μ-Synthesis Control of a Seismic Excited Building

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Abstract. In structural earthquake engineering, the earthquake protection of structures has considerably increased these last three decades since structural control appears as an efficient way for the energy dissipation due to dynamic loads. A number of papers investigated this field proving the potential of the active control to realize a significant reduction of the structures' response that conduct to an important rising of the human safety and seismic structures protection. However, the several control approaches applied did not automatically consider the uncertainties that exist in the realistic case. That means that there are some variations in the coefficients of the structure model as mass, stiffness or damping and even in the structure dynamics which should be tacked into account since it could affect the controller design. The motivation of this study is to perform the μ-synthesis, a robust control successfully applied in many other fields, for its ability to explicitly include all these aspects in the same time in the control design procedure. The quantification of the uncertainties as well as the interpretation of the performance requirements in appropriate weighting functions are one of the great advantages of this robust control technique. Hence, this paper presents through simulations the response of a three floors building structure subjected to a seismic excitation, modeled by Kanai Tajimi Filter, where the active component consists in an active bracing system (ABS) attached to the second floor. A robust controller is designed with considering parametric and dynamic uncertainties (structured uncertainties). The designed μ-controller demonstrates its efficiency to considerably attenuate the building response by providing a significant disturbance reduction over low control effort energy when considering the parametric and dynamic (structured) uncertainties acting simultaneously in the control system. The robustness of the controller is evaluated by varying the considered uncertainties to their worst case variation.

1 Introduction

The structural active control represents a field in permanent evolution such it knows these last few years the investigation of several papers using a new generation of controllers based on robust control methods as LQG, H_∞, adaptive control,…etc. [1–

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5]. Indeed, the earthquake protection of structures still constitutes a serious problem that looks for a way more and more efficient to dissipate the energy produced by a seismic load and raise the human safety and the structures protection even in presence of eventual uncertainties. This conducted the researchers to design controllers able to maintain the performance in the real conditions of implementation which implied to include in the control design procedure the maximum of practical considerations and model variations that often occur simultaneously. This means to take into account various aspects as the effect of sensors noise, time delay (induced by a mechanical force), physical actuator limits, and also errors in the parameters and dynamics of the structure and actuator models. Hence, the motivation of this work is to use the μ-synthesis, a robust control strategy, to explicitly incorporate the described aspects as robustness in stability and performance requirements. Based on the structured singular value ssv technique and the H_∞ bound computations, this control allows accounting explicitly for robustness to dynamic uncertainties, in the present case modeling errors in the actuator dynamic and to parametric uncertainties as, in the considered case, variations in mass stiffness and damping coefficients of the second floor of the structure model. In addition to robustness considerations, the μ-synthesis problem formulation poses the performance objectives as minimizing norm of some weight transfer functions [4, 5]. However, the quantification of the uncertainties as well as the interpretation of the performance requirements in appropriate weighting functions represent the critical step in the problem formulation of this multivariable control technique [6]. To evaluate the efficiency of the designed μ-controller, simulations are conducted on a building structure of three floors where the active component consists in an active bracing system (ABS) attached to the second floor. The seismic excitation is modelled by Kanai Tajimi filter attacked by a white noise [7, 8]. The resulting controller achieves closely similar performance in nominal and worst case of uncertainties variation and presents a great benefit of costing low control energy. However, the μ-synthesis technique presents the convenient of generating too high order controllers. By using a model reduction method as the balanced realization algorithm, a lower order controller is provided to achieve the same level of robust performances.

2 μ-Synthesis Control Theory

The μ-synthesis is a robust control technique since the formulation of its law explicitly takes into account the uncertainties in the system. It is based on the combination of the application of the μ-analysis which is in fact a stability robustness criterion, in one hand and H_∞ bound computations of some weight functions representing the performance specifications and limitations in another hand. The μ-synthesis is derived so that the algorithm called "*D-K* iterations" constitutes the effective mean to the computation of the controller. To truly model uncertainties, one should distinguish them from their origins. This leads to classify them into: parametric (real) uncertainty and dynamic (frequency dependent) uncertainty [4, 5]. The first class encompasses the uncertainties in system physical parameters such as: time constant, natural frequency…etc., in the structure case it is question of the mass, stiffness and damper. While the second one refers to those met by simplifying a complex model or in system neglected high frequency dynamics. There is an interesting formulation of uncertainties called structured uncertainties and formed by the combination of a block of real uncertainties and another of dynamic uncertainties. This formulation is particularly adopted in μ-synthesis theory because it is possible to make a link between the uncertainties and the physical system by means of the computation of a very useful tool called structured singular value μ which can be used in both stability analysis and synthesis of the control law.

Most uncertain systems are represented in the Linear Fractional Transformation (LFT) representation to extract explicitly the structured uncertainties (parametric and dynamic) from the system in a set called $\Delta(s)$ block. The LFT representation of Fig. (1) is adopted because it offers a prior knowledge of the studied process, indeed it reflects how the block $\Delta(s)$ of structured uncertainties affects the transfer matrix $M(s)$ representing the feedback structure of the system. An efficient way to study the robustness in stability and performance of any uncertain system consists to calculate its structured singular value (ssv) μ defined as

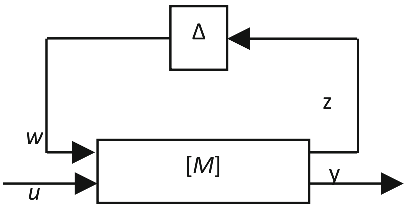

Fig. 1. Upper LFT representation performance

$$\mu_{\Delta}(M) = \frac{1}{\min_{\Delta \in \Delta}\{\bar{\sigma}(\Delta) : \det(I - M\Delta) = 0\}} \tag{1}$$

where,
Δ(s) represents the structured uncertainties block; $\bar{\sigma}$ is the maximum singular value; $M(s)$ is the transfer matrix of the feedback structure system and $s(= j\omega)$ is the Laplace variable in the frequency domain.

This is called μ-analysis and in fact, is just a Multi Input Multi Output (MIMO) extension of the Nyquist Stability Criterion in a Robust Stability Condition [9]. Hence, the system of Fig. (1) is internally stable for any structured Δ, with $\|\Delta\|_{\infty} \leq 1$, if and only if

$$\det(I - M(j\omega)\Delta(j\omega)) \neq 0, \forall\omega\forall\Delta \in \Delta, \|\Delta\|_{\infty} \leq \Leftrightarrow \mu(\mathrm{M}(j\omega)) : = \frac{\frac{1}{min\{\bar{\sigma}(\Delta(j\omega)) : \det(I - M(j\omega)\Delta(j\omega)) = 0\}}}{\Delta \in \Delta} < 1, \forall\omega \tag{2}$$

This being, the concept of the μ-synthesis technique is in fact very simple and tends to use the ssv μ_{Δ} such that it consists in designing a controller to a feedback uncertain system, as represented in Fig. (2) below, by combining the H_∞ control design and the diagonal scaling techniques from the ssv μ. Therefore, the parametric and non parametric uncertainties, modelled by the Δ block, are shaped under a specific form called *structured uncertainties* and are hence bounded as H_∞ norm of uncertain gains representing different perturbations acting in one or more inputs or outputs of the controlled system. Thus, the ssv μ constitutes the appropriate tool for analyzing the robustness (both stability and performance) of the system with the uncertainties hence structured though the μ *analysis* procedure and the control design will directly seek to minimize the μ value which represents the principal actor in the μ*-synthesis approach*.

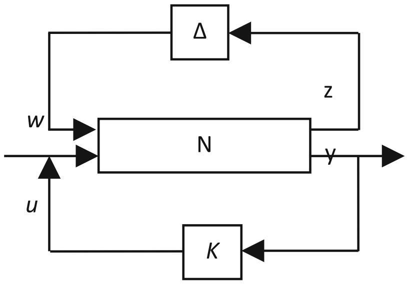

Fig. 2. Structure for controller synthesis

For $M = \mathcal{F}_\ell(N, K)$ a complex matrix, we can find a matrix D such as

$$\mu_\Delta(M) \leq \min_{D \in \mathcal{D}} \bar{\sigma}\left(DMD^{-1}\right) \tag{3}$$

with, $$\mathcal{D} = \left\{ \begin{bmatrix} d_1 I_{r1} & & & & & \\ & \ddots & & & & \\ & & d_s\, I_{rs} & & & \\ & & & \mathcal{D}_1 & & \\ & & & & \ddots & \\ & & & & & \mathcal{D}_F \end{bmatrix}_{\substack{D_i \in \mathbb{C}^{r_i \times r_i} \\ D_i = D_i^* > 0 \\ d_j \in \mathbb{R} \\ d_j > 0}} \right\}$$ N is the uncertain system, and K is the controller to be designed.

The μ-synthesis problem consists on designing a nominally stabilizing controller K solving the following minimization problem, which represents the μ upper bound over frequencies

$$\min_K \sup_\omega \min_{D(j\omega) \in \mathcal{D}} \bar{\sigma} D(\mathrm{j}\omega) \mathcal{F}_\ell(N, K)(j\omega) D^{-1}(\mathrm{j}\omega) \tag{4}$$

This problem is equivalent to a minimizing scaled H_∞ norm and can be expressed as

$$\min_K \min_{D, D^{-1} \in \mathcal{R}H_\infty, D(j\omega) \in \mathcal{D}} \left\| D \mathcal{F}_\ell(N, K) D^{-1} \right\|_\infty \tag{5}$$

The cost function Eq. (5) is no convex in regard to D and K, that's why the μ-*synthesis* is executed by means of an algorithm called *D-K iteration* to find a local minimum. Then, the μ*-analysis* procedure can be derived once the controller K is fixed, and constitutes a convex problem. However, if D is fixed the problem becomes convex and we are in face of an H_∞ optimal controller design. If the minimized μ value is less than 1, the obtained K is a robust stabilizing controller. To facilitate computation of μ controllers, an efficient procedure called *D-K iteration*, resumed in the following algorithm, is executed [10].

3 Formulation of the Building Design

In the present study, the system in question consists on a linear building structure subject to horizontal seismic excitation, governed by the equation of motion that can be written as

$$M\ddot{y}(t) + C\dot{y}(t) + K\dot{y}(t) = Hu(t) + E\ddot{u}_g(t) \tag{6}$$

where,

$y(t) = [x_1(t)x_2(t)\ldots x_i(t)]^T$ with $x_i(t)$ is the nth floor relative displacement with respect to ground; $\dot{y}(t)$ and $\ddot{y}(t)$ are the velocity and acceleration respectively; $u(t) = [u_1(t)u_2(t)\ldots u_i(t)]^T$, $u_i(t)$ is the r^{th} control force at the r^{th} floor, $\ddot{u}_g(t)$ is the seismic acceleration; $M, C, K \in \mathbb{R}^{nxn}$ are the mass, damping, and stiffness matrices of the structure, respectively. $H \in \mathbb{R}^{nxr}$ gives the location of the r controllers, and E is a vector denoting the influence of the external excitation namely the seismic solicitation. The location of the device control and the seismic influence are represented by $H = [\,0 \quad 1 \quad 0\,]^T$ and $E = [\,1 \quad 1 \quad 1\,]^T$. To express the Eq. (6) into the state space representation, we use the state vector $x(t) = [\,y(t) \quad \dot{y}(t)\,]$ such as it becomes

$$\dot{x}(t) = Ax(t) + Bu(t) + B_w w(t) \tag{7}$$

where,

A, B and B_w are system matrices defined as follows

$$A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix}, B = \begin{bmatrix} 0 \\ M^{-1}H \end{bmatrix}, B_w = -\begin{bmatrix} 0 \\ M^{-1}E \end{bmatrix},$$

For the numerical simulations, we consider a three-story building structure of one degree of freedom per floor, resulting in a total of a three degree of freedom as it is represented in Fig. (3). This model is chosen because it was the subject of an experimental verification and an active control has been applied to in [1]. The structure is subjected to horizontal seismic excitation producing a maximum energy deformation at the first floor. The active bracing system is connected between the first and the second floor to procure the active force assuring seismic protection. The structure is a building of rectangular shape with a floor area of 4.5 m x 3 m resulting in a total height of 9 m (3 m for each story), the masses from the bottom to top are 1000 kg, the stiffness are also identical for all floors and are equal to 1407 KN/m. The damping in the isolation system is so chosen to provide a damping ratio about 1.5% in each floor; this corresponds to damping coefficients of 1470 N.m/s2. The three natural frequencies are 7.5, 22.5 and 37.5 rd/s for each mode respectively.

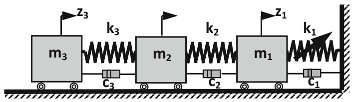

Fig. 3. A three mass-spring-damper system

4 Formulation of the Synthesis Design Problem

The example consists on a three floors building simulated by a three mass-spring-damper system and considers uncertainties in the structure parameter coefficients of the second floor since each variation on these parameters affect directly the two other floors. The variations is about 30% in mass, damper and stiffness of the second floor where the active bracing system (*ABS*) is placed to produce the force controlled by the feedback. As the displacement measurements are not so expensive to achieve than in the last years, they are used in this example for the control feedback.

The control objectives can be summarized as shown in Fig. (4), with two external sources of disturbances:

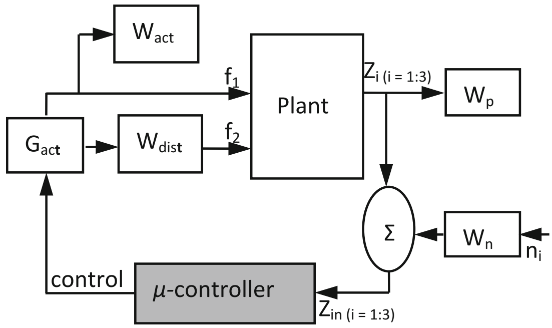

Fig. 4. A diagram of the structure model interconnection with the weighted functions (μ- control design objectives)

– Seismic excitation filter, so called Kanai Tajimi whose output has the frequency peak corresponding to the maximum energy in a set of near fault earthquakes, and input is white noise excitation. Hence f_2 is a disturbance modelled as a normalized signal d and shaped by a weighting transfer function W_{dist} [10].

$$W_{dist} = \frac{\sqrt{S_0} * \left(2 * z_z * s + w_g^2\right)}{s^2 + 2 * z_g * s + w_g^2} \tag{8}$$

where $w_g = 2\pi$ rd/s, $\xi = 0.3$, and $S_0 = \frac{0.03 * z_g}{\left(\pi * w_g * \left(4 * z_g^2 + 1\right)\right)}$ and s is the Laplace variable.

The model inputs are the seismic disturbance, f_2 and actuator force, f_1. The model outputs are the displacements of the three floors z_i avec $i = 1{:}3$. Since the objective is to regulate the fundamental mode response, a first order filter W_p is used in the controller design to weight the output (displacement z_1) of the first floor of the structure as follow

$$W_p = \propto \frac{a}{s+a} = 80\frac{8.5}{s+8.5} \tag{9}$$

with a = 8.5 rd/s determines the roll of frequency.

The dynamic of the ABS actuator used is represented by G_{act}, a first order transfer function modelling the nominal actuator dynamic as follow

$$G_{act} = \frac{K_{act}}{T_{act}s+1} \tag{10}$$

with $K_{act} = a_2$, $T_{act} = c/a_1$.k and $a_1 = 10$, $a_2 = 20$, $k = 1500$ and $c = 30000$.

G_{act} approximates the physical actuator dynamic and the variations between this model and the physical device can be represented as a family of actuator models by considering respectively 10% and 20% errors on K_{act} and T_{act}. These uncertainties are treated as neglected dynamic and modelled as multiplicative uncertainty shape by the weighting function W_{unc} and is obtained by a graphical trial/error approximation

$$W_{unc} = \frac{0.38s - 0.5475}{s+5.475} \tag{11}$$

The control objectives can then be reinterpreted as a disturbance rejection goal, where the impact of the signals of earthquake and noise on the structure is to be minimized. Then, the actual controller problem design that corresponds to the diagram of Fig. 2 can be formulated as

- The controller measures the displacements z_1, z_2, z_3 of the mass m_1, m_2 and m_3 and applies the control force f_1 via the actuator of G_{act} transfer function.
- The seismic disturbance is modelled by Kanai Tajimi second order filter attacked by a white noise dist and shaped by W_{dist} [7, 8]. The normalised disturbance signal f_2 is applied at the system input.
- The performance objective is to attenuate the disturbances by a factor of 80 below the frequency 8.5 rd/s.

5 Results of Numerical Simulations

Using the Matlab software computing [10], we developed a controller on the base of μ synthesis theory with displacement measurements feedback to actively control the described structure. This technique is able to treat the real case of a structure model affected by parametric and dynamic uncertainties commonly named structured uncertainties. The parametric uncertainty considered in this paper concerns stiffness, damper and mass of the second floor where the actuator is placed.

Hence, we treat particularly the parametric uncertainties of this floor by considering 30% in k_2, c_2 and m_2 respectively. The second uncertainty is inherent to the nature of the actuator command. The ABS actuator produces a mechanical control that introduces a non negligible dynamic, which constitutes an important aspect to include in our design to better perform the control of the structure and produce an inaccurate control force. Thus, the possible variations in the actuator model represent a dynamic uncertainty treated as input multiplicative error and modelled by W_{unc}.

The performance objective in the case of structural active control is primary to ensure a good compromise among reducing the response of the structure while limiting the control effort necessary to this aim. The μ-based controller obtained, is checked to verify its efficiency to reach this desired objectives. That tends to determine how large the μ value of the closed loop gain of the transfer from disturbance-to-model outputs (earthquake-to-floors' displacement and acceleration) can get for the specified structure uncertainties and then, verify that the μ bound is less than one to attest of the controller robustness. The same checking process is applied to the gain of the closed loop transfer from disturbance- to- controller output (earthquake-to-command) which relates to actuator control effort. Hence, Figs. (5) and (6) show the time domain representation of the first floor inter-story drift and acceleration of the structure responses in the open loop (without μ-control) and closed loop (with μ-control) respectively. We can easily observe from these figures that the structure responses to seismic load are significantly reduced by the μ-controller; it is about 77% for the inter-story drift with a mu value reaching 0.5 and 41% for the accelerations with mu equal to 1.02 which means that all the performance objectives are achieved. Note that, as a rule of thumb, when μ is near to 1, as it is obtained in the present case, the desired and effective closed loop bandwidths match closed.

For comparison purposes, the structural root mean square (*RMS)* quantities before and after control of the inter-story drifts, and accelerations of the three floors when the system uncertainties are varied to them nominal values obtained by μ-controller in Fig. (7). For validation purposes, he RMS values obtained in [1] are summarised in Table 1 and compared to those obtained by our μ-controller. We can easily observe from these evaluations (figures of time domain and *RMS* values) that the structure responses to seismic load are significantly reduced by the μ controller which provides a large reduction in comparison with [1].

The Fig. (8) shows the time domain response of the first floor inter-story drift in the nominal case and worst case scenario of parameters variation.

The controller achieves closely similar performance in nominal and worst case which traduces a maintaining of performances even in presence of severe degradation in the structure model.

For more detailed evaluation, the μ-value corresponding to how large the gain from disturbance-to-output norm get for the specified system uncertainties is computed. As μ is equal to 1.4, it is steel not far from 1.2 that confirms the robustness of the designed controller.

The μ-controller is also checked on the base of cost of force produced to realize the desired performance presented earlier, to this aim in Fig. (9) we plot the control effort in time domain for nominal uncertainties and we observe a low cost control reaching a maximum of 549 N.

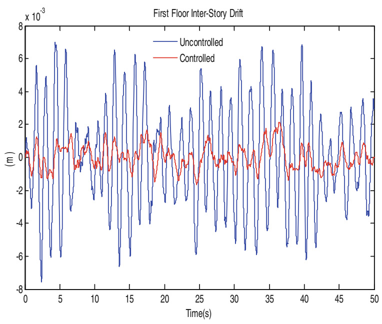

Fig. 5. Time domain representation of the first floor inter-story drift before and after control.

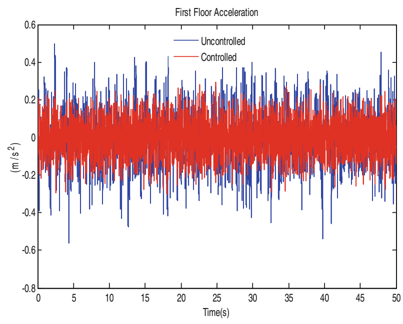

Fig. 6. Time domain representation of the first floor acceleration before and after control.

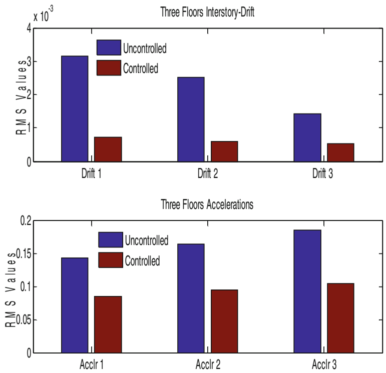

Fig. 7. RMS values of the floors inter-story drifts and accelerations before and after control.

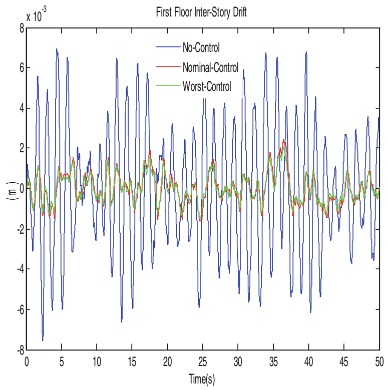

Fig. 8. Time domain representation of the first floor inter-story drift response in the nominal case (Nominal-Control) and worst case (Worst-Control) of uncertainties variation.

In practical point of view, the μ-synthesis technique presents the convenient of generating too high order controllers that can be avoided by using a model reduction realisation to find a lower order controller that achieves the same level of robust performance. Hence, a balanced realisation method is employed to look for a lower order controller without degrading the performance of the initial one. The Fig. (10) shows the robust performance as a function of the μ-controller order, where it is easily shown that from order equal to 15 (order 13 provides also closely similar performances), the reduced controller is able to reach the performance objectives.

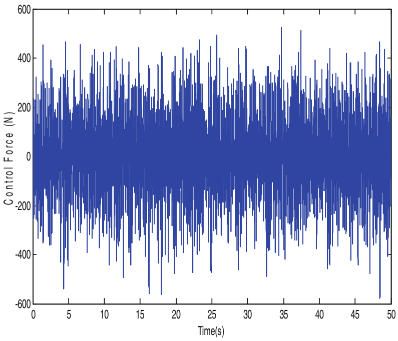

Fig. 9. Control effort produced by μ-controller

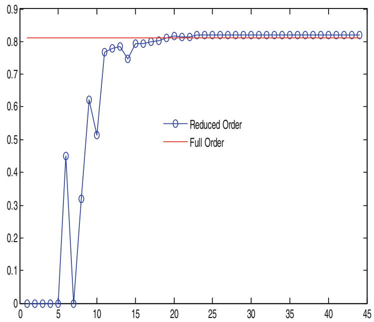

Fig. 10. Approximation of the reduced controllers order by comparing their robust performance margin.

6 Conclusions

This article is devoted to the performance evaluation of a robust control law, known as μ-synthesis, to actively control the response of a civil structure consisting on three stories building exposed to seismic effect and submitted to mixed uncertainties (parametric and dynamic). The obtained simulations results, allow emphasizing the following aspects of the μ-synthesis that further its application in active control structure:

- This control is particularly adapted to consider in the control design, weighting functions able to closely match the desired performance and compromise.
- With the mean of structured uncertainties, the μ-controller takes naturally into account the parametric and dynamic uncertainties affecting the structure, and provides similar vibration attenuation even in the worst case of uncertainties variation.
- This approach shows a great advantage of producing low control energy comparing to several control approaches in literature already performed while providing a significant reduction of structure response, average 70%–78%.

Finally, experimental tests shall be carried out in a future work to certify the actual simulations results and verify the usefulness of the designed μ-controller, which since a long time represents the principal criterion for evaluating the efficiency of an approach.

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