Chapter 17
Tutorial on Nonlinear System Identification

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Abstract Because nonlinearity is now a frequent occurrence in real-life applications, the practitioner should understand the resulting dynamical phenomena and account for them in the design process. This tutorial focuses on nonlinear system identification, which extracts relevant information about nonlinearity directly from experimental measurements. Specifically, the identification process is a progression through three steps, namely detection, characterization and parameter estimation. The tutorial presents these steps in detail and illustrates them using real aerospace structures.

Keywords Nonlinear vibrations • System identification • Detection • Characterization • Parameter estimation

17.1 Introduction

Mathematical modeling refers to the use of mathematical language to simulate the behavior of a ‘real world’ (practical) system. Its role is to provide a better understanding and characterization of the system. Theory is useful for drawing general conclusions from simple models, and computers are useful for drawing specific conclusions from complicated models. In the theory of mechanical vibrations, mathematical models—termed structural models—are helpful for the analysis of the dynamic behavior of the structure being modeled.

The demand for enhanced and reliable performance of vibrating structures in terms of weight, comfort, safety, noise and durability is ever increasing while, at the same time, there is a demand for shorter design cycles, longer operating life, minimization of inspection and repair needs, and reduced costs. With the advent of powerful computers, it has become less expensive both in terms of cost and time to perform numerical simulations, than to run a sophisticated experiment. The consequence has been a considerable shift toward computer-aided design and numerical experiments, where structural models are employed to simulate experiments, and to perform accurate and reliable predictions of the structure’s future behavior.

Even if we are entering the age of virtual prototyping [1], experimental testing and system identification still play a key role because they help the structural dynamicist to reconcile numerical predictions with experimental investigations. The term ‘system identification’ is sometimes used in a broader context in the technical literature and may also refer to the extraction of information about the structural behavior directly from experimental data, i.e., without necessarily requesting a model (e.g., identification of the number of active modes or the presence of natural frequencies within a certain frequency range). In this tutorial, system identification refers to the development (or the improvement) of structural models from input and output measurements performed on the real structure using vibration sensing devices.

Linear system identification is a discipline that has evolved considerably during the last 30 years [2, 3]. Modal parameter estimation—termed modal analysis—is indubitably the most popular approach to performing linear system identification in structural dynamics. The model of the system is known to be in the form of modal parameters, namely the natural frequencies, mode shapes and damping ratios. The popularity of modal analysis stems from its great generality; modal parameters can describe the behavior of a system for any input type and any range of the input. Numerous approaches have been developed for this purpose: Ibrahim time domain method [4], eigensystem realization algorithm [5], stochastic subspace identification method [6], polyreference least-squares complex frequency domain method [7] to name a few. It is, however, important to note that modal identification of highly damped structures or complex industrial structures with high modal density and
large modal overlap are now within reach. Unification of the theoretical development of modal identification algorithms was attempted in [8, 9], which is another sign of the maturity of this research field.

The focus in this tutorial is on structural system identification in the presence of nonlinearity. Nonlinearity is generic in Nature, and linear behavior is an exception. In structural dynamics, typical sources of nonlinearities are:

- Geometric nonlinearity results when a structure undergoes large displacements and arises from the potential energy. An illustration is the simple pendulum, the equation of motion of which is \( \ddot{\theta} + \omega_0^2 \sin \theta = 0 \); the nonlinear term \( \omega_0^2 \sin \theta \) represents geometric nonlinearity, since it models large angular motions. Large deformations of flexible elastic continua such as beams, plates and shells are also responsible for geometric nonlinearities [see, e.g., [10, 11]].
- Inertia nonlinearity derives from nonlinear terms containing velocities and/or accelerations in the equations of motion, and takes its source in the kinetic energy of the system (e.g., convective acceleration terms in a continuum and Coriolis accelerations in motions of bodies moving relative to rotating frames).
- A nonlinear material behavior may be observed when the constitutive law relating stresses and strains is nonlinear. This is often the case in foams [12–14] and in resilient mounting systems such as rubber isolators [15].
- Damping dissipation is essentially a nonlinear and still not fully modeled and understood phenomenon. The modal damping assumption is not necessarily the most appropriate representation of the physical reality, and its widespread use is to be attributed to its mathematical convenience. Dry friction effects (bodies in contact, sliding with respect to each other) and hysteretic damping are examples of nonlinear damping, see, e.g., [16–19]. It is important to note that dry friction affects the dynamics especially for small-amplitude motion, which is contrary to what might be expected by conventional wisdom.
- Nonlinearity may also result due to boundary conditions (for example, free surfaces in fluids, vibro-impacts due to loose joints or contacts with rigid constraints, clearances, imperfectly bonded elastic bodies), or certain external nonlinear body forces (e.g., magnetoelastic, electrodynamic or hydrodynamic forces). Clearance and vibro-impact nonlinearity possesses nonsmooth force-deflection characteristic and generally requires a special treatment compared with other types of nonlinearities [20].

Many practical examples of nonlinear dynamic behavior have been reported in the engineering literature. In the automotive industry, brake squeal which is a self-excited vibration of the brake rotor related to the friction variation between the pads and the rotor is an irritating but non-life-threatening example of an undesirable effect of nonlinearity [21]. Many automobiles have viscoelastic engine mounts which show marked nonlinear behavior: dependence on amplitude, frequency and preload. In an aircraft, besides nonlinear fluid-structure interaction, typical nonlinearities include backlash and friction in control surfaces and joints, hardening nonlinearities in the engine-to-pylon connection, and saturation effects in hydraulic actuators. In [22], a commercial airplane is described in which the propellers induced a subharmonic vibration of order 1/2 in the wings which produced a subharmonic of order 1/4 in the rudder. The oscillations were so violent that the effects on the airplane were catastrophic [23]. In mechatronic systems, sources of nonlinearities are friction in bearings and guideways, as well as backlash and clearances in robot joints. In civil engineering, many demountable structures such as grandstands at concerts and sporting events are prone to substantial structural nonlinearity as a result of looseness of joints. This creates both clearances and friction and may invalidate any linear model-based simulations of the behavior created by crowd movement. Nonlinearity may also arise in a damaged structure: fatigue cracks, rivets and bolts that subsequently open and close under dynamic loading or internal parts impacting upon each other.

With continual interest to expand the performance envelope of structures at ever increasing speeds, there is the need for designing lighter, more flexible, and consequently, more nonlinear structural elements. It follows that the demand to utilize nonlinear (or even strongly nonlinear) structural components is increasingly present in engineering applications. It is, therefore, rather paradoxical to observe that very often linear behavior is taken for granted in structural dynamics. Why is it so? It should be recognized that at sufficiently small-amplitude motions, linear theory may be accurate for modeling, although it is not always the case (e.g., dry friction). However, the main reason is that nonlinear dynamical systems theory is far less established than its linear counterpart. Indeed, the basic principles that apply to a linear system and that form the basis of modal analysis are no longer valid in the presence of nonlinearity. In addition, even weak nonlinear systems can exhibit extremely interesting and complex phenomena which linear systems cannot. These phenomena include jumps, bifurcations, saturation, subharmonic, superharmonic and internal resonances, resonance captures, limit cycles, modal interactions and chaos. Readers who look for an introduction to nonlinear oscillations may consult [23–26]. More mathematically inclined readers may refer to [27, 28].

This is not to say that nonlinear systems have not received considerable attention during the last decades. Even if, for years, one way to study nonlinear systems was the linearization approach [29, 30], many efforts have been spent in order to develop theories for the investigation of nonlinear systems in structural dynamics. A nonlinear extension of the concept of mode shapes was proposed in [31, 32] and further investigated in [33–36]. Weakly nonlinear systems were thoroughly analyzed...
using perturbation theory [23, 37–39]. Perturbation methods include for instance the method of averaging, the Lindstedt-Poincaré technique and the method of multiple scales and aim at obtaining asymptotically uniform approximations of the solutions. During the last decade or so, one has witnessed a transition from weakly nonlinear structures to strongly nonlinear structures (by strongly nonlinear systems, a system for which the nonlinear terms are the same order as the linear terms is meant) thanks to the extension of classical perturbation techniques [40, 41] and the development of new methodologies [20, 42–44].

Recently, a few studies proposed to take advantage of nonlinearities instead of ignoring or avoiding them, which represents an interesting shift in paradigm. For example, the concept of parametric resonance is exploited to design microelectromechanical oscillators with filtering capabilities in [45]. In [46–48], it is shown that essential (i.e., nonlinearizable) nonlinearity leads to irreversible nonlinear energy transfer phenomena between subsystems—termed nonlinear energy pumping. In [49], chaotic interrogation and phase space reconstruction are used to assess the strength of a bolted connection in a composite beam. In [50], the geometric shape of dynamic attractors is exploited to enhance small parametric variations in a system.

Focusing now on the development (or the improvement) of structural models from experimental measurements in the presence of nonlinearity, i.e., nonlinear system identification, one is forced to admit that there is no general analysis method that can be applied to all systems in all instances [see, e.g, previous overviews [51, 52]], as it is the case for modal analysis in linear structural dynamics. In addition, many techniques which are capable of dealing with systems with low dimensionality collapse if they are faced with system with high modal density. Two reasons for this failure are the inapplicability of various concepts of linear theory and the highly ‘individualistic’ nature of nonlinear systems. A third reason is that the functional \( S(x) \) which maps the input \( x(t) \) to the output \( y(t) = S(x(t)) \), is not known beforehand. For instance, the ubiquitous Duffing oscillator [53], the equation of motion of which is \( m\ddot{y}(t) + c\dot{y}(t) + ky(t) + ky^3(t) = x(t) \), represents a typical example of polynomial form of restoring force nonlinearity, whereas hysteretic damping is an example of nonpolynomial form of nonlinearity. This represents a major difficulty compared with linear system identification for which the structure of the functional is well defined.

Even if there is a difference between the way one did nonlinear system identification ‘historically’ and the way one would do it now, the identification process may be regarded as a progression through three steps, namely detection, characterization and parameter estimation, as outlined in Fig. 17.1. Once nonlinear behavior has been detected, a nonlinear system is said to be characterized after the location, type and functional form of all the nonlinearities throughout the system are determined. The parameters of the selected model are then estimated using linear least-squares fitting or nonlinear optimization algorithms depending upon the method considered.

Nonlinear system identification is an integral part of the verification and validation (V&V) process. According to [54], verification refers to solving the equations correctly, i.e., performing the computations in a mathematically correct manner, whereas validation refers to solving the correct equations, i.e., formulating a mathematical model and selecting the coefficients such that physical phenomenon of interest is described to an adequate level of fidelity. As stated in [55], one definition that captures many of the important aspects of model validation is taken from the simulation sciences literature:

The substantiation that a model within its domain of applicability possesses a satisfactory range of accuracy consistent with the intended application of the model [56]

Scope of the Presentation

The motivation behind this tutorial presentation is threefold. First, it is meant to provide a concise point of departure for researchers and practitioners alike wishing to assess the current state of the art in the identification of nonlinear structural models. Second, the tutorial intends to review several methods that have been proposed in the technical literature and to highlight some of the reasons that prevent these techniques from being applied to complex structures. The last goal is to identify future research needs which would help to ‘push the envelope’ in nonlinear system identification.
1. Detection

Yes or No?

Aim: detect whether a nonlinearity is present or not (e.g., Yes)

2. Characterization

What? Where? How?

Aim: a. determine the location of the non-linearity (e.g., at the joint)
   b. determine the type of the non-linearity (e.g., Coulomb friction)
   c. determine the functional form of the non-linearity
   [e.g., \( f_{NL}(y, \dot{y}) = \alpha \text{sign}(\dot{y}) \)]

3. Parameter estimation

How much?

Aim: determine the coefficient of the non-linearity (e.g., \( \alpha = 5.47 \))

\[ f_{NL}(y, \dot{y}) = 5.47 \text{sign}(\dot{y}) \text{ at the joint} \]

Fig. 17.1 Identification process

References
