Chapter 16
Classical Continua and Infinities

The physical theories of classical mechanics, electrodynamics and gravitation (relativity theory) have been developed alongside classical analysis. Thereby assumptions about the formal mathematical models for theoretical physics had to be made which were partly (to some degree of accuracy) corroborated empirically; and partly a mere convenience.

In particular, classical continuum physics employed mathematical objects – the continuum of real and complex numbers – which, from a logical, recursion theoretic, and algorithmic point of view, has turned out to be highly nontrivial, to say the least. For instance, as is argued in the Appendix A, with probability one, an arbitrary real number turns out to be incomputable, and even algorithmically incompressible – that is, random. Stated differently, almost all elements of a continuum are not attainable by any operational physical process. They require unlimited (in terms of computation space, time et cetera) resources.

When contemplating the use of nonconstructive means for physical models, two questions are imminent:

(i) Are these nonconstructive continuum models a faithful representation of the physical systems in the sense that they do not *underrepresent* – that is, do they systems? include and comprise essential operational features of these physical systems.

(ii) Are these nonconstructive continuum models a faithful representation of the physical systems in the sense that they do not overrepresent; that is, that they do not introduce entities, properties, capacities and features which have no correspondence in the empirical data? If they allege and suggest capacities – such as irreducible randomness and computability beyond the universal Turing-type – can these capacities be utilized and (technologically) harvested for "supertasks" [53, 187, 188, 188, 352, 530] which go beyond the finite capacities usually ascribed to physical systems?

(iii) What kind of verification, if any at all, can be given for nonconstructive means? The term "(non)constructive" is used here in its metamathematical meaning [63, 77, 78, 354].

K. Svozil, *Physical (A)Causality*, Fundamental Theories of Physics 192,
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Pointedly stated, if some theory is the *double* of some physical system (or *vice versa* [21]) we have to differentiate between properties of the theory and properties of the physical system. And we have to make sure that we do not over-represent physical facts by formalisms which contain elements which have no correspondence to the former. Because if we are not careful enough we fall pray of Jaynes' *Mind Projection Fallacy* mentioned in Sect. 9.5 (p. 42).

Another issue is the applicability of mathematical models or methods which somehow implicity or explicitly rely on infinities. For instance, Cantor's diagonalization technique which is often used to prove the undenumerability of the real unit interval relies on an infinite process [79] which is nonoperational. Again the issue of supertasks mentioned earlier arises. It may not be totally unjustified to consider the question of whether or not theoretical physics should allow for such infinities unsettled. The issue has been raised already by Eleatic philosophy [253, 331, 440], and may be with us forever.