Chapter 3
Reflexive Measurement

The vision that self-reflexivity may, through self-intervention amounting to paradoxical situations, impose some restrictions on the performance and the capacity of physical agents to know their own states, is a challenging one. It has continued to present a source of inspiration.

Before beginning a brief review of the subject, let me recall an anecdote of Bocca della Verità, the Mouth of Truth, a marble mask in the portico of Rome’s Santa Maria in the Cosmedin church. According to Rucker’s own account [437, p. 178], “Legend has it that God himself has decreed that anyone who sticks a hand in the mouth slot and then utters a false statement will never be able to pull the hand back out. But I have been there, and I stuck my hand in the mouth and said, “I will not be able to pull my hand back out.” (May God forgive me!)”

3.1 General Framework

In a very similar manner as discussed earlier in Chap. 2 one can identify $S$ with measurements $M$, and $P$ with the set of possible outcomes $O$ of these measurements. Alternatively, one may associate a physical state with $P$.

For the sake of construction of a “non-measurable self-inspection” relative to all operational capacities let us again closely follow the scheme involving the non-existence of fixed points. In particular, let us assume that it is not possible to measure properties without changing them. This can be formalized by introducing a disturbance function $\delta : O \rightarrow O$ without a fixed point, such that, for all $o \in O$, $\delta(o) \neq o$. Then we may construct a non-operational measurement $u : M \rightarrow O^M$ by forming

$$u(m) = \delta(g(m, m)),$$

with $g(m, m) = [f(m)](m)$.

Again, because, in a proof by contradiction, suppose that some operational measurement $h$ could express $u$; that is, $u(m_1) = h(m_1, m_2)$. But then, by identifying
$m = m_1 = m_2$, we would obtain $h(m, m) = \delta(h(m), m)$, thereby again clearly contradicting our definition of $\delta$.

In summary, there is a limit to self-inspection, as long as one deals with systems of sufficiently rich phenomenology. One of the assumptions has been that there is no empirical self-exploration and self-examination without changing the sub-system to be measured. Because in order to measure a subsystem, one has to interact with it; thereby destroying at least partly its original state. This has been formalized by the introduction of a “diagonal-switch” function $\delta : P \rightarrow P$ without a fixed point.

In classical physics one could argue that, at least in principle, it would be possible to push this kind of disturbance to arbitrary low levels, thereby effectively and for all practical purposes (fapp) eliminating the constraints on, and limits from, self-observation. One way of modelling this would be a double pendulum; that is, two coupled oscillators, one of them (the subsystem associated with the “observed object”) with a “very large” mass, and the other one of them (the subsystem associated with the “observer” or the “measurement apparatus”) with a “very small” mass.

In quantum mechanics, unless the measurement is a perfect replica of the preparation, or unless the measurement is not eventually erased, this possibility is blocked by the discreteness of the exchange of at least one single quantum of action. Thus there is an insurmountable quantum limit to the resolution of measurements, originating in self-inspection.

### 3.2 Earlier and More Recent Attempts

Several authors have been concerned about reflexive measurements, and, in particular, possible restrictions and consequences from reflexivity. Their vision has been to obtain a kind of inevitable, irreducible indeterminism; because determinate states might be provable inconsistent.

Possibly the earliest speculative note on intrinsic limits to self-perception is obtained in von Neumann’s book on the Mathematical Foundations of Quantum Mechanics, just one year after the publication of Gödel’s centennial paper [242] on the incompleteness of formal systems. Von Neumann notes that [554, Sect. 6.3, p. 438] “...the state of information of the observer regarding his own state could have absolute limitations, by the laws of nature.”¹ It is unclear if he had recursion theoretic incompleteness in mind when talking about “laws of nature.” Yet, von Neumann immediately dismissed this idea as a source of indeterminism in quantum mechanics and rather proceeded with the value indefiniteness of the state of individual parts of a system – comprising the object and the measurement apparatus combined – in (what Schrödinger later called) an entangled state (cf. the later Sects. 12.8 and 12.10).

¹German original [552, Sect. 6.3, p. 233], “...die Informiertheit des Beobachters über den eigenen Zustand könnte naturgesetzliche Schranken haben.”.
Probably the next author discussing similar issues was Popper who, in a two-part article on *indeterminism in quantum physics and in classical physics* [416, 417] mentions that, like quantum physics, even classical physics “knows a similar kind of indeterminacy, also due to ‘interference from within.’”

In what follows I shall just cite a few later attempts and survey articles with no claim of completeness. Indeed many authors appears to have had similar ideas independently; without necessarily being aware of each other. This is then reflected by a wide variety of publications and references. Many of the following references have already been discussed and listed in my previous reviews of that subject [499, 516].

Zwick’s *quantum measurement and Gödel’s proof* [595], cites, among other authors, Komar [317] and Pattee [384, p. 117] (cf. the quote on p. 4) as well as Lucas [345] (although the latter did only discuss related issues regarding minds-as-machines).

According to his own draft notes written on a TWA in-flight paper on Feb. 4–6, 1974 [568] Wheeler imagined adding “‘participant’ to ‘undecidable propositions’ to arrive at physics.” Alas, by various records (cf. from Bernstein [59, p. 140–141] and Chaitin [499, p. 112], including this Author’s private conversation with Wheeler), Gödel himself has not been very enthusiastic with regards to attempts to relate quantum indeterminism, and, in particular, with regards to quantum measurements, with logical incompleteness.

More recently, Breuer has published a series of articles [72–74] on the impossibility of accurate self-measurements. Lately Mathen [356, 357] as well as Szangolies [525, 526] have taken up this topic again.