. Pattern Anal. Machine Intell. 11, 674–693 (1989)
- [3] Mallat, S.G.: Multi frequency channel decomposition of images and wavelet models. IEEE Trans. Acoust., Speech, Signal Processing 37, 2091–2110 (1989)

- [4] Woods, J.W., O'Neil, S.D.: Sub band coding of images. *IEEE Trans. Acoust., Speech, Signal Processing* 34, 1278–1288 (1986)
- [5] Averbuch, A., Lazar, D., Israeli, M.: Image compression using wavelet transform and multiresolution decomposition. *IEEE Trans. Image Processing* 5, 4–15 (1996)
- [6] Gersho, A., Gray, R.M.: *Vector Quantization and Signal Compression*. Kluwer Academic Publishers, Dordrecht (1992)
- [7] Gray, R.M.: Vector quantization. *IEEE Acoust., Speech, Signal Processing Mag.* 1, 4–29 (1984)
- [8] Nasrabadi, N.M., King, R.: Image coding using vector quantization: A review. *IEEE Trans. Commun.* 36, 957–971 (1988)
- [9] Paliwal, K., Ramasubramian, V.: Comments on modified K means algorithm for vector quantizer design. *IEEE trans Image processing* 9(11), 1964–1967 (2000)
- [10] Kadono, S., Tahara, O., Okamoto, N.: Encoding of color still pictures wavelet transform and vector quantization. In: *Canadian Conference on Electrical and Computer Engineering*, vol. 2, pp. 931–936 (2001)
- [11] Lippmann, R.P.: An introduction to computing with neural network, pp. 36–54. *IEEE Computer Society Press*, Los Alamitos (1987)
- [12] Polycarpou, M.M., Ioannou, P.A.: Learning and Convergence Analysis of Neural Type Structured Networks. *IEEE Transactions on Neural Network* 3(2), 39–50 (1992)
- [13] Rao, K.R., Yip, P.: *Discrete Cosine Transform - Algorithms, Advantages, Applications*. Academic Press, London (1990)
- [14] Sonehara, N., Kawato, M., Miyake, S., Nakane, K.: Image Data Compression Using a Neural Network Model. In: *IJCNN con. IEEE cat. No. 89CH2765-6*, vol. 2, pp. 35–41 (1989)
- [15] Setiono, R., Lu, G.: Image Compression Using a Feedforward Neural Network. In: *IJCNN*, pp. 4761–4765 (1994)
- [16] Chen, W.H., Smith, C.H.: Adaptive Coding of Monochrome and Color Images. *IEEE Trans. Commun., COM-25*, 1285–1292 (1977)
- [17] Gimlett, J.I.: Use of Activity Classes in Adaptive Transform Image Coding. *IEEE Trans. Commun., COM- 23*, 785–786 (1975)
- [18] Antonini, M., Barlaud, M., Mathieu, P., Daubechies, I.: Image coding using wavelet transform. *IEEE Trans. Image Processing* 1, 205–220 (1992)
- [19] Shapiro, J.M.: Embedded image coding using zerotrees of wavelet coefficients. *IEEE Trans. Signal Processing* 41, 3445–3462 (1993)
- [20] Denk, T., Parhi, K.K., Cherkassky, V.: Combining neural networks and the wavelet transform for image compression. In: *Proc. ICASSP IEEE Int. Conf. Acoust., Speech, Signal Processing*, Minneapolis, Minn., pp. I-637–I-640 (1993)
- [21] Burges, J.C., Simard, Y.P., Malvar, H.S.: Improving wavelet compression with neural networks. In: *Proc. Data Compression Conf., Snowbird, Utah, March 27–29*, p. 486 (2001)
- [22] Max, J.: Quantizing for minimum distortion. *IEEE Trans. Inform.Theory*, 7–12 (1960)
- [23] Jang, J.-S.R., Sun, C.-T., Mizutani, E.: *Neuro-Fuzzy and Soft Computing*, pp. 426–427. Prentice-Hall, Englewood Cliffs (1997)
- [24] Bezdek, J.C.: *Pattern Recognition with Fuzzy Objective Function Algorithms*, New York, Plenum
- [25] Ruspini, E.H.: A New Approach to Clustering. *Information and Control* 15, 22–32 (1969)
- [26] Dunn, J.C.: A Fuzzy Relative of the ISODATA Process and its use in Detecting Compact Well Separated Clusters. *Journal of Cybernetics* 3(3), 32–57 (1973)

- [27] Bezdek, J.C., Ehrlich, R., Full, W.: FCM: The Fuzzy c-Means Clustering Algorithm. In: Computers and Geosciences, vol. 10(2-3), pp. 191–203 (1984)
- [28] Sasazaki, K., Ogasawara, H., Saga, S., maeda, J., Suzuki, Y.: Fuzzy vector Quantization of Images based on local fractal dimension. In: IEEE international conference on Fuzzy Systems, Canada, pp. 16–21 (2006)
- [29] Liu, H., Zhai, L., Gao, Y., Li, W., Zhou, J.: Image compression based on biorthogonal wavelet transform. In: Proceedings of ISCIT, pp. 578–581 (2005)
- [30] Premaraju, S., Mitra, S.: Efficient image coding using multi resolution wavelet transform and vector quantization. Image analysis and interpretation, pp. 135-140 (1996)