

Chapter 9

Alleys to Be Further Investigated and Open Questions

9.1 Positivity of the Hamiltonian

As we shall argue at length in Part II, it is quite likely that the gravitational force will be a crucial ingredient in resolving the remaining difficulties in the CAT theory. One of the various arguments for this is that gravity is partly based on the existence of local time translations, which are redefinitions of time that depend on the location in space. The generator for these transformations is the Hamiltonian density, which here must be a local operator. At the same time, it is important also for gravity theory to have a lower bound on energy density. Apparently, gravity hinges exactly on those two important demands on the Hamiltonian operator that are causing us some troubles, and so, conceivably, the problem of quantizing gravity and the problem of turning quantum mechanics into a deterministic theory for dynamics, will have to be solved together.

On the other hand one might argue that the non-locality of the quantum-induced Hamiltonian is exactly what we need to explain away Bell's theorem.

The exact position of the gravitational force in our theory is not completely clear. Therefore, one might hope that the inclusion of a gravitational force can be postponed, and that cellular automaton models exist also on flat space–time lattices. In Part II, we shall see that our *PQ* formalism, Chap. 16, allows us to split space-like coordinates into two parts: integers that specify the location of a point on a lattice, and fractional, or periodic, coordinates that could be used to position a point within one lattice cell, or else merely play the role of canonically conjugated variables associated to a discretized momentum variable. Here, however, accommodating for non-compact symmetries such as Lorentz invariance is extremely difficult.

The most obnoxious, recurring question will be that the Hamiltonians reproduced in our models, more often than not, appear to lack a lower bound. This problem will be further studied in Part II, Chaps. 14, 22, and in Sect. 19.1. The properties that seem to raise conflicts when combined, are

1. H must be constant in time:

$$\frac{d}{dt}H = 0, \tag{9.1}$$

2. H must be bounded from below:

$$\langle \psi | H | \psi \rangle \geq 0, \quad (9.2)$$

3. H should be extensive, that is, it must be the sum (or integral) of local terms:

$$H = \sum_{\vec{x}} \mathcal{H}(\vec{x}), \quad (9.3)$$

4. And finally, it must generate the equations of motion:

$$U(t) = e^{-iHt}, \quad (9.4)$$

or equivalently, for all states $|\psi\rangle$ in Hilbert space,

$$\frac{d}{dt} |\psi\rangle = -iH|\psi\rangle. \quad (9.5)$$

In standard quantum theories, one starts with just any classical model, and subjects it to the well-known procedure of “quantization”. One then usually has no difficulty finding an operator H that obeys these conditions. However, when we start with a classical model and search directly for a suitable Hamiltonian operator generating its evolution law (9.5), then it will rarely obey both (9.2) and (9.3).

All these equations are absolutely crucial for understanding quantum mechanics. In particular, the importance of the bound (9.2) is sometimes underestimated. If the bound would not have been there, none of the familiar solutions of the Schrödinger equation would be stable; any, infinitesimally tiny, perturbation in H would result in a solution with energy E that decays into a combination of two spatially separated solutions with energies $E + \delta E$ and $-\delta E$.

All solutions of the Schrödinger equation would have such instabilities, which would describe a world quite different from the quantum world we are used to.

This is the reason why we always try to find an expression for H such that a lower bound exists (which we can subsequently normalize to be exactly zero). From a formal point of view, it should be easy to find such a Hamiltonian. Every classical model should allow for a description in the form of the simple models of Sect. 2.2.1, a collection of cogwheels, and we see in Fig. 2.3 that we can subsequently adjust the constants δE_i so that the bound (9.2) exists, even if there are infinitely many cogwheels with unlimited numbers of teeth.

However, also the third condition is needed. $\mathcal{H}(\vec{x})$ is the Hamiltonian density. Locality amounts to the demand that, at distances exceeding some close distance limit, these Hamilton densities must commute (Eq. (5.22)).

One may suspect that the ultimate solution that obeys all our demands will come from quantizing gravity, where we know that there must exist a local Hamiltonian density that generates local time diffeomorphisms. In other treatises on the interpretation of quantum mechanics, this important role that might be played by the gravitational force is rarely mentioned.

Some authors do suspect that gravity is a new elementary source of ‘quantum decoherence’, but such phrases are hardly convincing. In these arguments, gravity is treated perturbatively (Newton’s law is handled as an additive force, while black

holes and scattering between gravitational waves are ignored). As a perturbative, long-range force, gravity is in no fundamental way different from electromagnetic forces. Decoherence [95] is a concept that we completely avoid here (see Sect. 3.5).

Since we have not solved the Hamiltonian positivity problem completely, we have no systematic procedure to control the kind of Hamiltonians that can be generated from cellular automata. Ideally, we should try to approximate the Hamiltonian density of the Standard Model. Having approximate solutions that only marginally violate our requirements will not be of much help, because of the *hierarchy problem*, see Sect. 8.2: the Standard Model Hamiltonian (or what we usually put in its place, its Lagrangian), requires *fine tuning*. This means that very tiny mismatches at the Planck scale will lead to very large errors at the Standard Model scale. The hierarchy problem has not been solved, so this indeed gives us another obstacle.

Regarding locality of the Hamiltonian, there may be another message hidden in quantum gravity when applied to black holes. Unfinished research indicates a non commutative property of the space–time coordinates, leading to uncertainty relations of the form

$$\delta x \cdot \delta t = \mathcal{O}(L_{\text{Pl}}^2), \tag{9.6}$$

where L_{Pl} is the Planck length. Regarding the Hamiltonian to be a generator of infinitesimal time translations (as opposed to evolution operators that produce finite shifts in time), one may suspect that the Hamiltonian requires δt to be infinitesimal. In that case, δx is large, so that one may expect the Hamiltonian density to be a non-local operator. The fact that locality is restored when δt is kept finite then implies that the cellular automaton may still be local.

9.2 Second Quantization in a Deterministic Theory

When Dirac arrived at the famous Dirac equation to describe the wave function of an electron, he realized that he had a problem: the equation allows for positive energy solutions, but these are mirrored by solutions where the energy, including the rest-mass energy, is negative. The relativistic expression for the energy of a particle with momentum \vec{p} (in units where the speed of light $c = 1$), is

$$E = \pm \sqrt{m^2 + \vec{p}^2}. \tag{9.7}$$

If a partial differential equation gives such an expression with a square root, it is practically impossible to impose conditions on the wave function such that the unwanted negative sign is excluded, unless such a condition would be allowed to be a non-local one, a price Dirac was not prepared to pay. He saw a more natural way out:

There are very many electrons, and the N -electron solution obeys Pauli’s principle: the wave function must switch sign under interchange of any pair of electrons. In practice this means that all electrons each must occupy different energy levels. Levels occupied by one electron cannot be reached by other

electrons. Thus, Dirac imagined that all negative energy levels are normally filled by electrons, so that others cannot get there. The vacuum state is by definition the lowest energy state, so the negative-energy levels are all occupied there. If you put an extra electron in a positive energy level, or if you remove an electron from a negative energy spot, then in both cases you get a higher energy state.

If an electron is removed from a negative energy level, the empty spot there carries a net energy that is positive now. Its *charge* would be opposite to that of an electron. Thus, Dirac predicted an *antiparticle* associated to the electron:¹ a particle with mass m_e and charge $+e$, where normal electrons have mass m_e and charge $-e$. Thus, the positron was predicted.

In cellular automata we have the same problem. In Sect. 14, it will be explained why we cannot put the edge of the energy spectrum of an automaton where we would like to put it: at zero energy, the vacuum state, which would then naturally be the lowest energy state. We see that locality demands a very smooth energy function. If we symmetrize the spectrum, such that $-\pi < E\delta t < \pi$, we get the same problem that Dirac had to cope with, and indeed, we can use the same solution: second quantization. How it works will be explained in Sect. 15. We take k fermionic particles, which each can occupy N states. If we diagonalize the U operator for each ‘particle’, we find that half of the states have positive energy, half negative. If we choose $k = \frac{1}{2}N$, the lowest energy state has all negative energy levels filled, all positive energies empty; this is the lowest energy state, the vacuum.

The excited states are obtained if we deviate slightly from the vacuum configuration. This means that we work with the energy levels close to the center of the spectrum, where we see that the Fourier expansions of Sect. 14 still converge rapidly. Thus, we obtain a description of a world where all net energies are positive, while rapid convergence of the Fourier expansion guarantees effective locality.

Is this then the perfect solution to our problem? Nearly, but not quite. First, we only find justifiable descriptions of second quantized fermions. The bosonic case will be more subtle, and is not yet quite understood² Secondly, we have to replace the Dirac equation by some deterministic evolution law, while a deterministic theory to be exposed in Sect. 15.2 describes sheets, not particles. We do not know how to describe local deterministic interactions between such sheets. What we have now, is a description of non-interacting particles. Before introducing interactions that are also deterministic, the sheet equations will have to be replaced by something different.

¹Dirac first thought that this might be the *proton*, but that was untenable; the mass had to be equal to the electron mass, and the positron and the electron should be able to annihilate one another when they come close together.

²But we made a good start: bosons are the energy quanta of harmonic oscillators, which we should first replace by harmonic rotators, see Chaps. 12.1–13. Our difficulty is to construct harmonically coupled chains of such rotators. Our procedures worked reasonably well in one space-, one time dimension, but we do not have a bosonic equivalent of the neutrino model (Sect. 15.2), for example.

Supposing this problem can be addressed, we work out the formalism for interactions in Sect. 22.1. The interaction Hamiltonian is obtained from the deterministic law for the interactions using a BCH expansion, which is not guaranteed to converge. This may be not a problem if the interaction is weak. We bring forward arguments why, in that case, convergence may still be fast enough to obtain a useful theory. The theory is then not infinitely accurate, but this is not surprising. We could state that, indeed, that problem was with us all along in quantum field theory: the perturbative expansion of the theory is fine, and it gives answers that are much more precise than the numbers that can be obtained from any experiment, but they are not infinitely precise, just because the perturbation expansion does not converge (it can be seen to be merely an *asymptotic* expansion). Thus, our theory reproduces exactly what is known about quantum mechanics and quantum field theory, just telling us that if we want a more accurate description, we might have to look at the original automaton itself.

Needless to emphasize, that some of the ideas brought forward here are mostly speculation, they should still be corroborated by more explicit calculations and models.

9.3 Information Loss and Time Inversion

A very important observation made in Sect. 7 is that, if we introduce information loss in the deterministic model, the total number of orthogonal basis elements of the ontological basis may be considerably reduced, while nevertheless the resulting quantum system will *not* show any signs of time irreversibility. The *classical* states however, referring to measurement results and the like, are linked to the original ontological states, and therefore *do* possess a thermodynamical arrow of time.

This may well explain why we have time non reversibility at large, classical scales while the microscopic laws as far they are known today, still seem to be perfectly time reversible.

To handle the occurrence of information loss at the sub-microscopic level, we introduced the notion of info-equivalence classes: all states that, within a certain finite amount of time evolve into the same ontological state $|\psi(t)\rangle$, are called info-equivalent, all being represented as the same quantum basis element $|\psi(0)\rangle$. We already alluded to the similarity between the info-equivalence classes and the *local gauge equivalence classes*. Could it be that we are therefore talking about the same thing?

If so, this would mean that local gauge transformations of some state represented as an ontological state, may actually describe different ontological states at a given time $t = t_0$, while two ontic states that differ from one another only by a local gauge transformation, may have the property that they both will evolve into the same final state, which naturally explains why observers will be unable to distinguish them.

Now we are aware of the fact that these statements are about something being fundamentally unobservable, so their relevance may certainly be questioned.

Nevertheless this suggestion is justifiable, as follows. One may observe that formulating quantum field theory *without* employing the local gauge-equivalence prin-

principle, appears to be almost impossible,³ so the existence of local gauge equivalence classes can be ascribed to the mathematical properties of these quantized fields. Only rarely can one replace a theory with local gauge equivalence by one where this feature is absent or of a totally different nature.⁴ Exactly the same can be said about ontological equivalence classes. They will be equally unobservable at large scales—by definition. Yet rephrasing the deterministic theory while avoiding these equivalence classes altogether may be prohibitively difficult (even if it is not principally excluded). So our argument is simply this: these equivalence classes are so similar in Nature, that they may have a common origin.

This then leaves an exciting question: general relativity is also based on a local gauge principle: the equivalence of locally curved coordinate frames. Can we say the same thing about that gauge equivalence class? Could it also be due to information loss? This would mean that our underlying theory should be phrased in a *fixed* local coordinate frame. General coordinate invariance would then be ascribed to the fact that the *information that determines our local coordinates* is something that can get lost. Is such an idea viable? Should we investigate this?

My answer is a resounding *yes!* This could clarify some of the mysteries in today's general relativity and cosmology. Why is the cosmological constant so small? Why is the universe spatially flat (apart from local fluctuations)? And, regarding our cellular automaton: How should we describe an automaton in curved space–time?

The answers to these questions are then: yes, our universe is curved, but the curvature is limited to local effects. We *do* have an important variable playing the role of a metric tensor $g_{\mu\nu}(\vec{x}, t)$ in our automaton, but it lives in a well-defined coordinate frame, which is flat. Take a gauge condition obeying $g_{i0} = g^{i0} = 0$, $i = 1, 2, 3$. Let then λ^i be the three eigenvalues of g^{ij} , the space-like components of the inverse metric (so that $(\lambda^i)^{-1} \equiv \lambda_i$ are the eigenvalues of g_{ij}). Let $\lambda^0 = |g^{00}|$. Then the local speed of light, in terms of the coordinates used, is given by $c^2 = |\vec{\lambda}|/\lambda^0$. We can impose an inequality on c : assume that the values of the metric tensor are constrained to obey $|c| \leq 1$. If now we write $g_{\mu\nu} = \omega^2(\vec{x}, t)\hat{g}_{\mu\nu}$, with a constraint on $\hat{g}_{\mu\nu}$ such as $\det(\hat{g}_{\mu\nu}) = -1$ (See Appendix B), then there is no limitation on the value of ω . This means that the universe can inflate towards any size, but on the average it may stay flat.

We continue this subject in Part II, Sect. 22.4.

9.4 Holography and Hawking Radiation

There is an other reason to expect information loss to be an inescapable feature of an ultimate theory of physics, which concerns microscopic black holes. Classically, black holes absorb information, so, one may have to expect that our classical, or 'pre-quantum' system also features information loss. Even more compelling is the

³The interactions would have to be kept quite weak, such as the electro-magnetic ones.

⁴Examples are known, in the form of *dual transformations*.

original idea of *holography* [81, 117]. It was Stephen Hawking’s important derivation that black holes emit particles [47, 48], due to quantum fluctuations near the horizon. However, his observation appeared to lead to a paradox:

The calculation suggests that the particles emerge in a thermal state, with perfect randomness, regardless how the black hole was formed. Not only in deterministic theories, but also in pure quantum theories, the arrangement of the particles coming out should depend to some extent on the particles that went in to form a black hole.

In fact, one expects that the state of all particles emerging from the black hole should be related to the state of all particles that formed the black hole by means of a unitary matrix, the *scattering matrix* S [100, 103].

Properties of this matrix S can be derived using physical arguments [106]. One uses the fact that particles coming out must have crossed all particles that went in, and this involves the conventional scattering matrix determined by their “Standard Model” interactions. Now since the centre-of-mass energies involved here are often much larger than anything that has been tested in laboratories, much stays uncertain about this theory, but in general one can make some important deductions:

The only events that are relevant to the black hole’s reaction upon the particles that went in, take place in the region very close to the event horizon. This horizon is two-dimensional.

This means that all *information* that is processed in the vicinity of a black hole, must be effectively represented at the black hole horizon. Hawking’s expression for the black hole entropy resulting from his analysis of the radiation clearly indicates that it leaves only one bit of information on every surface element of roughly the Planck length squared. In natural units:

$$S = \pi R^2 = \log(W) = (\log 2)^2 \log W; \quad W = 2^{\Sigma/4 \log 2}, \quad (9.8)$$

where $\Sigma = 4\pi R^2$ is the surface area of the horizon.

Where did the information that went into the bulk of space–time inside a black hole go? We think it was lost. If we phrase the situation this way, we can have holography without losing locality of the physical evolution law. This evolution law apparently is extremely effective in destroying information.⁵

Now we can add to this that we cannot conceive of a configuration of matter in space–time that is such that it contains *more* information per unit of volume than a black hole with radius corresponding to the total energy of the matter inside. Therefore the black hole obeys the *Bekenstein limit* [5]:

the maximum amount of information that fits inside a (spherical) volume V is given by the entropy of the largest black hole that fits inside V .

⁵Please do not confuse this statement with the question whether *quantum information* is lost near a black hole horizon. According to the hypothesis phrased here, quantum information is what is left if we erase all redundant information by combining states in equivalence classes. The black hole micro states then correspond to these equivalence classes. By construction, equivalence classes do not get lost. Recently, it was discovered that this requires *antipodal entanglement* [127].

Information loss in a local field theory must then be regarded in the following way (“holography”):

In any finite, simply connected region of space, the information contained in the bulk gradually disappears, but what sits at the surface will continue to be accessible, so that the information at the surface can be used to characterize the info-equivalence classes.

At first sight, using these info-equivalence classes to represent the basis elements of a quantum description may seem to be a big departure from our original theory, but we have to realize that, if information gets lost at the Planck scale, it will be much more difficult to lose any information at much larger scales; there are so many degrees of freedom that erasing information completely is very hard and improbable; rather, we are dealing with the question how exactly information is represented, and how exactly do we count bits of information in the info-equivalence classes.

Note that, in practice, when we study matter in the universe, the amount of energy considered is far less than what would correspond to a black hole occupying the entire volume of space. So in most practical cases, the Bekenstein limit is not significant, but we have to remember that, in those cases, we always consider matter that is still very close to the vacuum state.

Information loss is mostly a local feature; globally, information is preserved. This does mean that our identification of the basis elements of Hilbert space with info-equivalence classes appears to be not completely local. On the other hand, both the classical theory and the quantum theory naturally forbid information to be spread faster than the speed of light.

Let us end this section with our view on the origin of Hawking radiation. The physical laws near the horizon of a black hole should be derived from the laws controlling the vacuum state as seen by an observer falling in. This vacuum is in a single quantum state, but consists of myriads of ontological states, distinguishable by considering all conceivable fluctuations of the physical fields. Normally, all these states form a single equivalence class.

At the horizon, however, signals entering the black hole cannot return, so the mixture of the information that causes all these states to form a single equivalence class, is rearranged substantially by the presence of the black hole, so much so that, as seen by the distant observer, not a single equivalence class is experienced, but very many classes. Thus, the vacuum is replaced by the much larger Hilbert space spanned by all these classes. They together form the rich spectrum of physical particles seen to emerge from the black hole.

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