Feature Asymmetry of the Conformal Monogenic Signal

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Abstract. Local properties of image (phase, amplitude and orientation) can be estimated practically using quadrature filters kernel and can be easily represented in two dimensions using the monogenic signal. This powerful feature representation has given rise to robust phase-based edge detection. Nonetheless, it is limited to the class of intrinsically one-dimensional signals, such as lines and edges. All other possible local patterns such as corners and junction are of intrinsic dimension two. Our aim in this paper is to present a new edge detection method for extracting local features of any curved signal. It is based on the conformal monogenic signal which is in practical applications compatible with intrinsically one and two-dimensional signal. Using different filters, our model have been tested and compared with classical models and some recent ones. The preliminary results show that our detection technique is more efficient and more accurate.

Keywords: Conformal monogenic signal · Local phase information · Feature asymmetry · Edge detection

1 Introduction

Interestingly, several physiological experiences have suggested that image structures like lines, edges, junctions and orientations play an important role in the Human Visual System. Consequently, these features have always been considered as central in the analysis since the early days. Their detection has therefore been a fundamental operation that needed to be processed in a reliable and robust way. Feature detection has been extensively studied in the literature and still remains an active field of research [1,6,9–13,16].

Throughout the history of digital image processing, smoothing and differentiation have been subjects of intense study. A variety of optimal differential operators have been proposed to solve different computer vision problems. For instance, edges and lines detection have received a particular attention. Differentiation is highly sensitive to illumination variations and do not localize accurately or consistently. Indeed, the localization of gradient based features varies with scale of analysis. Amplitude-based techniques however are known to be

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sensitive to smooth shading and lighting variations. Furthermore, edges, corners and other features are not simple step changes in luminance. Hence, recent contributions have reached a high degree of sophistication [13]. These may include, for instance, statistical models based detection [9]; linear and non linear scale space methods and links to regularisation theory [11].

Amplitude-based operators (Gradient) are sensitive to intensity variations and do not localize accurately or consistently. To minimize these problems we need a feature operator that is maximally invariant to intensity and scale. An alternative approach to amplitude based techniques is the use of phase information. Theoretically, it is known that a wide range of feature types gives rise to points of high Phase Congruency (PC). Also,it has been shown that this model successfully explains a number of psychophysical effects on human feature perception. PC is a dimensionless quantity, invariant to contrast and allows to the features to be tracked over extended sequences more reliably. Indeed, phase is an underused local image attribute that can have many applications [3,4,10].

Local properties of image (phase, amplitude and orientation) can be estimated practically using *quadrature filters* kernel [2,5] and can be easily represented in two dimensions using the *monogenic signal* [7]. This powerful feature representation has given rise to robust phase based detectors contours, the most remarkable is the Feature Asymmetry (FA) [3,4,10]. Nonetheless, it is limited to the class of intrinsically one-dimensional signals.

Two dimensional images can be classified into local regions of different intrinsic dimensions as shown in Fig. 1. The intrinsic dimension expresses the number of degrees of freedom necessary to describe local structure. Constant signals are of intrinsic dimension zero (i0D), lines and edges are of intrinsic dimension one (i1D) and all other possible patterns such as corners and junction are of intrinsic dimension two (i2D). For a given two-dimensional signal f and local region $N \subseteq \mathbb{R}^2$:

$$i0D = \{f : f(\mathbf{x}_i) = f(\mathbf{x}_i)\forall \mathbf{x}_i, \mathbf{x}_j \in N\}, \tag{1}$$

$$i1D = \{f : f(\mathbf{x}) = g(\langle \mathbf{x}, \mathbf{y} \rangle)\forall \mathbf{x} \in N, \mathbf{y} \in \mathbb{R}^2 \text{ and } |\mathbf{y}| = 1\}\backslash i0D, \tag{2}$$

$$i2D = f \notin (i0D \cup i1D), \tag{3}$$

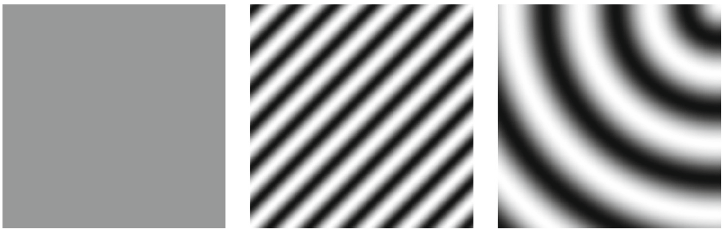


Fig. 1. Typical examples of a global intrinsic 2D signals. From left to right: a constant signal (i0D), an arbitrary rotated 1D signal (i1D) and a curved i2D signal. All signals displayed here preserve their intrinsic dimension globally. Figure from [18].

where $\langle \cdot, \cdot \rangle$ denotes the inner product and $g : \mathbb{R} \rightarrow \mathbb{R}$ represents the local structural feature function of 1D signal model such as $g(x) = a(x)\cos(\phi(x))$. Note that, in general, i2D signals can only be modeled by an infinite number of superimposed i1D signals [18].

In this paper, we present a novel way to compute the FA. It is based on the *conformal monogenic signal* which is a rotational invariant quadrature filter for extracting local features of any curved signal [18]. The conformal monogenic signal contains the introduced monogenic signal as a special case and combines scale space theory in one unified algebraic framework. The main advantage of the conformal monogenic signal in practical applications is its compatibility with intrinsically one-dimensional (i1D) and special intrinsically two-dimensional signal (i2D).

2 The Conformal Monogenic Signal

The local properties of a given 2D signal $f(\mathbf{x})$ can be estimated by the monogenic signal $f_M(\mathbf{x}) : \mathbb{R}^n \rightarrow \mathbb{R}^{n+1}$. It is defined in a space of dimension $n+1$ with sufficient degrees of freedom to represent the local characteristics of a signal in nD:

$$f_M(\mathbf{x}) = (f, \mathbf{f}_{\mathcal{R}})(\mathbf{x}). \tag{4}$$

The Riesz transform $\mathbf{f}_{\mathcal{R}}$ is considered as a generalised 1D Hilbert transform and preserves its most interesting properties, it is written by means of convolution as:

$$\mathbf{f}_{\mathcal{R}}(\mathbf{x}) = (\mathbf{h} * f)(\mathbf{x}) = \frac{\mathbf{x}}{A_{n+1}|\mathbf{x}|^{n+1}} * f(\mathbf{x}), \text{ with } A_{n+1} = \frac{2\pi^{\frac{n+1}{2}}}{\Gamma(\frac{n+1}{2})}. \tag{5}$$

For the particular case of a two-dimensional signal ($n = 2$) the generalised Hilbert transform kernel $\mathbf{h}$ is given by:

$$\mathbf{h}(\mathbf{x}) = (h_1, h_2)(\mathbf{x}) \ , \text{ and } h_i = \frac{x_i}{2\pi|\mathbf{x}|^3}. \tag{6}$$

The local properties turned out to be invariant with respect to rotations and translation, due to the rotational equivalent and the linear shift invariance properties of the Riesz transform. These properties make the monogenic signal as a powerful feature representation. Nonetheless, it is limited to the class of intrinsically one-dimensional signals. Thus, the conformal monogenic signal is considered as a generalisation for analysing i1D and i2D local feature for two-dimensional signal.

2.1 The Conformal Space

As introduced by Wietzke and Sommer [18], the main idea of the conformal monogenic signal is to lift up 2D signals to an appropriate conformal space with more degrees of freedom compared to the 2D monogenic signal. Since, line and circle of the two-dimensional signal domain are mapped to circles on the sphere in conformal space, with center $(0, 0, \frac{1}{2})^T$ and radius $\frac{1}{2}$:

$$\mathbb{S} := \left\{ \xi \in \mathbb{R}^3 : \xi_1^2 + \xi_2^2 + (\xi_3 - \frac{1}{2})^2 = \frac{1}{4} \right\}. \tag{7}$$

The sphere $\mathbb{S}$ touches the Euclidean plan $\mathbb{R}^2$ such that its south-pole coincides with the origin $(0, 0, 0)^T$. This projection is conformal and can be inverted by $\mathcal{C}^{-1}$ for all elements of $\mathbb{S} \in \mathbb{R}^3$:

$$\mathcal{C}^{-1}(\mathbf{x}) = \frac{1}{x_1^2 + x_2^2 + 1} \begin{pmatrix} x_1 \\ x_2 \\ x_1^2 + x_2^2 \end{pmatrix}, \tag{8}$$

for $\mathbf{x} = (x_1, x_2)^T \in \mathbb{R}^2$ whereas the projection from $\mathbb{S}$ to $\mathbb{R}^2$ is given by:

$$\mathcal{C}(\omega) = \frac{1}{1 - \omega_3} \begin{pmatrix} \omega_1 \\ \omega_2 \end{pmatrix}, \text{ with } \omega \in \mathbb{S}. \tag{9}$$

Lines and circles of the 2D signal domain will be mapped to circles on the sphere. All planes corresponding to circles remain unchanged. That is the reason why the conformal monogenic signal models i1D line and all kinds of curved i2D signals which can be locally approximated by circles.

The spherical embedding f_c of the circular signal f with respect to a point on interest $\mathbf{x} \in \mathbb{R}^2$ is given by:

$$f_c(\mathbf{x}; \xi) = \begin{cases} f(\mathcal{C}(\xi) + \mathbf{x}) & \text{for } \xi \in \mathbb{S}, \\ 0 & \text{else}. \end{cases} \tag{10}$$

Now the conformal monogenic signal can be introduced by:

$$\begin{aligned} f_{CM}(\mathbf{x}) &= (f_c, \mathbf{f}_{\mathcal{C}})(\mathbf{x}; \mathbf{0}), \\ &= (f_c, h_1 * f_c, h_2 * f_c, h_3 * f_c)(\mathbf{x}; \mathbf{0}). \end{aligned} \tag{11}$$

We are now able to estimate the i1D and i2D local amplitude, orientation and phase of curved 2D signal respectively by:

$$a(\mathbf{x}) = \sqrt{f_c^2(\mathbf{x}; \mathbf{0}) + \sum_{i=1}^{3} (h_i * f_c)^2(\mathbf{x}; \mathbf{0})}, \tag{12}$$

$$\theta(\mathbf{x}) = \arctan 2\left(h_2 * f_c, h_1 * f_c\right)(\mathbf{x}; \mathbf{0}), \tag{13}$$

and

$$\phi(\mathbf{x}) = \arctan 2\left(\sqrt{\sum_{i=1}^{3} (h_i * f_c)^2(\mathbf{x}; \mathbf{0})}, f_c(\mathbf{x}; \mathbf{0}) \right). \tag{14}$$

3 Feature Asymmetry

In practical applications, the local properties are estimated using a pair of bandpass quadrature filters. Indeed, the detection of local properties by the conformal monogenic signal assumes that the signal consists of few frequencies, that is bandlimited. A real image consist of a wide range of frequencies, therefore a set of bandpass filters needs to be combined with the conformal monogenic signal. Equation (11) becomes:

$$f^s_{CM}(\mathbf{x}) = (\mathcal{Q} * f_c, \mathcal{Q} * \mathbf{f}_{\mathcal{C}})(\mathbf{x}; \mathbf{0}), \tag{15}$$

where $\mathcal{Q}(\mathbf{x}; s)$ is the spatial domain representation of an isotropic bandpass filter and $s > 0$ is a scaling parameter. Thus, the monogenic signal can be represented by a scalar valued even and vector valued odd filtered responses, with the following simple tick:

$$\begin{aligned} even &= (\mathcal{Q} * f_c)(\mathbf{x}; \mathbf{0}), \\ \mathbf{odd} &= (\mathcal{Q} * h_1 * f_c, \mathcal{Q} * h_2 * f_c, \mathcal{Q} * h_3 * f_c)(\mathbf{x}; \mathbf{0}). \end{aligned}$$

Several families of quadrature pairs have been proposed and applied in the literature. Most authors have not provided a reasonable justification for the use of a particular family apart from simplicity of use or the satisfaction of the zero DC condition. In [2], the authors introduce a new generalised pairs of quadrature filters, and after comparison, they concluded that the widely used log-Gabor kernels are probably not a very good choice in the case of feature detection. They showed that Derivative of Derivative/Difference of α scale space family (ASSD and DoSS respectively) has better properties. (see Fig. 2). A 2D isotropic ASSD kernel is defined in Fourier domain by

$$\mathcal{Q}_{ASSD}(\mathbf{u}) = \begin{cases} n_c \mathbf{u}^a \exp\left(-(s\mathbf{u})^{2\alpha}\right) & \text{if} \quad \mathbf{u} \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{16}$$

where the frequency coordinate $\mathbf{u} = (u_1, u_2)$, $\alpha \in]0, 1]$, n_c is a normalisation constant. The derivative parameter $a \in \mathbb{R}^+$ meaning we are using fractional order derivatives.

3.1 The Edge Detection Measure

Step edge detection is performed using the *feature asymmetry* measure (FA) of Kovesi [10] redefined using the conformal monogenic signal presented previously. To identify step edges essentially involves finding points where the absolute value of the local phase is 0° at a positive edge and 180° at a negative edge. In other words, the difference between the odd and the even filter responses is large. We define the new multiple scales feature asymmetry by:

$$FA_{CM} = \frac{\sum_s \lfloor |\mathbf{odd}_s| - |even_s| - T_s \rfloor}{\sum_s \sqrt{even_s^2 + |\mathbf{odd}_s|^2 + \varepsilon}}. \tag{17}$$

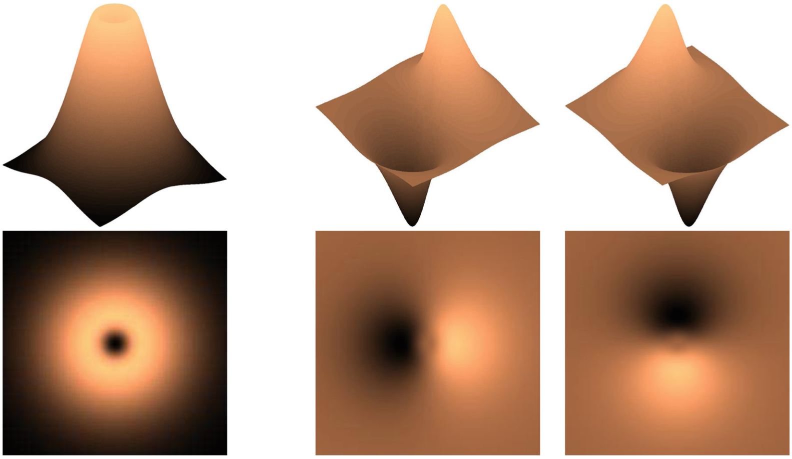


Fig. 2. 2D DoSS kernels in Fourier domain for certain parameters. From left to right: the isotropic even part and the pair composing the odd part.

where $\lfloor \cdot \rfloor$ denotes zeroing of negative values and T is a noise threshold estimate [10]. Here, ε is used to avoid zero division. The FA_{CM} takes values in $[0, 1]$, close to zero in smooth regions and close to one near boundaries.

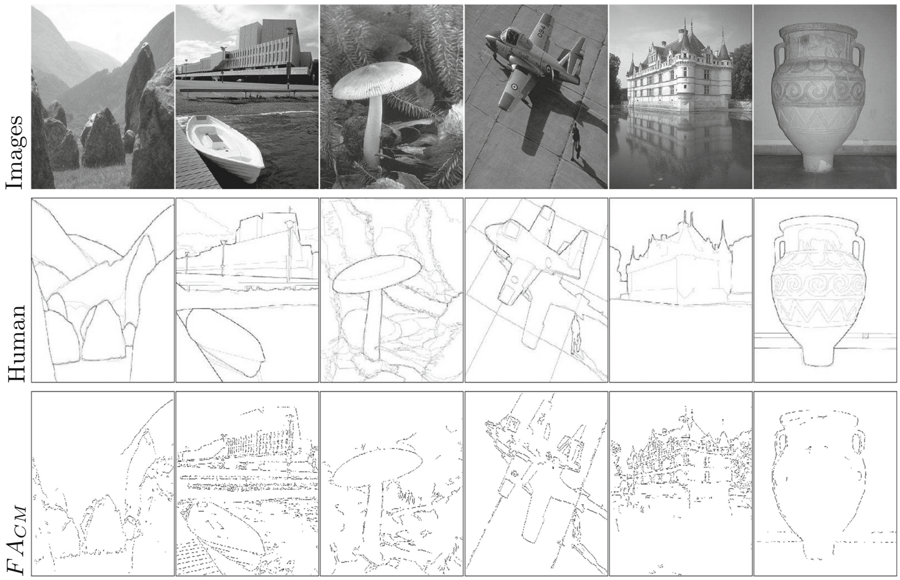
4 Results and Discussion

To evaluate the performance of the proposed approach, we made comparisons between manual delineation of Berkeley Segmentation Database [1] and the automatic results. One band-pass filter has been chosen for these tests, namely the α-Scale Spaces Derivative filter [2] (ASSD). This filter was chosen for its parametric nature of $\alpha \in]0, 1]$ which makes it possible to find the classic filters like the Derivative (Difference) of Gaussian ($\alpha = 1$) and Poisson filter ($\alpha = 0.5$).

The evaluation of this method is carried out by Precision-Recall curves which are obtained by varying the detection threshold. There is however, an interesting point on the curves defined by the measure $F = 2\frac{\text{Precision}\cdot\text{Recall}}{\text{Precision}+\text{Recall}}$. Thus, the location of the maximum of this measure along the curve defines the optimum threshold and provides a summary score. Table 1 reports a summary of results obtained on the database of 500 images of Berkeley. It should be noted that the most interesting measure is the F-measure (ODS), the other ones are involved only to bring more precision. Table 1 summarises the comparison results between the proposed method FA_{CM} and the classical models as well as some newer and more sophisticated models [1,8,16]. An overview of these results is illustrated in Fig. 3. It is easy to see that the FA_{CM} approach significantly exceeds, in terms of performance, the remaining methods, irrespective of the selected filter (Gaussian or Poisson). Since the FA_{CM} detector performed by conformal monogenic signal combines all intrinsic dimensions in one framework, including curved edges

Table 1. Summary of comparison results. Represented on the table, the best score over the image dataset ODS (Optimal Dataset Scale), the best score per image OIS (Optimal Image Scale), as well as the area below the precision-recall curve AR (Average Precision).

Edge detection method	ODS	OIS	AR
Manual	0.80	0.80	–
FA_{CM}	0.61	0.64	0.54
Felzenszwalb and Huttenlocher [8]	0.61	0.64	0.65
Arbelaez et al. [1]	0.60	0.64	0.58
Sharon et al. [16]	0.56	0.59	0.54
Canny [6]	0.60	0.63	0.58
Marr and Hildreth [12]	0.57	0.59	0.21
Prewitt [14]	0.51	0.54	0.38
Sobel and Feldman [17]	0.51	0.53	0.38
Roberts [15]	0.50	0.53	0.73



(a) (b) (c) (d) (e) (f)

Fig. 3. Edge detection results on the BSDS500 benchmark. From top to bottom: original images, corresponding manual segmentations, results obtained by the FA_{CM} method.

and lines. It could be an interesting alternative for the gradient or the Laplace operator.

The FA_{CM} can be computed efficiently and easily implemented into existing low level image processing steps of Computer Vision applications. Furthermore, it can be calculated with all the advantages of rotationally invariant local phase based approaches, robustness against brightness and contrast changes, and without the need of any partial derivatives. Hence, lots of numerical problems of partial derivatives on discrete grids can be avoided.

It is natural to think tackle the proposed approach with more recent and efficient ones, such as the model called GPB-owt-ucm of [1]. However, such methods are in a quite developed and sophisticated level, and take into account the texture, the multi-scale framework and the presence of noise in images. To be at the same level and get better detection, it is interesting to include a component for treating texture. We can note also that according to the experiments carried out on the Berkeley database, the measure F depends on the size and type of the selected images sample. Thus, we plan to experiment our approach by other publicly available datasets.

5 Conclusion

We proposed in this paper a new idea in isotropic 2D phase based edge detection. This new Conformal Monogenic Feature Asymmetry detector combines all intrinsic dimensions in one framework, including curved edges and lines. Using different filters, we tested and compared our approach with conventional models and some newer models. It appears that our results are in the same order as those of the state of the art. Although these results are introductory, they seem to be promising. Indeed, this method, simple to implement and easily expandable to higher dimensions, opens up new perspectives. Furthermore, it can be computed efficiently and easily implemented with all the advantages of rotationally invariant local phase based approaches, robustness against brightness and contrast changes, and without the need of any partial derivatives. Hence, lots of numerical problems of partial derivatives on discrete grids can be avoided. In short, it could be an interesting alternative for the gradient based method. More applications of the FA_{CM} such as edge detection on three-dimensional data will be part of our future work.

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