An Application to the Treatment of Geophysical Images through Orthogonal Projections

Sixto Romero and Francisco Moreno

Escuela Politécnica Superior La Rábida, Ctra. Palos de la Frontera s/n, 21819 Palos de la Frontera, Huelva, Spain
sixto@uhu.es, franmo@uhu.es

Abstract. The present work provides the generalization to the approach proposed by Frei and Chen to square masks of any dimension for line and edge detection in digital images. It is completed with the application of the designed algorithm to the image of a archaeological site, that to our judgement permit us to establish an correlation between the mathematical results and results of the archaeologicals explorations

1 Introduction

When we try to extract information from an image whose definition does not allow to recognize lines, edges, or isolated points, the first step to take is segmentation. This process consists in dividing the image into its parts. The result will depend on the treatment given to the analysed image.

Segmentation algorithms for monochrome images [6] are based on the two basic properties of grey-level values: discontinuity and similarity. In the first case the image partition is based on abrupt changes in grey levels and is used for the location of isolated points, lines and edges [5]. In the second case, thresholding, region growing, and region splitting and merging is used. For the detection of the three basic types of discontinuities, isolated points, lines and edges, we will use the usual way of applying a suitable mask. This technique consists in treating each pixel of the original image and creating a new image. To do this, using 3×3 masks as an example, we will change the center pixel grey level, which matches the central cell of the mask, following the equation [3]

$$R = p_1 z_1 + p_2 z_2 + \cdots + p_9 z_9 \tag{1}$$

This pixel is assigned the grey level given by R. $p_i,\ i = 1 \ldots 9$ represent the coefficients according to the mask type and $z_i,\ i = 1 \ldots 9$, represent grey levels of the pixels that make up the nine cells, according to Fig. [1]

2 Using Multimasks for Line and Edge Detection

We will focus on 3× 3 masks for line and edge detection. Let's consider the chosen pixel and the eight encompassing pixels as a nine-component vector representing the nine grey levels

$$\mathrm{z} = (z_1, z_2, z_3, \cdots, z_9)^T$$

M. Bubak et al. (Eds.): ICCS 2004, LNCS 3039, pp. 213–220, 2004.

p_1	p_2	p_3
p_4	p_5	p_6
p_7	p_8	p_9

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

Fig. 1. A 3×3 mask

whose component z_5 represents the grey level value for that pixel and another vector, also with nine components, representing the mask coefficients

$$\mathrm{p} = (p_1, p_2, p_3, \cdots, p_9)^T$$

In a matrix form, [1] can be written

$$R = \mathrm{p}^T \mathrm{z}. \quad \mathrm{p}, \mathrm{z} \in \mathbb{R}^9$$

By properly choosing the mask coefficients so that they represent orthogonal vectors we can split up our 9-dimensional vector space into three 4-, 4- and 1-dimensional orthogonal subspaces. This approach was first proposed by Frei and Chen [8] in 1977. The first two subspaces will be called edge and line subspaces, and the last one will be called measure subspace. Frei and Chen suggested that a probability measure [4] for a pixel to belong to an edge or a line could be given by the value of the angle forming vector z, which represents each pixel, with its orthogonal projection onto each subspace. The smaller the angle, the closest the vector to its corresponding subspace and thereby the most likely that the vector belongs to the subspace. Starting from this proposal, our work will consist in calculating the projection matrix which generalizes the masks usually used by researchers. If we treat initially the masks proposed by Frei and Chen as orthogonal vectors for each subspace, and therefore basis, and if we have into account that their components sum up zero, we can choose as a basis vector for the measure subspace a vector whose components are all equal to the unit and therefore orthogonal to the remaining eight, given that we use the usual scale product. The bases for the orthogonal edge and line subspaces are given by the mask coefficients in Fig. 2, where the first four are $\mathrm{p}_1, \mathrm{p}_2, \mathrm{p}_3, \mathrm{p}_4$, suitable for edge detection; the next four $\mathrm{p}_5, \mathrm{p}_6, \mathrm{p}_7, \mathrm{p}_8$ are appropriate for line detection, and the last one, u, is added to complete a basis of the vector space $\mathbb{R}^9$.

p_1

1	$\sqrt{2}$	1
0	0	0
−1	$-\sqrt{2}$	−1

p_2

1	0	−1
$\sqrt{2}$	0	$-\sqrt{2}$
1	0	−1

p_3

0	−1	$\sqrt{2}$
1	0	−1
$-\sqrt{2}$	1	0

p_4

$\sqrt{2}$	−1	0
−1	0	1
0	1	$-\sqrt{2}$

p_5

0	1	0
−1	0	−1
0	1	0

p_6

−1	0	1
0	0	0
1	0	−1

p_7

1	−2	1
−2	4	−2
1	−2	1

p_8

−2	1	−2
1	4	1
−2	1	−2

u

1	1	1
1	1	1
1	1	1

Fig. 2. Orthogonal masks suggested by Frei and Chen

2.1 Projection Matrices for 3×3 Grids

According to Frei and Chen's approach, we consider edge, line and measure subspaces formed respectively by vectors of Fig. [2], $\mathcal{B} \equiv \{p_1, p_2, p_3, p_4\}$, $\mathcal{L} \equiv \{p_5, p_6, p_7, p_8\}$ y $\mathcal{U} \equiv \{u\}$. We calculate the projection matrices on each one of them, $P_{\mathcal{B}}$, $P_{\mathcal{L}}$ y $P_{\mathcal{U}}$. These matrices will be obtained through the matrix product, for each of them, given by

$$\begin{aligned} P_{\mathcal{B}} &= B(B^T B)^{-1} B^T \\ P_{\mathcal{L}} &= L(L^T L)^{-1} L^T \\ P_{\mathcal{U}} &= U(U^T U)^{-1} U^T \end{aligned} \qquad (2)$$

where the columns of these matrices represent the vectors of each subspace. For $n = 9$, the following result is obtained

$$P_{\mathcal{B}} = (b_{ij}) = \begin{cases} b_{ii} = \frac{1}{2} & \forall i \neq \frac{n+1}{2} \\ b_{i,n-i} = -\frac{1}{2} & \forall i \neq \frac{n+1}{2} \\ b_{\frac{n+1}{2},\frac{n+1}{2}} = 0 & \\ b_{ij} = 0 & \text{otherwise} \end{cases}$$

$$P_{\mathcal{L}} = (l_{ij}) = \begin{cases} b_{ii} = \frac{7}{18} & \forall i \neq \frac{n+1}{2} \\ f_{i,n-i} = -\frac{7}{18} & \forall i \neq \frac{n+1}{2} \\ f_{\frac{n+1}{2},\frac{n+1}{2}} = \frac{8}{9} & \\ f_{ij} = -\frac{1}{9} & \text{otherwise} \end{cases}$$

$$P_{\mathcal{U}} = (u_{ij}) \qquad / \qquad u_{ij} = \frac{1}{9} \quad \forall i,j = 1 \cdots n.$$

If we use the respective $3, 5, 7, \cdots,$ dimensional square masks, the results are generalized giving rise to vector spaces of dimensions equal to the square of those numbers: $9, 25, \cdots$.

3 Projection Matrices on Edge and Line Subspaces

We will prove the aforementioned generalization with the two following theorems.

3.1 Projection Matrix onto the Edge Subspace

Theorem 31 *Let E, $\dim E = n$, $n = 2k + 1$, $k \in \mathbb{N}$ be an Euclidean space and let $\mathcal{B} \subset E$ be the k-dimensional vector subspace, generated by the array $\langle \boldsymbol{b}_1, \boldsymbol{b}_2, \cdots, \boldsymbol{b}_k \rangle$ of basis vectors of $\mathcal{B}$, whose symmetric components are opposed and their sum is zero. In these circumstances, the projection matrix onto the $\mathcal{B}$ subspace is the matrix $P_{\mathcal{B}} \in \mathcal{M}_n(\mathbb{R})$ given by the following expression*

$$P_{\mathcal{B}} = (b_{ij}) = \begin{cases} b_{ii} = \frac{1}{2} & \forall i \neq \frac{n+1}{2} \\ b_{i,n-i} = -\frac{1}{2} & \forall i \neq \frac{n+1}{2} \\ b_{\frac{n+1}{2},\frac{n+1}{2}} = 0 & \\ b_{ij} = 0 & \text{otherwise} \end{cases} \qquad (3)$$

Proof. The projection matrix onto the vector subspace $P_{\mathcal{B}}$ is, according to 2, $P_{\mathcal{B}} = B(B^TB)^{-1}B^T$ where the columns of the B matrix are the vectors $\langle \boldsymbol{b}_1, \boldsymbol{b}_2, \cdots, \boldsymbol{b}_k \rangle$. Being a projection matrix, it will follow

$$P_{\mathcal{B}}B = B$$

and then

$$P_{\mathcal{B}}B - I_nB = \Theta \Rightarrow (P_{\mathcal{B}} - I_n)B = \Theta$$

thereby B is the matrix of eigenvectors corresponding to the eigenvalue $\lambda = 1$ of the $P_{\mathcal{B}}$ matrix. It is known from linear algebra that, according to the spectral theorem [1], every symmetric matrix is orthogonally and reciprocally diagonalizable. It is proved that the $\lambda = 1$ eigenvalue, of k algebraic multiplicity, matches the k eigenvectors which are precisely the array $\{\boldsymbol{b}_1, \boldsymbol{b}_2, \cdots, \boldsymbol{b}_k\}$. In order to prove this, we just need to solve the equation

$$(P_{\mathcal{B}} - I_n)\boldsymbol{b} = \mathbf{0}$$

the result is an undetermined system of $k+1$ equations whose solutions are the mentioned vectors with the structure suggested in theorem 31, i.e., the opposed symmetric components and the sum of them all, which is null, given that the center component is null -the number of components is odd.

3.2 Projection Matrix onto the Line Subspace

Theorem 32 *Let E, $\dim E = n,\ n = 2k+1,\ k \in \mathbb{N}$ be an Euclidean space and let $\mathcal{L} \subset E$ be the k dimensional vector subspace, generated by the array $\langle \boldsymbol{l}_1, \boldsymbol{l}_2, \cdots, \boldsymbol{l}_k \rangle$ of basis vectors of $\mathcal{B}$, whose symmetric components are opposed and their sum is zero. In these circumstances the projection matrix onto the $\mathcal{L}$ subspace is the matrix $P_{\mathcal{L}} \in \mathcal{M}_n(\mathbb{R})$ given by the following expression*

$$P_{\mathcal{L}} = (l_{ij}) = \begin{cases} l_{ii} = \frac{n-2}{2n} & \forall i \neq \frac{n+1}{2} \\ l_{i,n-i} = -\frac{n-2}{2n} & \forall i \neq \frac{n+1}{2} \\ l_{\frac{n+1}{n},\frac{n+1}{n}} = \frac{n-1}{n} & \\ l_{ij} = -\frac{1}{n} & \text{otherwise} \end{cases} \tag{4}$$

Proof. The projection matrix onto the vector subspace $P_{\mathcal{L}}$ is, according to 2, where the columns of the L matrix are vectors $P_{\mathcal{L}} = L(L^TL)^{-1}L^T$. Being a projection matrix, it will follow

$$P_{\mathcal{L}}L = L$$

and then

$$P_{\mathcal{L}}L - I_nL = \Theta \Rightarrow (P_{\mathcal{L}} - I_n)L = \Theta$$

thereby L is the matrix of eigenvectors corresponding to the eigenvalue $\lambda = 1$ of the matrix $P_{\mathcal{L}}$. According to the spectral theorem [1], we know that every symmetric matrix is orthogonally and reciprocally diagonalizable. It is proved that the $\lambda = 1$ eigenvalue, of k algebraic multiplicity, matches the k eigenvectors which are precisely the array $\{\boldsymbol{l}_1, \boldsymbol{l}_2, \cdots, \boldsymbol{l}_k\}$. In order to prove this, we just need to solve the equation

$$(P_{\mathcal{L}} - I_n)\boldsymbol{l} = \mathbf{0}$$

the result is an undetermined system of $k + 1$ equations whose solutions are the mentioned vectors with the structure suggested in theorem 32, i.e., the opposed symmetric components are equal and the sum of them all is null, therefore the center component is equal and opposed to the sum of the remaining components.

4 Algorithm for Line and Edge Detection

In the Frei and Chen's approach it is use the specifics masks and we prove, 31 and 32, that the masks aren't important, well the projections matrices only depend to the mask dimension that we can to use. The proposed algorithm can be summed up in the following steps:

Step 1. Reading of the image and calculation of the average of typical deviations for each pixel and their neighbors, according to the dimensions of the selected grid. For the computational expense not to be excessively high, a random number of the image pixels are chosen, $\pm 0.02\%$ approximately.

Step 2. Each pixel is read and its typical deviation and that of their neighbor is made, according to the dimensions of the selected grid, and this is compared to the average obtained previously and multiplied by a factor chosen at will. This way we decide whether the pixel is a candidate to be an edge pixel, a line pixel or a uniform region pixel.

Step 3. Finally, with the candidates to be edge or line pixels, and according to the approach suggested by Frei and Chen, we calculate the norms of their projections, and comparing both of them we decide whether they are edge or line pixels[2].

4.1 Application of the Algorithm to Determine Lines and Edges in a Geophysical Image

The suggested algorithm will allow to obtain lines and edges on a digital image without modifying the rest of the image. We apply masks of any dimension, odd and larger than one, and we will obtain different results depending on our interest. The Fig. 3 a) show the original image obtained with program PROSPECT [7] in graphic format standard: BMP, JPG, GIFF, ... susceptible of the studied by any commercial software implemented in MAPLE or MATLAB, for example.

5 Conclusions

We think that the more important conclusions are: is not necessary to use any mask and the original image remain the same except the lines and edges. For streets images, highways, flat, buildings, cadasters, face photographs of persons, animal and things, it is relatively easy the edges detection. When it is considered to study a geophysical image the problem adopts a high degree of complexity. The present work shows a particular application in the one which is obtained a map from anomalies distribution electrical corresponding to the site from Cerro El Palmarón. With the different applied models have been detected important alignments related to the walls of the constructions of the period. It is provided, from the sight point of the images interpretation a structural anomalies plan that it has served as guide for the ulterior excavation. In this way the DIP represents an important tool for the historical restitution of the cited site.

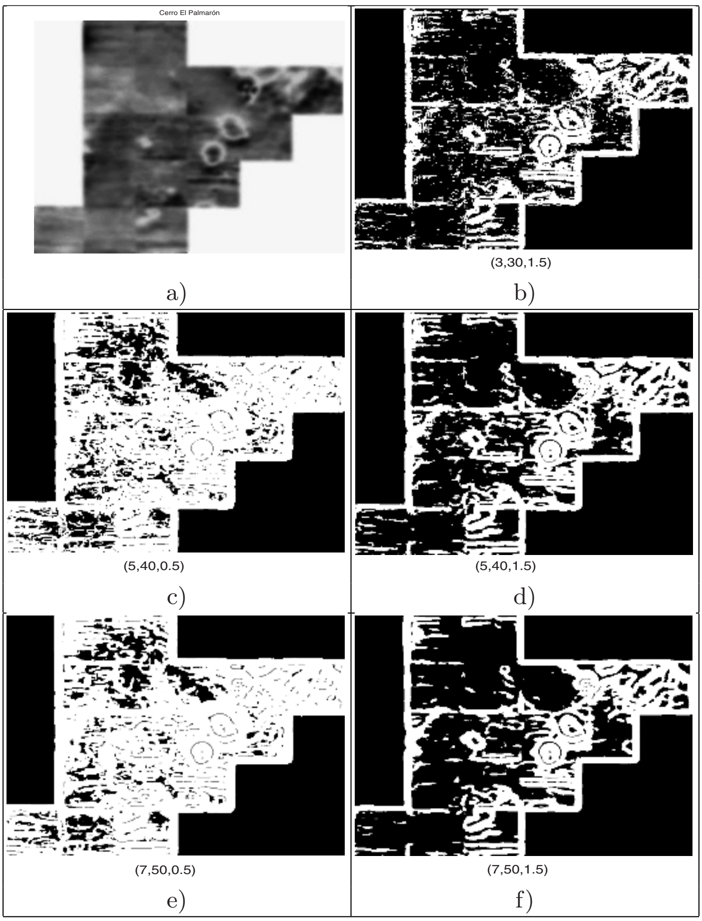

Fig. 3. a) Original image. b)c)d)e) y f) Images transformed for detections edges and lines

References

1. Noble B. , Daniel Janes W.: Applied Linear Algebra. Prentice-Hall, New York (1982)
2. Burton H. Wiejak, J.S.: Convolution with separable mask for early image processing. Computer Vision, Graphics and Image Processing **32** (1985) 279-290
3. González C.Rafael, Woods Richard E.:Digital Image Processing. Addison-Wesley, Massachusetts (1992)
4. Park D.J.: Edge detection in noisy images based on the co-ocurrence matrix. Pattern Recognition **27** (1994) 765–774
5. Gómez Lopera J.F.: An analysis of edge detection by using the jensen-shannon divergence . Journal of Mathematical Imaging and Vision **13** (2000) 35–56
6. Pratt William K.: Digital Image Processing. John Wiley & Sons, INC, New York (2001)
7. Romero S.: Modelización Matemática y Tratamiento Digital de Imágenes Geofísicas Aplicadas a la Restitución Histórica: Programa PROSPECT. IGIDL-Lisboa **1** (2000) 273–274
8. Frei W., Chen C.C..: Fast Boundary Detection: A Generalization and a New Algorithm. IEEE Trans. Computer **26** (1977) 988-998