

Constant Weight Codes with Package CodingTheory.m in Mathematica

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Abstract. The author offers the further development of package CodingTheory.m [1] in the direction of research of properties and parameters of Constant weight codes (lower and upper bounds) based on works [2], [3] and also using the Table of Constant Weight Binary Codes (online version, Neil J. A. Sloane: Home Page <http://www.research.att.com/~njas/codes/Andw/>) and the table of upper bounds on $A(n, d, w)$ (which in many cases also gives lower bounds) maintained by Erik Agrell, Alexander Vardy and Kenneth Zeger, and which is an electronic supplement to their paper Upper bounds for constant-weight codes, <http://www.s2.chalmers.se/~agrell/bounds/cw.html>. The offered package allows to carry out the comparative analysis of parameters of new codes with classical upper bounds such as Johnson bound, Linear programming (Ph Delsarte) bound ..., and also with already available classes of codes. As an example we consider a some construction of codes as union of two codes with parameters $(n, 2a, a + b)$ and $(m, 2b, a + b)$, that in some cases gives codes the best than codes obtained with use Juxtaposing .

1 Introduction

An (n, d, w) constant weight binary code is a binary code of length n , code distance d in which all code words have the same number, w , of “ones.” We will denote the maximal possible size of (n, d, w) constant weight code by $A(n, d, w)$. The most important and interesting problem is finding the largest possible size $A(n, d, w)$ of a (n, d, w) constant weight code (hereafter called optimal codes). The results of code searching used to be put in tables of optimal codes. The first lower bound appeared in 1977 in the book of MacWilliams and Sloane ([4], pp.684-691). A table of binary constant weight codes of length $n \leq 28$ with explicit constructions for most of the 600 codes was presented in the encyclopedic work of Brouwer, Shearer, Sloane, and Smith [2]. Today Neil J. A. Sloane presents his table of constant weight codes online and performs continual updates. There is a companion table of upper bounds on $A(n, d, w)$ (which in many cases also gives lower bounds) maintained by Erik Agrell, Alexander Vardy and Kenneth Zeger, and which is an electronic supplement to their paper Upper bounds for constant-weight codes. The fragment of Sloane’s table of constant weight codes with parameters $20 \leq n \leq 70$, $d = 18$ and $10 \leq w \leq 16$ looks as

follows (the point means that the appropriate code is optimal code and index give the key till example “j” means Juxtaposing (see below))

n,w	10	11	12	13	14	15	16
20	2.	2	1	1	1	?	?
...
26	2.	2.	2.	2.	2	?	?
27	3. ^j	3. ^j	3. ^j	3. ^j	3	?	?
28	3.	3.	3.	4. ^j	4. ^j	?	?
...
69	18. ^{z18}	?	?	?	?	?	?
70	21. ^{z18}	?	?	?	?	?	?
n,w	10	11	12	13	14	15	16

2 The Package “Constant Weight Codes”

The package ” Constant Weight Codes” is a file written in MATHEMATICA and will be read into MATHEMATICA with the commands.

```
In[1] := <<ConstantWeightCodes`
```

The package consists of two parts: one part with Table of Lower bounds on $A(n, d, w)$ based on the Sloane’s table with explanations

```
In[2] := LowerboundsCWC[[26,12,10]]
Out[2]=33
```

We can find information about this lower bound using command

```
In[3] := LowerboundsCWC[[26,12,10,information]]
Out[3]= K. J. Nurmela, M. K. Kaikkonen and P. R. J.
Östergård, New constant weight codes..., IEEE Trans.
Info. Theory 43 (1997), 1623-1630.
```

and the second part contains commands describing the classical upper bounds of constant weight codes. The information on the list of possible upper bounds can be received using the command `?*Bound*`.

```
In[4] := ?*Bound*
Out[4]=
BergerBound JohnsonBound SecondJohnsonBound LPBound ...
```

The complete information about a command is received by using the command `? Name`.

```
In[5] := ? LPBound
Out[5]= LPBound [n,d,w] give Linear programming upper
bound ( Ph. Delsarte ) on the size for constant weight
code n-length of code, d- code distance, d is even
number and w- weight of the code.
```

We can see also as far as the code (see In[2], In[3], lower bound) differs from known upper bounds

```
In[6] := FreimanBound[26,12,10] BergerBound[26,12,10]
Out[6]= 195 189
```

If the number of code vectors on a code coincides with one of the upper bound this give, that the code is optimal code

```
In[7] := OptimalCWC[[58,18,10]] OptimalCWC[[70,18,10]]
Out[7]= ? 21
```

We also have an opportunity to make changes in a package, in case of occurrence of new codes with parameters it is better known

```
In[8] := LowerboundCWC[[58,18,10]]=12;
OptimalCWC[[58,18,10]]=12;
```

```
LowerboundCWC[[58,18,10,information]]= "Gashkov I. Some
optimal constant weight code, The Karlstad workshop on
Applied mathematics; September 7-11, 2003";
```

```
DeleteFile["LowerboundCWC", "OptimalCWC"]; Save[Lowerbound
dCWC", LowerboundCWC]; Save["OptimalCWC", OptimalCWC];
```

Now we will show how, using MATHEMATICA, we can, base on Sloane's table (see above) update this table. Trivial values we can obtain from well knows theorem 1. [2]

Theorem 1.

- a) If d is odd, $A(n, d, w) = A(n, d+1, w)$.
- b) $A(n, d, w) = A(n, d, n-w)$
- c) $A(n, d, w) = 1$ if $2w < d$
- d) If $d = 2w$ then $A(n, d, w) = [n/w]$, where $[n/w]$ is greatest integer less then or equal n/w .
- e) $A(n, 2, w) = \binom{n}{w}$.

By juxtaposing two codes (the method of construction a new codes by placing them side by side) we can obtain lower bound $A(n1 + n2, d1 + d2, w1 + w2) \geq \min\{A(n1, d1, w1), A(n2, d2, w2)\}$. Codes obtain with use juxtaposing we shall denote through $m(n1, d1, w1) + (n2, d2, w2)$ where m is $\min\{A(n1, d1, w1), A(n2, d2, w2)\}$. The command Juxtaposing[...] changes the table of codes supplementing with its codes obtained above by a stated method, in a case if the "j" code has the greater or equal

number of code vectors, than already available in the table. As example we chose the parameters of the table $20 \leq n \leq 70, d = 18$ and $10 \leq w \leq 16$.

```
In[9] := LowerboundsCWC = Juxtaposing[LowerboundsCWC];
```

We chose the parameters of the table $20 \leq n \leq 70, d = 18$ and $10 \leq w \leq 16$.

```
In[10] := n1=20;n2=20;w1=10;w2=16;d=18;
Table[Take[LowerboundsCWC[[n,d]],{w1,w2}],{n,n1,n2}]
Out[10]=
(
  2(2,2,1) + (18,16,9)    2(2,2,1) + (18,16,10)    1    ...    1
  2(2,2,1) + (19,16,9)    2(2,2,1) + (19,16,10)    ...    ...    1
  ...                      ...                      ...    ...    ...
  18 Optimal              42(24,6,4) + (45,12,7)    ...    ...    ...
)
```

We can give the next generation of this table of lower bounds, using the following theorem.

Theorem 2. If $A(n1, 2a, a + b) = M1$ and $A(n2, 2b, a + b) = M2$ then $A(n1 + n2, 2(a + b), \min\{n1, n2\}) \geq \min\{M1, M2\} + 1$

Proof. Let $n1 \geq n2$. We placing two codes $(n1, 2a, a + b)$ and $(n2, 2b, n2 - a - b)$ side-by-side. Using Juxtaposing we obtain code with parameters $(n1 + n2, 2(a + b), n2)$. But we can add to code one binary vector more $v = (0, 0, \dots, 0, 1, 1, \dots, 1)$, where the first $n1$ coordinates are equal 0, the last $n2$ equal 1. We can calculate distance between any vectors belonging to code $(n1 + n2, 2(a + b), n2)$ and vector v . We can calculate this distance as the sum of two distances: the first distance is the distance between an any vector of a code $(n1, 2a, a + b)$ and a zero vector, the second is the distance between an any vector of a code $(n2, 2b, n2 - a - b)$ and a “one “ vector (vector consisting of 1).

Example: Codes with parameters $(6, 4, 3)$ and $(4, 2, 3)$ consist of the same number of vectors.

Code (6, 4, 3)	Code (4, 2, 3)	Code (10, 6, 4)
V1= (1, 1, 1, 0, 0, 0)	(1, 1, 1, 0)=W1	(1, 1, 1, 0, 0, 0, 0, 0, 0, 1)
V2= (0, 0, 1, 1, 1, 0)	(1, 1, 0, 1)=W2	(0, 0, 1, 1, 1, 0, 0, 0, 1, 0)
V3= (0, 1, 0, 1, 0, 1)	(0, 1, 1, 1)=W3	(0, 1, 0, 1, 0, 1, 1, 0, 0, 0)
V4= (1, 0, 0, 0, 1, 1)	(1, 0, 1, 1)=W4	(1, 0, 0, 0, 1, 1, 0, 1, 0, 0)
		(0, 0, 0, 0, 0, 0, 1, 1, 1, 1)

We shall denote such codes through $m(n1, 2a, a + b) \cup (n2, 2b, a + b)$ where m is $\min\{M1, M2\} + 1$. We can transform the table of constant weight codes taking into

account the theorem 2 and we obtain a new codes

- $(13, 6, 8) \cup (20, 10, 8) \rightarrow A(33, 16, 13) \geq 18$
- $(13, 6, 8) \cup (21, 10, 8) \rightarrow A(34, 16, 13) \geq 19$
- $(16, 8, 9) \cup (19, 10, 9) \rightarrow A(35, 18, 16) \geq 17$
- $(13, 6, 9) \cup (23, 12, 9) \rightarrow A(36, 18, 13) \geq 11$

$$\begin{aligned}
 &(13, 6, 9) \cup (24, 12, 9) \rightarrow A(37, 18, 13) \geq 14 \\
 &(8, 2, 6) \cup (31, 10, 6) \rightarrow A(39, 12, 8) \geq 9 \\
 &(14, 6, 9) \cup (25, 12, 9) \rightarrow A(39, 18, 14) \geq 26 \\
 &(14, 6, 9) \cup (26, 12, 9) \rightarrow A(40, 18, 14) \geq 27 \\
 &(14, 6, 9) \cup (27, 12, 9) \rightarrow A(41, 18, 14) \geq 29 \\
 &(11, 2, 8) \cup (57, 14, 8) \rightarrow A(68, 16, 11) \geq 58
 \end{aligned}$$

Above the carried out computer calculations prove, that codes that we obtain using the theorem 2 it is impossible to present as juxtaposing two codes.

2 (2,2,1)+(18,16,9)	2 (2,2,1)+(18,16,10)	1	1	1	1	1
2 (2,2,1)+(19,16,9)	2 (2,2,1)+(19,16,10)	2 (2,2,1)+(19,16,11)	1	1	1	1
2 (2,2,1)+(20,16,9)	2 (2,2,1)+(20,16,10)	2 (2,2,1)+(20,16,11)	2 (2,2,1)+(20,16,12)	1	1	1
2 (2,2,1)+(21,16,9)	2 (2,2,1)+(21,16,10)	2 (2,2,1)+(21,16,11)	2 (2,2,1)+(21,16,12)	2 (2,2,1)+(21,16,13)	1	1
2 (2,2,1)+(22,16,9)	2 (2,2,1)+(22,16,10)	2 (2,2,1)+(22,16,11)	2 (2,2,1)+(22,16,12)	2 (2,2,1)+(22,16,13)	2 (2,2,1)+(22,16,14)	1
2 (2,2,1)+(23,16,9)	2 (2,2,1)+(23,16,10)	2 (2,2,1)+(23,16,11)	2 (2,2,1)+(23,16,12)	2 (2,2,1)+(23,16,13)	2 (2,2,1)+(23,16,14)	2 (2,2,1)+(23,16,15)
2 (2,2,1)+(24,16,9)	2 (2,2,1)+(24,16,10)	2 (2,2,1)+(24,16,11)	2 (2,2,1)+(24,16,12)	2 (2,2,1)+(24,16,13)	2 (2,2,1)+(24,16,14)	2 (2,2,1)+(24,16,15)
3 (3,2,1)+(24,16,9)	3 (3,2,1)+(24,16,10)	3 (3,2,1)+(24,16,11)	3 (3,2,1)+(24,16,12)	3 (3,2,1)+(24,16,13)	3 (3,2,1)+(24,16,14)	3 (3,2,1)+(24,16,15)
3 (3,2,1)+(25,16,9)	3 (3,2,1)+(25,16,10)	3 (3,2,1)+(25,16,11)	4 (4,2,1)+(24,16,12)	4 (4,2,2)+(24,16,12)	4 (4,2,3)+(24,16,12)	3 (3,2,1)+(25,16,15)
3 (3,2,1)+(26,16,9)	3 (3,2,1)+(26,16,10)	3 (3,2,1)+(26,16,11)	4 (4,2,1)+(25,16,12)	4 (4,2,1)+(25,16,13)	4 (4,2,2)+(25,16,13)	4 (4,2,3)+(25,16,13)
3 (3,2,1)+(27,16,9)	3 (3,2,1)+(27,16,10)	5 (10,6,4)+(20,12,8)	5 (10,6,4)+(20,12,9)	5 (10,6,4)+(20,12,10)	6 (10,6,5)+(20,12,10)	5 (10,6,4)+(20,12,12)
3 (3,2,1)+(28,16,9)	3 (3,2,1)+(28,16,10)	5 (10,6,4)+(21,12,8)	5 (4,2,2)+(27,16,11)	6 (7,4,3)+(24,14,11)	6 (4,2,2)+(27,16,13)	6 (4,2,2)+(27,16,14)
3 (3,2,1)+(29,16,9)	4 (4,2,1)+(28,16,10)	5 (5,2,1)+(27,16,11)	6 (7,4,3)+(25,14,10)	7 (7,4,3)+(25,14,11)	8 (4,8,7)+(18,10,8)	8 (4,8,7)+(18,10,9)
3 (3,2,1)+(30,16,9)	4 (4,2,1)+(29,16,10)	6 (7,4,3)+(26,14,9)	7 (7,4,3)+(26,14,10)	9 (15,8,6)+(18,10,8)	11 (11,6,5)+(22,12,10)	11 (11,6,5)+(22,12,11)
4 (4,2,1)+(30,16,9)	4 (4,2,1)+(30,16,10)	6 (4,2,2)+(30,16,10)	8 (8,4,3)+(26,14,10)	10 (11,6,5)+(23,12,9)	12 (15,8,7)+(19,10,8)	15 (15,8,7)+(19,10,9)
4 (4,2,1)+(31,16,9)	5 (5,2,1)+(30,16,10)	7 (7,4,3)+(28,14,9)	10 (15,8,6)+(20,10,7)	12 (16,8,6)+(19,10,8)	16 (16,8,6)+(19,10,9)	17 New(16,8,9)U(19,10,8)
4 (4,2,1)+(32,16,9)	6 (6,2,1)+(30,16,10)	9 (12,6,4)+(24,12,8)	11 New(13,6,9)U(23,12,9)	16 (16,8,6)+(20,10,8)	17 (17,8,6)+(19,10,9)	22 (12,6,6)+(24,12,10)
4 (4,2,1)+(33,16,9)	6 (6,2,1)+(31,16,10)	9 (12,6,4)+(25,12,8)	14 New(13,6,9)U(24,12,9)	17 (17,8,6)+(20,10,8)	22 (12,6,6)+(25,12,9)	24 (13,6,6)+(24,12,10)
4 (4,2,1)+(34,16,9)	7 (7,2,5)+(21,10,6)	10 (13,6,4)+(25,12,8)	16 (16,8,6)+(22,10,7)	18 (13,6,5)+(25,12,9)	25 (13,6,6)+(25,12,9)	28 (10,4,4)+(28,14,12)
4 (4,2,1)+(35,16,9)	7 (7,2,5)+(22,10,6)	13 (13,6,4)+(26,12,8)	16 (16,8,6)+(23,10,7)	26 New(14,6,9)U(25,12,9)	28 (14,6,5)+(25,12,10)	30 (16,8,8)+(23,10,8)
5 (5,2,1)+(35,16,9)	8 (12,6,4)+(28,12,7)	13 (13,6,4)+(27,12,8)	17 (17,8,6)+(23,10,7)	27 New(14,6,9)U(26,12,9)	28 (14,6,5)+(26,12,10)	38 (20,8,6)+(20,10,10)
5 (5,2,1)+(36,16,9)	8 (13,6,4)+(28,12,7)	14 (14,6,4)+(27,12,8)	20 (16,6,4)+(25,12,9)	29 New(14,6,9)U(27,12,9)	39 (14,6,6)+(27,12,9)	39 (14,6,6)+(27,12,10)
5 (5,2,1)+(37,16,9)	9 (18,8,5)+(24,10,6)	16 (20,8,5)+(22,10,7)	21 (18,8,6)+(24,10,7)	39 (15,6,5)+(27,12,9)	39 (14,6,6)+(28,12,9)	45 (19,8,7)+(23,10,9)
5 (5,2,1)+(38,16,9)	9 (12,6,4)+(31,12,7)	16 (20,8,5)+(23,10,7)	24 (19,8,6)+(24,10,7)	39 (15,6,5)+(28,12,9)	42 (15,6,5)+(28,12,10)	52 (19,8,7)+(24,10,9)
5 (4,2,2)+(40,16,8)	10 (19,8,5)+(25,10,6)	20 (21,8,5)+(23,10,7)	28 (19,8,6)+(25,10,7)	39 (16,6,5)+(28,12,9)	48 (16,6,5)+(28,12,10)	56 (20,8,7)+(24,10,9)
6 (6,2,1)+(39,16,9)	12 (19,8,5)+(26,10,6)	21 (21,8,5)+(24,10,7)	28 (19,8,6)+(26,10,7)	40 (20,8,6)+(25,10,8)	56 (21,8,6)+(24,10,9)	72 (20,8,7)+(25,10,9)
6 (6,2,1)+(40,16,9)	13 (20,8,5)+(26,10,6)	21 (21,8,5)+(25,10,7)	28 (19,8,6)+(27,10,7)	48 (21,8,6)+(25,10,8)	56 (21,8,6)+(25,10,9)	80 (20,8,7)+(26,10,9)
6 (6,2,1)+(41,16,9)	14 (20,8,5)+(27,10,6)	23 (23,8,5)+(24,10,7)	36 (20,8,6)+(27,10,7)	54 (21,8,6)+(26,10,8)	72 (22,8,6)+(25,10,9)	91 (21,8,7)+(26,10,9)
6 (6,2,1)+(42,16,9)	16 (20,8,5)+(28,10,6)	24 (24,8,5)+(24,10,7)	37 (20,8,6)+(28,10,7)	56 (21,8,6)+(27,10,8)	78 (20,8,7)+(28,10,8)	118 (21,8,7)+(27,10,9)
7 (7,2,1)+(42,16,9)	16 (20,8,5)+(29,10,6)	25 (19,8,6)+(30,10,6)	37 (21,8,6)+(28,10,7)	66 (22,8,6)+(27,10,8)	78 (21,8,7)+(28,10,8)	120 (21,8,7)+(28,10,9)
7 (7,2,1)+(43,16,9)	20 (21,8,5)+(29,10,6)	28 (19,8,6)+(31,10,6)	39 (20,8,6)+(30,10,7)	77 (22,8,6)+(28,10,8)	78 (22,8,7)+(28,10,8)	132 (23,8,7)+(28,10,9)
7 (7,2,1)+(44,16,9)	21 (21,8,5)+(30,10,6)	31 (20,8,6)+(31,10,6)	40 (20,8,6)+(31,10,7)	77 (23,8,6)+(28,10,8)	91 (25,8,6)+(26,10,9)	152 (23,8,7)+(28,10,9)
8 (8,2,1)+(44,16,9)	21 (21,8,5)+(31,10,6)	31 (20,8,6)+(32,10,6)	42 (21,8,6)+(31,10,7)	78 (24,8,6)+(28,10,8)	100 (25,8,6)+(27,10,9)	132 (24,8,7)+(28,10,9)
8 (8,2,1)+(45,16,9)	23 (23,8,5)+(30,10,6)	31 (21,8,6)+(32,10,6)	42 (22,8,6)+(31,10,7)	78 (25,8,6)+(28,10,8)	118 (26,8,6)+(27,10,9)	132 (25,8,7)+(28,10,9)
9 (9,2,1)+(45,16,9)	24 (24,8,5)+(30,10,6)	31 (22,8,6)+(32,10,6)	42 (23,8,6)+(31,10,7)	78 (26,8,6)+(28,10,8)	130 (26,8,6)+(28,10,9)	167 (22,8,7)+(32,10,9)
?	30 (25,8,5)+(31,10,6)	35 (21,8,6)+(35,10,6)	42 (25,8,6)+(31,10,7)	78 (27,8,6)+(28,10,8)	130 (28,8,6)+(28,10,9)	198 (23,8,7)+(33,10,9)
?	30 (25,8,5)+(32,10,6)	35 (22,8,6)+(35,10,6)	42 (26,8,6)+(31,10,7)	?	?	232 (23,8,7)+(34,10,9)
?	31 (27,8,5)+(31,10,6)	35 (23,8,6)+(35,10,6)	42 (27,8,6)+(31,10,7)	?	130 (26,8,6)+(32,10,9)	253 (23,8,7)+(35,10,9)
?	31 (27,8,5)+(32,10,6)	38 (15,6,5)+(44,12,7)	42 (14,6,6)+(45,12,7)	?	130 (26,8,6)+(33,10,9)	253 (23,8,7)+(36,10,9)
?	31 (28,8,5)+(32,10,6)	42 (15,6,5)+(45,12,7)	45 (15,6,6)+(45,12,7)	?	130 (26,8,6)+(34,10,9)	254 (25,8,7)+(35,10,9)
?	31 (29,8,5)+(32,10,6)	45 (16,6,5)+(45,12,7)	45 (16,6,6)+(45,12,7)	78 (13,2,2)+(48,16,12)	130 (26,8,6)+(35,10,9)	257 (26,8,7)+(35,10,9)
?	31 (27,8,5)+(35,10,6)	45 (17,6,5)+(45,12,7)	45 (17,6,6)+(45,12,7)	78 (13,2,2)+(49,16,12)	130 (26,8,6)+(36,10,9)	273 (27,8,7)+(35,10,9)
?	33 (28,8,5)+(35,10,6)	45 (18,6,5)+(45,12,7)	48 (15,6,6)+(48,12,7)	78 (13,2,2)+(50,16,12)	130 (11,2,3)+(52,16,12)	278 (27,8,7)+(36,10,9)
?	34 (29,8,5)+(35,10,6)	48 (16,6,5)+(48,12,7)	56 (15,6,6)+(49,12,7)	91 (14,2,3)+(50,16,12)	137 (32,8,6)+(32,10,9)	296 (28,8,7)+(36,10,9)
?	35 (30,8,5)+(35,10,6)	48 (16,6,5)+(49,12,7)	56 (16,6,6)+(49,12,7)	100 (15,2,2)+(50,16,12)	137 (32,8,6)+(33,10,9)	300 (29,8,7)+(36,10,9)
?	37 (22,6,4)+(44,12,7)	56 (17,6,5)+(49,12,7)	56 (17,6,6)+(49,12,7)	100 (15,2,2)+(51,16,12)	150 (33,8,6)+(33,10,9)	320 (30,8,7)+(36,10,9)
11 (11,2,1)+(56,16,9)	38 (23,6,4)+(44,12,7)	56 (18,6,5)+(49,12,7)	56 (18,6,6)+(49,12,7)	105 (15,2,2)+(52,16,12)	150 (33,8,6)+(34,10,9)	327 (30,8,7)+(37,10,9)
12 (12,2,1)+(56,16,9)	40 (23,6,4)+(45,12,7)	56 (19,6,5)+(49,12,7)	57 (11,4,5)+(57,14,8)	120 (16,2,2)+(52,16,12)	163 (34,8,6)+(34,10,9)	362 (31,8,7)+(37,10,9)
18 Optimal z18	42 (24,6,4)+(45,12,7)	56 (20,6,5)+(49,12,7)	57 (12,4,5)+(57,14,8)	130 (17,2,2)+(52,16,12)	163 (34,8,6)+(35,10,9)	372 (32,8,7)+(37,10,9)
21 Optimal z18	45 (25,6,4)+(45,12,7)	57 (13,4,4)+(57,14,8)	60 (22,8,6)+(48,10,7)	130 (17,2,2)+(53,16,12)	178 (35,8,6)+(35,10,9)	403 (32,8,7)+(38,10,9)

In particular, codes that were obtain using the theorem 2 is close to optimal codes. For instance

$$\begin{aligned}
 28 &\geq A(35,18,16) \geq 17; \quad 12 \geq A(36,18,13) \geq 11 \\
 15 &\geq A(37,18,13) \geq 14; \quad 39 \geq A(39,18,14) \geq 26
 \end{aligned}$$

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