Analysis on Coupled Line Cameras Using Projective Geometry

Jinwoo Lee[1], Joo-Haeng Lee[2,3], and Junho Kim[1(✉)]

[1] School of Computer Science, Kookmin University, Seoul, Republic of Korea
junho@kookmin.ac.kr

[2] Human-Robot Interaction Laboratory, ETRI, Daejeon, Republic of Korea

[3] Computer Software, University of Science and Technology, Daejeon, Republic of Korea

Abstract. Recently coupled line cameras (CLCs) have been introduced to calibrate camera parameters solely from a single rectangle with an unknown aspect ratio in the captured image. Even as CLCs are highly related to projective geometry, they have not been analyzed from the framework of projective geometry. In this study, we revisit CLCs using the concepts of projective geometry, such as cross-ratio and projective harmonic conjugate. Finally, we present novel and efficient algorithms to handle off-centered rectangles in CLCs and provide the experimental results of CLC-based reconstructions by using the proposed algorithm for real images.

Keywords: Coupled line cameras · Rectangle · Projective geometry · Cross-ratio · Projective harmonic conjugate

1 Introduction

Camera calibration is a fundamental tool for estimating the parameters of a given camera from its captured images. The intrinsic parameters depend on the internal characteristics of the camera, such as focal length, principal point, and distortion coefficients, whereas the extrinsic parameters tell us the external 3D pose of the camera against a scene, such as its orientation and position. Because the camera parameters are the basis for analyzing the scene geometry from camera images, they have been extensively studied in computer vision and related areas, including computer graphics and augmented reality.

Recently coupled line cameras (CLCs) have been introduced in [1–3] to estimate the extrinsic parameters of a camera solely from a *single rectangle* with an *unknown aspect ratio* in the captured image. Based on the assumption that the image is captured with a pinhole camera, CLCs provide an analytic solution for the camera pose by reconstructing the projective structure defined between the camera and the plane containing the rectangle. Moreover, it is possible to find the aspect ratio of the unknown rectangle in a scene.

CLCs find the camera pose by using the perspective length distortions captured from two line cameras, each of which belongs to one of two diagonals

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in a projected rectangle. By coupling two line cameras a geometrically possible solution space for the camera pose is defined, with an assumption that the center of the projected rectangle is at the center of the image. Because the center of the projected rectangle does not lie on the center of the image in practice, a proxy rectangle is constructed on the center of the image prior to CLCs that is projective similar [1] or congruent [3] to the given off-centered rectangle. Even as CLCs are highly related to projective geometry, especially in defining the line camera and finding a proxy rectangle, CLCs have not been analyzed from the framework of projective geometry.

In this study, we revisit CLCs using the concepts of projective geometry [4] and propose a novel and efficient algorithms to handle off-centered rectangles in CLCs. We firstly show that the lengths of four partial diagonals in any perspectively similar rectangles at arbitrary positions are analytically defined in an up-to-scale manner, by utilizing *projective harmonic conjugate*. Finally, we analytically obtain the lengths of four partial diagonals at the image center directly from an off-centered rectangle and apply them to CLCs for camera calibrations.

2 Related Work and Background

2.1 Related Work

There are several approaches to exploit the geometry of image quadrilateral for camera calibration. According to [5], we can reconstruct a scene rectangle from a single image without explicit correspondence information between an image quadrilateral and a scene rectangle. However, we need to specify intrinsic parameters such as focal length, which can be computed using a method based on the concept of the image of absolute conic (IAC). Once the rectangle is reconstructed, the external camera parameters can be computed using the method such as [6]. In other approaches [7,8], we need to specify the aspect ratio of a scene rectangle to calibrate the camera. Otherwise, at least two images are required for reconstruction.

In contrast to previous work, CLCs [1–3] provide the geometry of an unknown rectangle from an image qaudrilateral without prior information on the aspect ratio of the rectangle and the focal length of the camera. The initial conception of CLCs in [1] followed the successful formulation of its dual configuration on CLPs (coupled line projectors) that provided a solution to the projector pose estimation problem for a projected quadrilateral in the scene [9]. Through a series of progressive refinements [1–3], CLCs achieved a solution for the problem of estimating camera pose from the single image of an unknown scene rectangle. The solution is unique in that the projective reconstruction and rectangle determination can be performed simultaneously without prior knowledge of correspondences or camera parameters.

The imaging geometry of CLCs is assumed to be a pinhole projection model, which contributes to simplify the underlying geometric configuration and leads to the derivation of simple analytic solutions. The optimized analytic form in [2] is based on the observation of two intersecting solution spheres of line cameras.

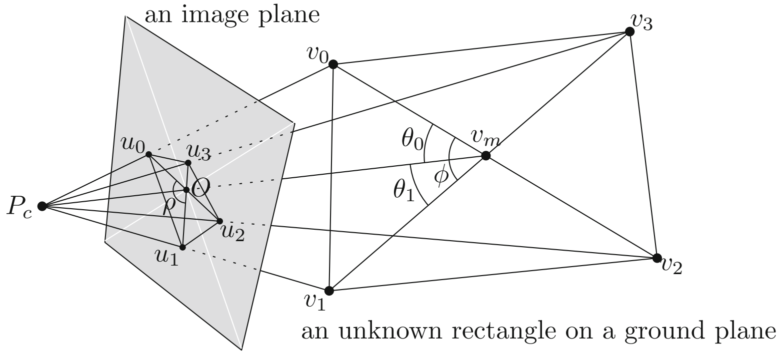


Fig. 1. Configuration of a coupled line camera. A unknown rectangle G with vertices v_0, v_1, v_2, v_3 is projected on an image plane of a camera p_c as a quadrilateral Q with vertices u_0, u_1, u_2, u_3 while the principal axis passes through the centers of G and Q.

This was further developed as generalized coupled line cameras (G-CLCs) in [3] to handle any arbitrary scene quadrilateral other than a rectangle. Note that no internal camera parameter is needed for projective reconstruction and the rectangle determination. If required, we can apply a standard calibration method such as in [6,10,11]. This is quite different from the previous method in [12] that requires a prior knowledge of internal camera parameters to reconstruct the scene rectangle.

The CLCs solution is largely composed of three steps: (1) From the given vertices of an image quadrilateral Q_g, a centered proxy quadrilateral Q is inferred using the geometric properties of parallel lines in perspective. The computation is solely composed of line-line intersection operations [3] (see Fig. 3). (2) For the centered proxy quadrilateral Q found, the analytic solutions of CLCs in [2] are applied to reconstruct the centered scene rectangle G and to find the position of the principal point p_c in the coordinate system defined by the centered scene rectangle G. (3) Once the projective structure defined by G and p_c is reconstructed, the geometry of the target scene rectangle Q_g can be geometrically determined using the method presented in [3]. In [1], a homography between Q and G was used to find G_g from Q_g.

This paper focuses on the first step of the CLCs solution that infers a centered proxy quadrilateral from a given off-centered quadrilateral in an image. We propose analytic solutions that define diagonal parameters for CLC-based reconstruction without explicitly inferring a centered proxy quadrilateral.

2.2 Background

In this section, we briefly introduce the CLCs and several projective geometry tools to analyze the CLCs based on projective geometry. We refer the readers to [1–3] for more information about CLCs and [4] for details on projective geometry.

Coupled Line Cameras. Figure 1 shows a configuration of CLCs. A rectangle G (i.e., $v_0v_1v_2v_3$) is projected on the image plane of a camera p_c as a quadrilateral Q (i.e., $u_0u_1u_2u_3$) while the principal axis passes through the centers of G and Q. In this configuration, lengths of partial diagonals of a rectangle are $|v_i - v_m| = 1$ $(i = 0, 1, 2, 3)$.

CLCs decompose the projective structure problem into two diagonal problem as *line cameras*. For the line camera, camera pose parameters such as the length of principal axis $\overline{p_c v_m} = d$ and a camera orientation $\angle p_c v_m v_i = \theta_i$, can be represented by lengths of projected partial diagonals $l_i = |u_i - u_m|$ $(i = 0, 1, 2, 3)$, as follows:

$$\cos\theta_i = \left(\frac{(l_i - l_{(i+2)})}{(l_i + l_{(i+2)})}\right) d = \alpha_i d \tag{1}$$

where $i = 0, 1$.

In the *pose equation* represented as Eq. (1), the possible positions of the camera are defined on a sphere based on *line division coefficient* α_i, for each diagonal $(i = 0, 1)$.

To geometrically merge two line cameras with an compliant scale, CLCs introduces a *coupling constraint*:

$$\beta = \frac{l_1}{l_0} \tag{2}$$

Using the geometric configuration specified in Eqs. (1) and (2), the length d of the principal axis can be computed as follows.

$$\begin{aligned} d &= \sqrt{\frac{A_0}{A_1}} \\ A_0 &= (1-\alpha_1)^2\beta^2 - (1-\alpha_0)^2 \\ A_1 &= \alpha_0^2(1-\alpha_1)^2\beta^2 - (1-\alpha_0)^2\alpha_1^2 \end{aligned} \tag{3}$$

Once the length of principal axis is found, θ_i can be computed using the pose equation. Next, the θ_i and an angle of the diagonals $\rho = \angle u_0 O u_1$ are employed to compute a diagonal angle $\phi = \angle v_0 v_m v_1$ as follows.

$$\cos\phi = \cos\rho \sin\theta_0 \sin\theta_1 + \cos\theta_0 \cos\theta_1 \tag{4}$$

Finally, the CLC-based reconstruction computes the camera position p_c as follows.

$$p_c = \frac{d(\sin(\phi)\cos(\theta_0), \cos(\theta_1) - \cos(\phi)\sin(\theta_0), \sin(\rho)\sin(\theta_0)\sin(\theta_1))}{\sin(\phi)} \tag{5}$$

Note that p_c is relatively defined in the coordinates defined by the reconstructed rectangle G.

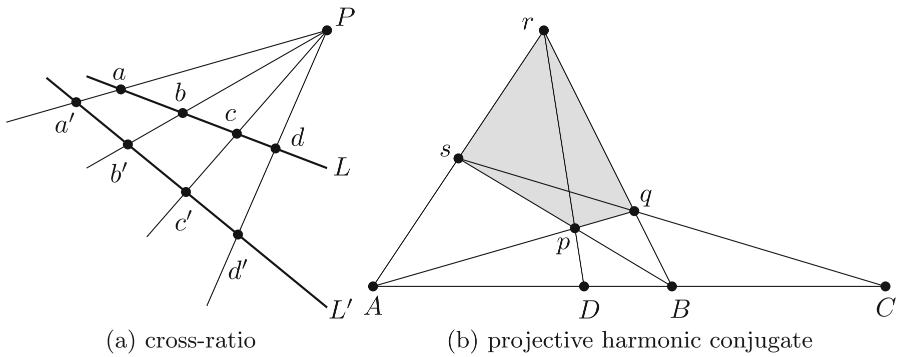


Fig. 2. Projective geometric properties used in analyzing CLCs

Projective Geometry. The cross-ratio is a ratio between the distances defined by four collinear points on a projective line (see Fig. 2(a)). When four lines starting from a point P intersect a line L at the four points a, b, c, d, the cross-ratio of the four points is defined as

$$(a, b; c, d) = \frac{|a, c|}{|a, d|} / \frac{|b, c|}{|b, d|}, \tag{6}$$

where $|a, c|$ denotes the directed distance from a to c.

The cross-ratio is the only invariant property that does not affect the projective transformation, meaning that the cross-ratio always holds true for any four collinear points a', b', c', d' on another line L', as follows.

$$(a, b; c, d) = (a', b'; c', d') = \frac{|a, c|}{|a, d|} / \frac{|b, c|}{|b, d|} = \frac{|a', c'|}{|a', d'|} / \frac{|b', c'|}{|b', d'|} \tag{7}$$

The projective harmonic conjugate comes from a special case of the cross-ratio induced from a quadrilateral (see Fig. 2(b)). Considering a quadrilateral $pqrs$, we can define two intersection points A and B, each of which joins a pair of opposite sides in $pqrs$. Once we define a line passing through A and B, we can further define C and D from the intersections between the line of AB and each diagonal of $pqrs$. In this case, D is called the harmonic conjugate of C, and the cross-ratio of four points $(A, B; C, D)$ is then defined as follows:

$$(A, B; C, D) = \frac{|A, C|}{|A, D|} / \frac{|B, C|}{|B, D|} = -1 \tag{8}$$

Let us consider that the arbitrary quadrilateral $pqrs$ is a projected rectangle, i.e., an image of a rectangle in 3D captured from a pinhole camera. Then, the four points A, B, C, D are four vanishing points on a vanishing line defined from the projected rectangle. Moreover, with projective harmonic conjugate one vanishing point can be calculated from the other three vanishing points. In this study, we

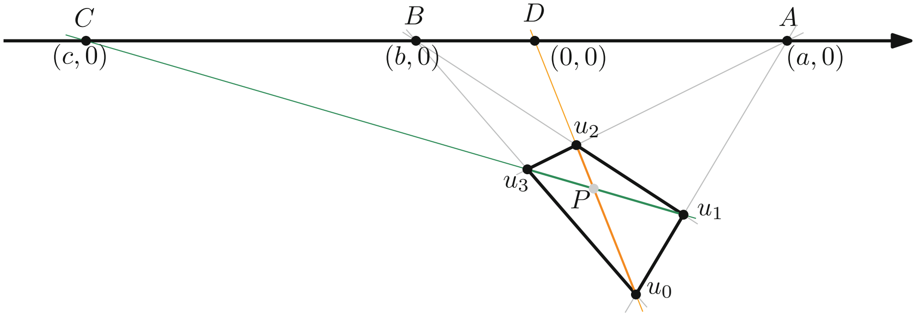


Fig. 3. Problem setting: The quadrilateral $u_0u_1u_2u_3$ is an image of an unknown planar rectangle. We can consider the four vanishing points A, B, C, D, on a vanishing line, where A and B are the infinity points about two pairs of parallel sides in the rectangle and C and D are the infinity points about two diagonal directions of the rectangle.

utilize this property to analyze the special relationships between the lengths of four partial diagonals in any perspectively similar rectangles at arbitrary positions.

3 Analysis

In this section, we analyze the projective structure on the image of a rectangle observed from a pinhole camera. We also show some special relationships among the lengths of the four partial diagonals in any perspectively similar rectangles at arbitrary positions, by using cross-ratios and projective harmonic conjugates.

Let us consider a quadrilateral $u_0u_1u_2u_3$, which is a projective image of a planar rectangle $v_0v_1v_2v_3$ with an unknown ratio in 3D, as shown in Fig. 3. Then, we can define a vanishing point A as the intersection of the extensions of a pair of opposite sides, $\overline{u_0u_1}$ and $\overline{u_2u_3}$. In a similar manner, we can obtain a vanishing point B defined by another pair of opposite sides, $\overline{u_1u_2}$ and $\overline{u_3u_0}$. Next, by defining a vanishing line passing through A and B, we further consider two more vanishing points C and D, which are the infinity points on the two diagonal directions of the rectangle. Notice that any projective similar rectangles with $u_0u_1u_2u_3$ on the same plane share the same vanishing points, A, B, C, and D, on the same vanishing line.

For the sake of simplicity, we define a new coordinate system in the image space, in a way that its origin is located on the vanishing point D, its x-axis is defined along with the vanishing line, and its y-axis is perpendicular to the x-axis. We represent the coordinates of other three vanishing points, as $A = (a, 0)$, $B = (b, 0)$, and $C = (c, 0)$, respectively, as shown in Fig. 3. Because the set of the four vanishing points, A, B, C, D, is a harmonic range, c can be represented by a, and b, from the projective harmonic conjugate, as follows.

$$c = \frac{2ab}{a+b} \tag{9}$$

Now, we analyze the length relationships among the four partial diagonals in a quadrilateral $u_0u_1u_2u_3$, whose projective center point is $P = (p^x, p^y)$. The center P of a projective image of a rectangle can be computed as the intersection point between the diagonals, as shown in Fig. 3. The length of a partial diagonal of the quadrilateral is represented as l_i, where $i = 0, 1, 2, 3$.

$$l_i = |u_i P| \tag{10}$$

Let us consider the distance from the projective center P and two vanishing points C and D, defined by two diagonals. We define the length between P and D as l_+ and the length between P and C as l_-.

$$l_+ = |DP| = \sqrt{(p^x)^2 + (p^y)^2} \tag{11}$$

$$l_- = |CP| = \sqrt{(p^x - c)^2 + (p^y)^2} \tag{12}$$

Let us represent the unit vectors from P to D as e_+ and from P to C as e_+.

$$e_+ = \frac{DP}{|DP|} = \frac{DP}{l_+} \tag{13}$$

$$e_- = \frac{CP}{|CP|} = \frac{CP}{l_-} \tag{14}$$

Now, we can represent the coordinates of u_i, in terms of the coordinate of P, the lengths of partial diagonals, l_i, and l_+, l_-, as follows ($i = 0, 1, 2, 3$).

$$u_0 = \begin{bmatrix} u_0^x \\ u_0^y \end{bmatrix} = P + l_0 e_+ = P + DP\frac{l_0}{l_+} = \frac{l_+ + l_0}{l_+}\begin{bmatrix} p^x \\ p^y \end{bmatrix} \tag{15}$$

$$u_1 = \begin{bmatrix} u_1^x \\ u_1^y \end{bmatrix} = P + l_1 e_- = P + CP\frac{l_1}{l_-} = \frac{l_- + l_1}{l_-}\begin{bmatrix} p^x \\ p^y \end{bmatrix} - \frac{l_1}{l_-}\begin{bmatrix} c \\ 0 \end{bmatrix} \tag{16}$$

$$u_2 = \begin{bmatrix} u_2^x \\ u_2^y \end{bmatrix} = P - l_2 e_+ = P - DP\frac{l_2}{l_+} = \frac{l_+ - l_2}{l_+}\begin{bmatrix} p^x \\ p^y \end{bmatrix} \tag{17}$$

$$u_3 = \begin{bmatrix} u_3^x \\ u_3^y \end{bmatrix} = P - l_3 e_- = P - CP\frac{l_3}{l_-} = \frac{l_- - l_3}{l_-}\begin{bmatrix} p^x \\ p^y \end{bmatrix} + \frac{l_3}{l_-}\begin{bmatrix} c \\ 0 \end{bmatrix} \tag{18}$$

Since u_0 and u_1 share the vanishing point A, the relation $(u_0 \times u_1) \cdot A = 0$ holds true for their homogeneous coordinates and we can obtain the following:

$$(u_0^y u_1^x - u_0^x u_1^y) = a(u_0^y - u_1^y) \tag{19}$$

By using Eqs. (15) and (16), we can represent Eq. (19) with a length relationship, as follows:

$$c(l_+ + l_0)l_1 = a(l_+ l_0 - l_- l_0) \tag{20}$$

By applying the projective harmonic conjugate in Eqs. (9) to (20), we further simplify Eq. (20) as follows.

$$2b(l_+ + l_0)l_1 = (a + b)(l_+ l_1 - l_- l_0) \tag{21}$$

If we apply the similar processes with Eqs. (19) and (20) to the other three sides of the quadrilateral, we can express the length of partial diagonals, l_1, l_2 and l_3, in terms of a, b, l_0, l_+, l_-, as follows.

$$\begin{aligned} l_1 &= \frac{(a+b)l_- l_0}{(a-b)l_+ - 2bl_0} \\ l_2 &= \frac{l_+ l_0}{l_+ + 2bl_0} \\ l_3 &= \frac{(a+b)l_- l_0}{(a-b)l_+ + 2al_0} \end{aligned} \tag{22}$$

Notice that for a given quadrilateral $u_0u_1u_2u_3$, the lengths about a, b, l_+, l_- are fixed. As a result, we can say that the up-to-scale dependencies exists among the lengths of partial diagonals in the quadrilateral, meaning that l_1, l_2, l_3 can be defined from l_0, as shown in Eq. (22).

For any projective similar rectangle to a given quadrilateral $u_0u_1u_2u_3$, the lengths about a, b are fixed because they share the vanishing points. In that case, once we compute l_+ and l_- based on the center of a similar rectangle, we can define l_1, l_2, l_3 from l_0, in an up-to-scale manner.

4 Algorithm

In this section, we provide specific algorithms to apply our analysis in Sect. 3 to CLCs for camera calibration. For a given off-centered quadrilateral in an image, the goal of the algorithms is to define the lengths of four partial diagonals about a projective similar or congruent rectangle placed on the image center.

CLCs using a similar rectangle: For a given off-centered quadrilateral $u_0u_1u_2u_3$, we first compute the four vanishing points A, B, C, and D, and measure the directed distances a and b, as shown in Fig. 3. Next, for the center point of an image, O, we measure l_+ and l_-, as $|DO|$ and $|CO|$, respectively. We set l_0 as a proper value (e.g., 1), and compute l_1, l_2, and l_3 from Eq. (22). We also compute the angle $\angle COD$ to obtain ρ as defined in [1]. Because the lengths of partial diagonals l_0, l_1, l_2, and l_3 and the angle between diagonals ρ are computed for a *centered*-case, we can perform the camera calibration with a CLC-based reconstruction.

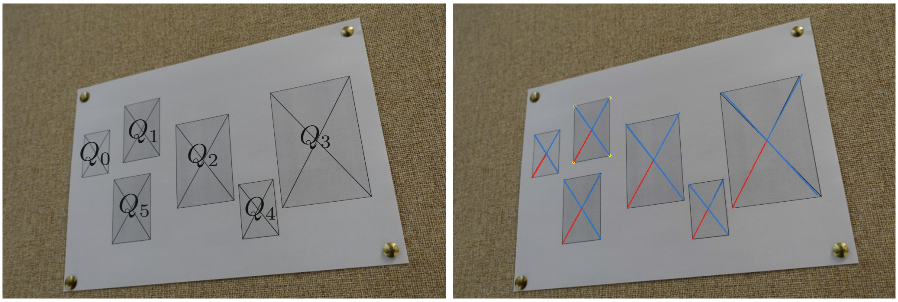
CLCs using a congruent rectangle: We have to explicitly compute l_0, to define the projective congruent rectangle placed at the image center from a given quadrilateral $u_0u_1u_2u_3$. First, we consider a new vanishing point F, defined from P and O as follows.

$$F = (P \times O) \times (A \times B) \tag{23}$$

Next, we compute u_0', the projective translation about u_0, defined in the *centered* congruent rectangle, as follows.

$$u_0' = (u_0 \times F) \times (C \times O) \tag{24}$$

Finally, we measure l_0 as $|u_0'O|$ and perform the same steps for the case of similar rectangles.



(a) (b)

Fig. 4. Experimental verification using Eq. (22): (a) We draw a number of similar rectangles on a plane and take a picture of it in a perspective viewpoint. (b) To experimentally verify the length relationships among the partial diagonals, we calculate the lengths of l_1, l_2, l_3, denoted as blue, from the measured length of l_0, denoted as red, in each quadrilateral. In this experiment, we use the four corner points of Q_1 (i.e., yellow dots) to define a vanishing line (Color figure online).

5 Experiments

Figure 4 shows an experimental verification of our analysis discussed in Sect. 3. First, we draw several similar rectangles on a plane and take a picture of the plane to get the images of the similar rectangles (i.e., the quadrilaterals Q_0, Q_1, $\cdots$, Q_5 as shown in Fig. 4). Next, we *measure* the lengths of partial diagonals, l_0, l_1, l_2, l_3, for each quadrilateral in the image space. Next, we select one of the quadrilaterals (e.g., Q_1 in Fig. 4) and compute the directed distances a and b

Table 1. Error analysis

Quad	Method	l_0	l_1	l_2	l_3	Length error
Q_0	measured	64.14	53.00	63.13	51.62	0.985 %
	calculated	-	52.89	63.71	50.41	
Q_1	measured	85.96	73.24	85.09	67.42	0.536 %
	calculated	-	72.21	85.18	67.69	
Q_2	measured	126.66	117.00	121.71	104.02	1.529 %
	calculated	-	114.96	124.96	104.53	
Q_3	measured	175.32	181.04	166.81	154.16	3.102 %
	calculated	-	172.79	172.06	151.34	
Q_4	measured	78.71	77.62	76.48	71.84	2.096 %
	calculated	-	75.20	78.06	70.89	
Q_5	measured	93.54	84.38	92.18	77.70	0.741 %
	calculated	-	83.01	92.62	77.42	

(a) original images (b) rectified images

Fig. 5. Rectified result for real images. (a) Selected quadrilaterals are denoted as yellow, and their perspectively congruent quadrilaterals are denoted as red. (b) Quadrilaterals and images are rectified by CLC-based reconstruction (Color figure online).

defined on a vanishing line, as described in Sect. 4. Finally, for each quadrilateral Q_i, we *calculate* l_1, l_2, l_3, from a measured l_0 by using the length relationships in Eq. (22).

Table 1 shows the errors in l_1, l_2, l_3, between measured lengths and calculated ones. We normalize the length differences based on l_0, in providing the errors. Notice that all errors are approximately 3 % although we have no prior knowledge of the intrinsic parameters of the camera and the aspect ratio of the similar rectangles. Thus, the CLC-based camera calibrations based on our analysis provides accurate results in most cases.

Figure 5 shows the image rectification based on CLC-based reconstruction by using the proposed method. In Fig. 5(a), We select a quadrilateral, denoted as yellow, assuming that it corresponds to an image of an unknown rectangle. Then, we calculate the lengths of partial diagonals for a perspectively congruent quadrilateral, denoted by red, with the given quadrilateral, by using the algorithm in Sect. 4. Then, we perform the CLC-based reconstruction by using Eqs. (1), (2), (3) and (4), to identify the position of the camera and the diagonal angle of a rectangle. Once the diagonal angle of the rectangle ϕ is identified, we can represent the vertices of the rectangle as $v_0 = (1, 0, 0)$, $v_1 = (\cos\phi, \sin\phi, 0)$, $v_2 = -v_0$, $v_3 = -v_1$. Finally we compute a homography H between the quadrilateral and the rectangle, and rectified an image of a quadrilateral included in the whole image using a homography mapping. Figure 5(b) shows the rectified results.

6 Conclusion

In this study, we revisited CLCs using the concepts of projective geometry. In this analysis, we showed that four lengths of partial diagonals about an image of an unknown planar rectangle are highly related to the cross-ratio and projective harmonic conjugate defined on its vanishing points. Using this observation, we provided an analytic solution to define the lengths of four partial diagonals in any perspectively similar rectangles at arbitrary positions in an image. Finally, we applied the solution to compute the partial diagonal lengths of a perspectively similar or congruent case at the center from a given off-centered quadrilateral. In the experimental results, we showed the numerical errors and some rectification results using CLC-based reconstructions for camera calibrations.

Although we analyzed CLCs based on projective geometry, we still need to consider a projective similar or congruent rectangle at the image center. For future work, we would like to investigate the analytic solutions for CLCs, where the camera calibration can be solely from an off-centered quadrilateral in a given image.

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