An Effective Modeling of Single Cores Prostheses Using Geometric Techniques

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Abstract. There has been a great demand for the artificial teeth prostheses that are made of materials sintered at 1500^oC, such as Zirconia ceramic. It is, however, very difficult for dental technicians to manually fabricate the prostheses with such materials since their degree of strength is very high. For that reason, the prostheses of strong materials have been fabricated by using CNC (computer numerical control) machines. The most important subject in the CNC fabrication is how accurately the prostheses can be modeled in three-dimensional space according to dentists' requirements. In this paper, we propose effective methods for modeling single cores, such as single caps and Conus cores, which are the principal ones of the artificial teeth prostheses. Our approach employs the 2D Minkowski sum, a developed ZMap algorithm, and other geometric techniques such as the tiling of contours. We also present and analyze the 3D visual examples of the single cores modeled by the proposed methods.

1 Introduction

Computers have been diversely applied to the area of dental surgery. Some examples are the digitalization of dental data, the 3D visualization of dental information, the automatic fabrication of artificial teeth using the CAD/CAM, and the measurement of accuracy during impression or die castings [2,3,5,9,10,14,15]. The reconstruction of artificial teeth prostheses is one of the most important processes in dental treatment. Most of the prostheses are still fabricated manually by dental technicians, but the manual labor causes various problems. The accuracy of the prostheses depends on the skills of dental technicians. Sometimes the prostheses cannot be fabricated within a limited time since the manual fabrication takes long time. Furthermore, there has lately been an increasing demand for prostheses that are made of materials sintered at 1500^oC, such as Zirconia ceramic, which cannot be handled by human hands. In order to resolve these problems, CAD/CAM techniques began to be applied to the fabrication systems of the prostheses [5,15].

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In general, the fabrication of artificial teeth prostheses is composed of four successive processes. First, a plaster cast is fabricated to make a pattern for a patient's teeth. This process is the same as the one that has been performed in the previous dental surgery, while the following other processes can be performed by using CAD/CAM techniques. Second, the 3D surface of the teeth is modeled by scanning the fabricated plaster cast with 3D scanners. Third, exploiting the 3D information of the teeth model, they design the 3D models that exactly correspond to the prostheses satisfying dentists' requirements. Finally, the designed models of prostheses can be fabricated by cutting ceramic materials with milling machines. When artificial teeth prostheses are fabricated, the most important subject is how accurately the prostheses can be modeled according to dentists' requirements. For the accurate modeling of the prostheses, we may be able to use the existing 3D modeling tools such as 3DMax, Maya, CATIA, and SolidWorks. But it may be impossible or it takes too long to model the prostheses with the existing tools so that the required conditions can be satisfied. Hence, we need to develop a dedicated 3D modeling tool for the efficient and accurate modeling of the prostheses.

Artificial teeth prostheses are composed of *core* prostheses and *crown* prostheses [13]. The crown prostheses are visible when the artificial teeth prostheses are put in. On the other hand, the core prostheses directly touch the original teeth ground by dentists, and the important function of them is to increase the adhesive strength between the original teeth and the crown prostheses. A *single core* means the prosthesis that is composed of one piece, and the typical examples are single caps and single Conus cores. In this paper, we first analyze the requirements that are necessary for modeling the prostheses of single cores, and then propose the methods for effectively modeling the single cores using geometric techniques [1,8] such as 2D Minkowski Sum, a developed ZMap algorithm, and the tiling of contours. We also provide and analyze the practical examples of the single cores modeled by using our techniques.

2 Requirements Analysis

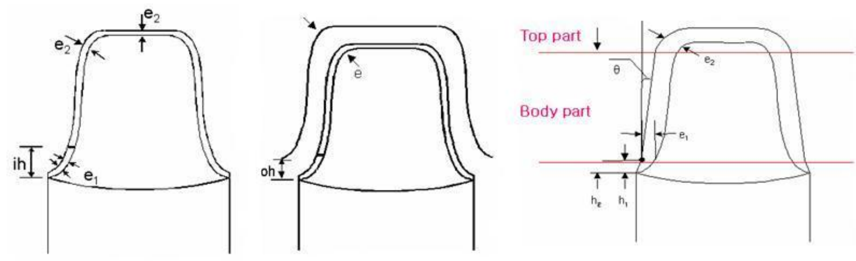
If there are any problems with a tooth due to decay, non-vitality, etc., a dentist grinds the original tooth, designs artificial tooth prostheses, and then fabricates the prosthesis put in above the ground tooth called the *abutment*. When a dentist grinds the original tooth, she/he makes the shape of an abutment so that the artificial teeth prosthesis can normally be mounted on it. The prosthesis is modeled after the coping design so that it is secure and it adheres to the abutment. A result of the coping design is just the single cap. The surfaces in both sides of the single cap adhere to the abutment and the crown, respectively. The surface adhering to the abutment is called the *inner surface*, while the surface adhering to the crown is called the *outer surface*.

With the sectional diagrams of a single cap, we explain the requirements for single caps in the area of prosthodontics. Fig. 1(a) illustrates the values that are required for guaranteeing the adhesive strength of the inner surface of the single cap. The inner surface is determined by expanding the surface of the abutment; the one part within a fixed height ih from the margin is expanded as much as a particular value e_1, while the other part is expanded as much as another particular value e_2. Fig. 1(b) illustrates the values that are required for the outer surface of the single cap. The outer surface is determined by expanding a part of the inner surface; the part above a fixed height oh from the margin is also expanded as much as a particular value e. The space between abutments may be wide when a large number of teeth are lost and only a few original teeth survive. The wide space causes the surviving original tooth to sustain a great load and the abutments are easily damaged. An alternative plan for resolving the problem was proposed by Korber [6], which is a method of dental treatment using the new prosthesis called the *Conus*. The Conus is designed to avoid the damage to the abutments and to sustain the load additionally with the mucosa of the mouth. The Conus core has an inner surface and an outer surface, as a single cap has the two surfaces. The inner surface should well adhere to the abutments. Hence, the inner surfaces of the Conus cores can be modeled by expanding the surface of the abutment the same as those of single caps were done: the one part within a fixed height h from the margin and the other parts expanded as much as particular values respectively. The outer surface of a single Conus core should be able to control the power sustaining the crown, and it has to be completely fixed at the time of the Conus's function. It also should be able to make the surviving teeth almost never get the stress. The values for satisfying these requirements are illustrated in Fig. 1(c), which is the sectional diagram of a Conus core. For determining the outer surface, the one part of the abutment within a fixed height h_1 from the margin is increasingly expanded to a particular value e_1, and then another part between the heights h_1 and h_2 from the margin is modeled as a conic type. The conic angle θ of the conic type is very important due to separation and adhesion between Conus cores and crown, which is recommended as 4^o, 6^o, and 9^o in the area of prosthodontics [13]. These two parts are defined as the *body part*, while the remaining part above the body part is defined as the *top part*. The outer surface corresponding to the top part is also determined by expanding the inner surface as much as a particular value e_2 similarly to the one of a single cap.

3 Modeling of Single Cores and Its Examples

3.1 The Inner Surface of Single Cores

For designing the inner surface of the single core for a given abutment, we have to first scan the given abutment impression, and then extract the tooth model lying above the margin from the scanned 3D data. The 3D scanners of the touch type were used for the scanning, and we choose a small value $0.05mm$ as input intervals on X- and Y-axes for guaranteeing the accuracy of the 3D data. In general, the tooth model has the same characteristics as a *terrain model* has since the abutment is scanned by the 3D scanner of touch type. Hence, the modeling of the inner surfaces of single cores can be transformed into the



(a) (b) (c)

Fig. 1. Designing requirements (a) the inner surfaces of a single core (b) the outer surfaces of a single cap (c) the outer surfaces of a single Conus cap

problem of expanding terrain models as much as given fixed values. In other words, the modeling can be formulated as the problem of Minkowski sum [1] in the plane.

MinkowskiSum$(\mathcal{T}, e) = \{\bar{v} + \bar{w} \parallel \bar{v} \in \mathcal{T},\ \bar{w} \in \mathcal{S}\}$, where $\mathcal{S}$ is an arbitrary sphere with radius e, whose center exists on the surface of $\mathcal{T}$. In our case of modeling the inner surfaces of a single core, e_1 is assigned to e if the height value for a point v is less than a given value h. Otherwise, we assign e_2 to e.

The Minkowski sum of a general model in 3D space can be solved by applying the results of Lozano-Perez [7] or Ghosh [4]. Lozano-Perez decomposes an original object into convex objects, and computes convex Minkowski sums for each pair of the convex objects. As a result, the original problem is represented with the union of all the convex Minkowski sums, but it is difficult to accurately implement the decomposition and union operations for this method. Assuming that the intersection of line/line, plane/line, and plane/plane can be exactly computed, Ghosh proposed an algorithm for computing the Minkowski sum of a general model by using the unions of face/face, face/edge, face/vertex and edge/edge Minkowski sums. This method is numerically error-prone when floating point arithmetic is used as well as it takes heavy load in the union operations.

In order to make the problems simpler, we review the characteristics of the tooth model and its inner surface in detail. In short, let $\mathcal{M}$ and $\mathcal{TM}$ represent a detected margin and an extracted tooth model lying above the margin respectively. For increasing the adhesive strength of a single core to be designed, its inner surface is determined by expanding the one part of $\mathcal{TM}$ within the height $0.5mm$ from $\mathcal{M}$ as much as $0.0120mm$, and expanding the other part as much as $0.0125mm$. From the observation that the expanding values are much smaller than the input interval $0.05mm$ on X- and Y-axes, we are going to solve the problem of modeling the inner surface by transforming it into the problem of 2D Minkowski sum [1]. Without the loss of generality, a given $\mathcal{TM}$ can be regarded as a terrain located at XY plane of the 3-axis coordinate system. After determining the minimum and maximum values of Y coordinates of $\mathcal{TM}$, we construct the 2D polygonal lines that have the X and Z coordinate values sequentially in the input interval $0.05mm$ on Y-axis. Geometrically speaking, the polygonal lines are the intersection of the tooth surface and the planes that are parallel with XZ plane. Clearly, the end points of the 2D polygonal lines are on $\mathcal{M}$. Before applying the Minkowski sum to the constructed polygonal lines, a particular contour $\mathcal{C}$ of $\mathcal{TM}$ has to be computed to lie at a fixed height h from $\mathcal{M}$. We can obtain $\mathcal{C}$ by simultaneously traversing the edges and vertices of $\mathcal{M}$ and $\mathcal{TM}$. Then $\mathcal{TM}$ can be divided into two parts with $\mathcal{C}$, which are expanded by e_1 and e_2 respectively, as discussed in Sect. 2. For efficiency, we determine where the vertices of $\mathcal{TM}$ lie between the two parts of $\mathcal{TM}$ during computing $\mathcal{C}$. After completing the vertex marking, we process the Minkowski sum operation with respect to the constructed polygonal lines.

3.2 The Outer Surfaces of a Single Cap

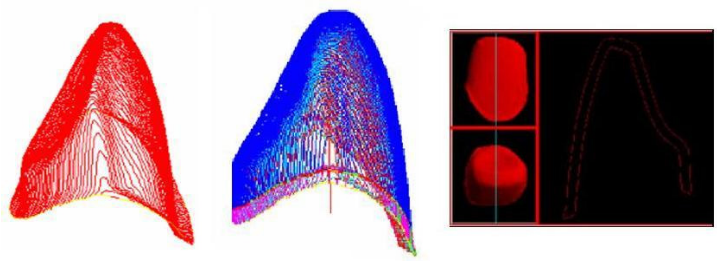
The outer surface of a single cap is modeled by expanding a part of $\mathcal{TM}$ above a fixed height h as much as a particular value e. If we apply the techniques in Sect. 3.1 to this model, the expansion does not occur in the points that have the minimum and maximum values of Y-coordinate. This is because the outer surface to be modeled includes the part of $\mathcal{TM}$ above the height h. Furthermore, since the expansion value $0.4mm$ is relatively large with respect to the input interval $0.05mm$ on X- and Y-axes, the polygonal lines obtained by applying the 2D Minkowski sum may be composed of the vertices that have the intervals greater than $0.05mm$.

For avoiding the limitation caused by a large expansion value, we develop a ZMap algorithm that can be applied to the part of $\mathcal{TM}$ above the height h. Our ZMap algorithm performs a geometric transformation for the set Vtx of all original vertices of $\mathcal{TM}$ above the height h. For each vertex $\bar{v} = (x, y, z) \in Vtx$, it builds the sphere with the origin $\bar{v}$ and the radius e, and then it finds the set $Vtx^n(\bar{v})$ of vertices that are contained in the sphere. For each vertex $\bar{v}^n = (x^n, y^n, z^n) \in Vtx^n(\bar{v})$, we calculate the Z coordinate value zz such that the point (x^n, y^n, zz) is on the sphere. Comparing the value z^n with the calculated value zz, it is replaced with the value zz if only $z^n < zz$.

Fig. 2(a) shows a set of polygonal lines of the tooth model lying above the margin that was detected by Yoo's algorithm [12]. Fig. 2(b) is the inner surface and the outer surface obtained from the given model, where the fixed height is $0.5mm$, and the expansion values below and above the height are $0.012mm$ and $0.0125mm$ respectively. Since a tooth model has the characteristics of a terrain, it is clear that the intersection takes the shape of a polygon called a cross-sectional polygon. Fig. 2(c) presents the cross-sectional polygon of the single cap modeled like Fig. 2(b) and the XZ-plane containing the center of the margin.

3.3 The Outer Surfaces of a Single Conus Core

The outer surface of a single Conus core was divided into the top part and body part as defined in Sect. 2. The top part can be modeled by employing the ZMap algorithm in Sect. 3.2, but the body part has to be modeled carefully



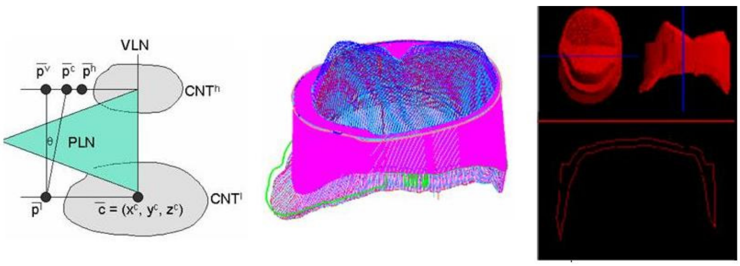
(a) (b) (c)

Fig. 2. An example of modeling a single cap (a) an original tooth model above a detected margin (b) a single cap model with the height $0.5mm$ and the two expansion values $0.012/0.0125mm$ below/above the height respectively (c) the visualization of a cross-sectional polygon

for satisfying the condition of a given conic angle. We develop a method for constructing the surface of the body part with 3D contours, which is known as the *tiling* problem. Fig. 3(a) shows the techniques of treating the body part for a given conic angle θ. First we determine the center $\bar{c} = (x^c, y^c, z^c)$ of $\mathcal{M}$, and the vertical line VLN passing the point $\bar{c}$. Next we extract the 3D contours of $\mathcal{TM}$ at the points of the fixed heights h_1 and h_2. The extracted contours are denoted by $\mathcal{C}^l$ and $\mathcal{C}^h$ which are called the *low region* and the *high region* of the body part respectively. The two contours $\mathcal{C}^l$ and $\mathcal{C}^h$ are, respectively, expanded as much as the values that dentists recommend, which were represented as e_1 and e_2 in Fig.3(a). After defining the plane PLN defined by $\bar{c}$, VLN, and a vertex on $\mathcal{M}$, we find the intersection point $\bar{p}^l$ ($\bar{p}^h$) between PLN and the expansion of $\mathcal{C}^l$ ($\mathcal{C}^h$). Let $\bar{p}^v$ be the point that has X/Y coordinates of $\bar{p}^l$ and Z coordinate of $\bar{p}^h$. Then, the line determining the conic angle θ from $\bar{p}^l$ will intersect the line segment between $\bar{p}^v$ and $\bar{p}^h$. This intersection point is represented as $\bar{p}^c$ in Fig. 3(a), and the intersection is iteratively computed around 360^o by rotating PLN with a fixed angle about the axis of VLN. Finally, we can describe an algorithm for modeling the body part by triangulating the line segments that connect all the found points $\bar{p}^l$, $\bar{p}^c$, and $\bar{p}^h$.

If we apply the modeling techniques in the above with the same conic angle to every region of $\mathcal{TM}$, however, the thickness of the modeled Conus will not be uniform since the inclined angle of $\mathcal{TM}$ is varied. In this paper, we divide the surface of an abutment into four regions by considering separation and adhesion between Conus cores and crown. The four conic angles which may be different from each other are assigned to each of the four regions. The assignment of different conic angles may cause another problem in the surface of the modeled Conus. A folding phenomenon may occur in the polygonal lines connecting the set $\{\bar{p}^c_i\}$ of intersection points that are obtained with different conic angles. Hence, this phenomenon will occur also on the surface of the body part that is formed with the point sets $\{\bar{p}^l_i\}$, $\{\bar{p}^c_i\}$, and $\{\bar{p}^h_i\}$. In order to resolve the folding phenomenon, we employ the B-Spine approximating curve algorithm to the consecutive points p_i^c for all $i = 1, \cdots, n$. The B-Spline approximating curve is iteratively generated until the folding phenomenon disappears [8,11].

Fig. 3(b) illustrates the single Conus core modeled for an abutment, where the fixed height is $0.3mm$, the conic angles in four regions are 10^o, 2^o, 2^o and 4^o, and the height of the high region of the body part is $0.6mm$. Fig. 3(c) shows the cross-sectional polygon generated by intersecting the modeled single Conus core with the XZ-plane containing the center of the margin.



(a) (b) (c)

Fig. 3. An example of modeling a single Conus core (a) the body part of a single Conus core (b) the single Conus model with two height values $0.3/0.6mm$ and four conic angles $10^o/2^o/2^o/4^o$ (c) the visualization of a cross-sectional polygon

4 Conclusion

This paper analyzed the requirements for modeling single caps and single Conus cores, which are the mostly used artificial teeth prostheses. Based on the analyzed requirements, we developed geometric algorithms for effectively modeling the prostheses. We adopted 2D Minkowski sum to modeling the inner surfaces of single cores, while a ZMap algorithm was developed for modeling the outer surfaces of single caps. The outer surfaces of single Conus cores were modeled by using the combination of the Minkowski sum of contours and the tiling of the expanded contours. We also presented the examples of the single cores modeled through simple interactions. In the future, it is required to develop more techniques for the efficient modeling of other prostheses such as bridge cores, single crowns, bridge crowns, etc.

References

1. Berg M.D., Kreveld W.V., Overmars M., Schwarzkopf O., Computational Geometry - Algorithms and Applications, Springer-Verlag, (1997).
2. Besimo C., Jeger C., Guggenheim R., Marginal Adaptation of Titanium Frameworks produced by CAD/CAM Techniques, The International Journal of Prosthodontics **10**, 6, (1997), 541-546.

3. Duret F., Blouin J.L., Duret B., CAD/CAM in dentistry, Jorunal Am. Dent. Assoc.**117**, 11, (1988), 715-720.
4. Ghosh P., A Unified Computational Framework for Minkowski Operations, Computer and Graphics **17**, 4, (1993), 357-378.
5. Gurke S., Generation of Tooth Models for Ceramic Dental Restorations, The 4th International Conference on Computer Integrated Manufacturing, Singapore, (Oct 1997).
6. Korber K.H., Konuskronen, Das rationelle Teleskopsystem, Einfuhrung Inklinik und Technik, Auflage, Huthig, Heidel-berg, (1983).
7. Lozano-Perez T., Spatial Planning: A Configuration Space Approach, IEEE Transaction on Computers **32**, 2, (1983), 108-120.
8. Meyers D., Skinner S., and Sloan K., Surfaces from Contours, ACM Transactions on Graphics **11**, 3, (1992), 228-258.
9. Rekow D. E., The Minnesota CAD/CAM System DentiCAD, Technical report, University of Minnesota, (1989).
10. Rekow D. E., CAD/CAM in Dentistry:Critical Analysis of Systems, In Computers in Clinical Dentistry, Quintessence Co. Inc., (1991), 172-185.
11. Yamagichi F., Curves and Surfaces in Computer Aided Geometric Design, Springer-Verlag, (1988).
12. Yoo K.H., An Efficient Algorithm for Extracting the Margin of a Tooth, Manuscript, (2003).
13. Yoon C.G., Kang D.W., Chung S.M., State-of-arts in Fixed Prosthodontics, Jongii Press, Korea, (1999).
14. Willer J., Rossbach A., and Weber H., Computer Assisted Milling of Dental Restorations using a new CAD/CAM Data Acquisition System, The International Journal of Prosthodontics **80**, 3, (1988) 346-353.
15. Jef M. van der Zel, Ceramic-fused-to-metak Restorations with a new CAD/CAM system, Quintessence International **24**, 11, (1993) 769-778.