Chapter 23
Fundamental Processes

Ever since J.C. Maxwell formulated his unifying electromagnetic theory in 1873, the phenomenon of electromagnetic radiation has fascinated the minds of theorists as well as experimentalists. The idea of displacement currents was as radical as it was important to describe electromagnetic waves. It was only 14 years later when G. Hertz in 1887 succeeded to generate, emit and receive again electromagnetic waves, thus, proving experimentally the existence of such waves as predicted by Maxwell's equations. The sources of the radiation are oscillating electric charges and currents in a system of metallic wires. In this text, we discuss the generation of electromagnetic radiation emitted by free electrons from first principles involving energy and momentum conservation as well as Maxwell's equations.

23.1 Radiation from Moving Charges

Analytical formulation of the emission of electromagnetic radiation posed a considerable challenge. Due to the finite speed of light one cannot make a snapshot to correlate the radiation field at the observer with the position of radiating charges. Rather, the radiation field depends on the position of the radiating charges some time earlier, at the retarded time, when the radiation was emitted. Already 1867 L. Lorenz included this situation into his formulation of the theory of electromagnetic fields and introduced the concept of retarded potentials. He did, however, not offer a solution to the retarded potentials of a point charge. Liénard [1] in 1898 and independently in 1900 Wiechert [2] derived for the first time expressions for retarded potentials of point charges like electrons. These potentials are now called the Liénard-Wiechert potentials relating the scalar and vector potential of electromagnetic fields at the observation point to the location of the emitting charges and currents at the time of emission. Using these potentials, Liénard was able to

H. Wiedemann, *Particle Accelerator Physics*, Graduate Text in Physics,
https://doi.org/10.1007/978-3-319-18317-6_23

calculate the energy lost by electrons while circulating in a homogenous magnetic field.

In 1907 [3, 4] and 1912 [5] Schott formulated and published his classical theory of radiation from an orbiting electron. He was primarily interested in the spectral distribution of radiation and hoped to find an explanation for atomic radiation spectra. Verifying Liénard's conclusion on the energy loss, he derived the angular and spectral distribution and the polarization of radiation. Since this classical approach to explain atomic spectra was destined to fail, his paper was forgotten and only 40 years later were many of his findings rediscovered.

23.1.1 Why Do Charged Particles Radiate?

Before we dive into the theory of electromagnetic radiation in more detail we may first ask ourselves why do charged particles radiate at all? Emission of electromagnetic radiation from charged particle beams (microwaves or synchrotron radiation) is a direct consequence of the finite velocity of light. A charged particle in uniform motion through vacuum is the source of electric field lines emanating from the charge radially out to infinity. While the charged particle is at rest or moving uniformly these field lines also are at rest or in uniform motion together with the particle. Now, we consider a particle being suddenly accelerated for a short time. That means the field lines should also be accelerated. The fact that the particle has been accelerated is, however, still known only within the event horizon in a limited area close to the particle. The signal of acceleration travels away from the source (particle) only at the finite speed of light. Field lines close to the charged particle are directed radially toward the particle, but far away, the field lines still point to the location where the particle would be had it not been accelerated. Somewhere between those two regimes the field lines are distorted and it is this distortion travelling away from the particle at the speed of light what we call electromagnetic radiation. The magnitude of these field distortions is proportional to the acceleration.

In a linear accelerator, for example, electrons are accelerated along the linac axis and therefore radiate. The degree of actual acceleration, however, is very low because electrons in a linear accelerator travel close to the velocity of light. The closer the particle velocity is to the velocity of light the smaller is the actual acceleration gained from a given force, and the radiation intensity is very small. In a circular accelerator like a synchrotron, on the other hand, particles are deflected transversely to their direction of motion by magnetic fields. Orthogonal acceleration or the rate of change in transverse velocity is very large because the transverse particle velocity can increase from zero to very large values in a very short time while passing through the magnetic field. Consequently, the emitted radiation intensity is very large. Synchrotron radiation sources come therefore generally in form of circular synchrotrons. Linear accelerators can be the source of intense synchrotron radiation in conjunction with a transversely deflecting magnet.

23.1.2 Spontaneous Synchrotron Radiation

Charged particles do not radiate while in uniform motion, but during acceleration a rearrangement of its electric fields is required and this field perturbation, traveling away from the charge at the velocity of light, is what we observe as electromagnetic radiation. Free accelerated electrons radiate similarly to those in a radio antenna, although now the source (antenna) is moving. Radiation from a fast moving particle source appears to the observer in the laboratory as being all emitted in the general direction of motion of the particle. This forward collimation is particularly effective for highly relativistic electrons where most of the radiation is concentrated into a small cone around the forward direction with an opening angle of $1/\gamma$, typically 0.1 to 1 mrad, where γ is the particle energy in units of its rest mass.

Radiation can be produced by magnetic deflection in a variety of ways. Whether it be a single kick-like deflection or a periodic right-left deflection, the radiation characteristics reflect the particular mode of deflection. Specific radiation characteristics can be gained through specific modes of deflections. Here, we will only shortly address the main processes of radiation generation and come back later for a much more detailed discussion of the physical dynamics.

In an undulator the electron beam is periodically deflected transversely to its direction of motion by weak sinusoidally varying magnetic fields, generating periodic perturbations of the electric field lines. A receiving electric field detector recognizes a periodic variation of the transverse electromagnetic field components and interprets this as quasi monochromatic radiation. In everyday life periodic acceleration of electrons occurs in radio and TV antennas and we may receive these periodic field perturbations with a radio or TV receiver tuned to the frequency of the periodic electron motion in the emitting antenna. The fact that we consider relativistic electrons is not fundamental, but we restrict ourselves in this text to high energy electrons only.

To the particle the wavelength of the emitted radiation is equal to the undulator period length (λ_{p}) divided by γ due to relativistic Lorentz contraction. In a stationary laboratory system, this wavelength appears to the observer further reduced by another factor 2γ due to the Doppler effect. The undulator period length of the order of centimeters is thus reduced by a factor γ^2 (10^6–10^8) to yield short wavelength radiation in the VUV and x-ray regime. The spectral resolution of the radiation is proportional to the number of undulator periods N_{p} and its wavelength can be shifted by varying the magnetic field. Most radiation is emitted within the small angle of $(\gamma\sqrt{N_{\mathrm{p}}})^{-1}$.

Increasing the magnetic field strength causes the pure sinusoidal transverse motion of electrons in an undulator to become distorted due to relativistic effects generating higher harmonic perturbations of the electron trajectory. Consequently, the monochromatic undulator spectrum exhibits higher harmonics and changes into a line spectrum. For very strong fields, many harmonics are generated which eventually merge into a continuous spectrum from IR to hard x-rays. In this extreme, we call the source magnet a wiggler magnet. The spectral intensity varies little over a broad wavelength range and drops off exponentially at photon energies higher than the critical photon energy, $\varepsilon_{\text{crit}} \propto B\gamma^2$. Changing the magnetic field, one may vary the critical photon energy to suit experimental requirements. Compared to bending magnet radiation, wiggler radiation is enhanced by the number of magnet poles N_{pol} and is well collimated within an angle of $1/\gamma$ to say $10/\gamma$, or a few mrad.

A bending magnet is technically the most simple radiation source. Radiation is emitted tangentially to the orbit similar to a search light while well collimated in the non-deflecting, or vertical plane. The observer at the experimental station sees radiation from only a small fraction of the circular path which can be described as a piece of a distorted sinusoidal motion. The radiation spectrum is therefore similar to that of a wiggler magnet while the intensity is due to only one pole. Because bending magnets define the geometry of the electron beam transport system or accelerator, it is not possible to freely choose the field strength and the critical photon energy is therefore fixed. Sometimes, specially in lower energy storage rings, it is desirable to extend the radiation spectrum to higher photon energies into the x-ray regime. This can be accomplished by replacing one or more conventional bending magnet with a superconducting magnet or superbends at much higher field strength. To preserve the ring geometry the length of these superbends must be chosen such that the deflection angle is the same as it was for the conventional magnet that has been replaced. Again, superbends are part of the ring geometry and therefore the field cannot be changed.

A more flexible version of a radiation hardening magnet is the wavelength shifter. This is a magnet which consists of a high field central pole and two weaker outside poles to compensate the deflection by the central pole. The total deflection angle is zero and therefore the field strength can be chosen freely to adjust the critical photon energy. It's design is mostly based on superconducting magnet technology, particularly in low energy accelerators, to extend (shift) the critical photon energy available from bending magnets to higher values.

A variety of more complicated magnetic field arrangements have been developed to primarily generate circularly or elliptically polarized radiation. In such magnets horizontal as well as vertical magnetic fields are sequentially employed to deflect electrons into some sort of helical motion giving raise to the desired polarization effect.

23.1.3 Stimulated Radiation

The well defined time structure and frequency of undulator radiation can be used to stimulate the emission of even more radiation. In an optical klystron [6] coherent radiation with a wavelength equal to the fundamental undulator wavelength enters an undulator together with the electron beam. Since the electron bunch length is much longer than the radiation wavelength, some electrons loose energy to the radiation field and some electrons gain energy from the radiation field while interacting with the radiation field. This energy modulation can be transformed into a density modulation by passing the modulated electron beam through a dispersive section. This section consists of deflecting magnetic fields arranged in such a way that the total path length through the dispersive section depends on the electron energy. The periodic energy modulation of the electron bunch then converts into a periodic density modulation. Now we have microbunches at a distance of the undulator radiation wavelength. This microbunched beam travels through a second undulator where again particles can loose or gain energy from the radiation field. Due to the microbunching, however, most particles are concentrated at phases where there is only energy transfer from the particle to the radiation field, thus providing a high gain of radiation intensity.

In a more efficient variation of this principle, radiation emitted by electrons passing through an undulator is recycled by optical mirrors in such a way that it passes through the same undulator again together with another electron bunch. The external field stimulates more emission of radiation from the electrons, and is again recycled to stimulate a subsequent electron bunch until there are no more bunches in the electron pulse. Generating from a linear accelerator a train of thousands of electron bunches one can generate a large number of interactions, leading to an exponential growth of electromagnetic radiation. Such a devise is called a free electron laser or short FEL.

23.1.4 Electron Beam

In this text we consider radiation from relativistic electron beams only. Such beams can be generated efficiently by acceleration in microwave fields. The oscillatory nature of microwaves makes it impossible to produce a uniform stream of particles, and the electron beam is modulated into bunches at the distance of the microwave wavelength. The bunched nature of the electron beam and the fact that these bunches circulate in a storage ring determines the time structure and spectrum of the emitted radiation. Typically, the bunch length in storage rings is 30–100 ps at a distance of 2–3 ns depending on the rf-frequency.

During the storage time of the particle beam, the electrons radiate and it is this radiation that is extracted and used in experiments of basic and applied research. Considering, for example, only one bunch rotating in the storage ring, the experimenter would observe a light flash at a frequency equal to the revolution frequency f_{rev}. Because of the extremely short duration of the light flash many harmonics of the revolution frequency appear in the light spectrum. At the low frequency end of this spectrum, however, no radiation can be emitted for wavelength longer than about the dimensions of the metallic vacuum chamber surrounding the electron beam. For long wavelengths the metallic boundary conditions for electromagnetic fields cannot be met prohibiting the emission of radiation. Practically, useful radiation is observed from storage rings only for wavelengths below the microwave regime, or for $\lambda \lesssim 1$ mm.

23.2 Conservation Laws and Radiation

The emission of electromagnetic radiation from free electrons is a classical phenomenon. We may therefore use a visual approach to gain some insight into conditions and mechanisms of radiation emission. First, we will discuss necessary conditions that must be met to allow an electron to emit or absorb a photon. Once such conditions are met, we derive from energy conservation a quantity, the Poynting vector, relating energy transport or radiation to electromagnetic fields. This will give us the basis for further theoretical definitions and discussions of radiation phenomena.

The emission of electromagnetic radiation involves two components, the electron and the radiation field. For the combined system energy-momentum conservation must be fulfilled. These conservation laws impose very specific selection rules on the kind of emission processes possible. To demonstrate this, we plot the energy versus momentum for both electron and photon. In relativistic terms, we have the relation $\gamma = \sqrt{1 + (\beta\gamma)^2}$ between energy γ and momentum $\beta\gamma$. For consistency in quantities used we normalize the photon energy to the electron rest energy, $\gamma_\mathrm{p} = \varepsilon_\mathrm{p}/mc^2$, where $\varepsilon_\mathrm{p} = \hbar\omega$ is the photon energy and mc^2 the electron rest mass while the normalized photon momentum is $\beta_\mathrm{p}\gamma = \hbar k_\mathrm{p}/mc^2$. Similarly, we express the speed of light by $\beta_\mathrm{p} = c_\mathrm{p}/c = 1/n$ where $n > 1$ is the refractive index of the medium surrounding the photon. With these definitions and assuming, for now, vacuum as the medium ($n = 1$) the location of a particle or photon in energy-momentum space is shown in Fig. 23.1(left).

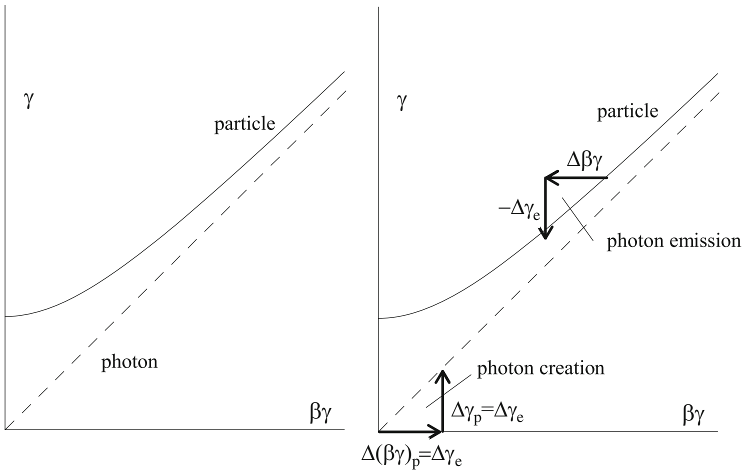


Fig. 23.1 Energy-momentum relationship for particles and photons (*left*). Violation of energy or momentum conservation during emission and absorption of electromagnetic radiation by a free electron travelling in perfect vacuum ($\beta_\mathrm{p} = 1$) (*right*)

Energy and momentum of a particle are related such that it must be located on the "particle"-line in Fig. 23.1(left) while a photon is always located on the "photon"-line. Transfer of energy between particle and photon must obey energy-momentum conservation. In Fig. 23.1(right) we apply this principle to a free electron in vacuum emitting (absorbing) a photon. To create a photon the electron would have to loose (gain) an amount of momentum which is numerically equal to the energy gained (lost) by the photon. Clearly, in this case the electron would end up at a location off the "particle"-line, thus violating momentum conservation. That cannot be, and such a process is therefore not permitted. A free electron in vacuum cannot emit or absorb a photon without violating energy-momentum conservation.

23.2.1 Cherenkov Radiation

We have been careful to assume an electron in perfect vacuum. What happens in a material environment is shown in Fig. 23.2. Because the refractive index $n > 1$, the phase velocity of radiation is less than the velocity of light in vacuum and with $\beta = 1/n$, the "photon"-line is tilted towards the momentum axis.

Formally, we obtain this for a photon from the derivative $\mathrm{d}\gamma/\mathrm{d}(\beta\gamma)$ which we expand to $\frac{\mathrm{d}\gamma}{\mathrm{d}(\beta\gamma)} = \frac{\mathrm{d}\gamma}{\mathrm{d}\omega}\frac{\mathrm{d}\omega}{\mathrm{d}k}\frac{\mathrm{d}k}{\mathrm{d}(\beta\gamma)}$ and get with $\gamma = \hbar\omega/mc^2, k = n\frac{\omega}{c}$, and the

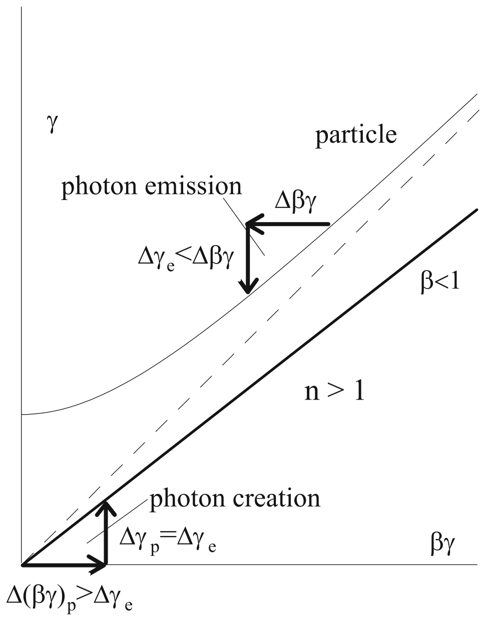


Fig. 23.2 Energy and momentum conservation in a refractive environment with $n > 1$

momentum $\beta\gamma = \frac{\hbar}{mc}k$, the derivative

$$\frac{\mathrm{d}\gamma_{\mathrm{p}}}{\mathrm{d}\,(\beta\gamma)_{\mathrm{p}}} = \frac{1}{n} < 1\,, \tag{23.1}$$

where we have added the subscript $_{\mathrm{p}}$ to differentiate between photon and electron parameters.

The dispersion function for a photon in a material environment has a slope less than unity as shown in Fig. 23.2. In this case, the numerical value of the photon momentum is less than the photon energy, analogous to the particle case. To create a photon of energy γ_{p} we set $\gamma_{\mathrm{p}} = -\Delta\gamma = -\beta\Delta\beta\gamma$ from (1.30), where from (23.1) the photon energy $\gamma_{\mathrm{p}} = \frac{1}{n}\,(\beta\gamma)_{\mathrm{p}}$ and get from both relations $(\beta\gamma)_{\mathrm{p}} = -n\beta\Delta\beta\gamma$. Because of symmetry, no momentum transverse to the particle trajectory can be exchanged, which means radiation is emitted uniformly in azimuth. The change in longitudinal momentum along the trajectory is $-\Delta\beta\gamma = \left.(\beta\gamma)_{\mathrm{p}}\right|_{\parallel} = (\beta\gamma)_{\mathrm{p}}\cos\theta$. In a dielectric environment, free electrons can indeed emit or absorb a photon although, only in a direction given by the angle θ with respect to the electron trajectory. This radiation is called Cherenkov radiation, and the Cherenkov angle θ is given by the Cherenkov condition

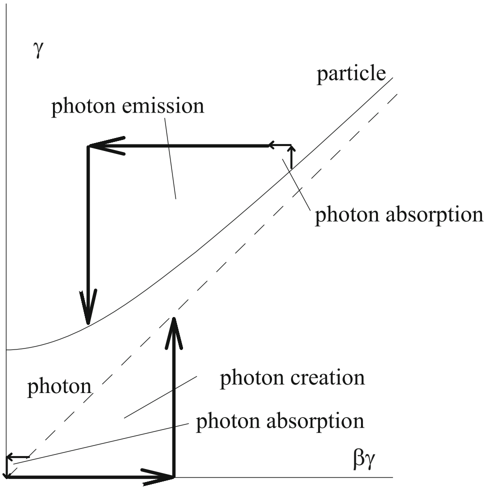
$$n\beta\cos\theta = 1\,. \tag{23.2}$$

Note, that this condition is not the same as saying whenever an electron passes though a refractive medium with $n > 1$ there is Cherenkov radiation. The Cherenkov condition requires that $n\beta > 1$ which is, for example, not the case for an electron beam of less than 20 MeV traveling through air.

23.2.2 Compton Radiation

To generate electromagnetic radiation from free electrons in vacuum without violating energy-momentum conservation, we may employ the Compton effect which is the scattering of an incoming photon by the electron. In energy-momentum space this process is shown in Fig. 23.3. The electron, colliding head-on with an incoming photon absorbs this photon and emits again a photon of different energy. In this process it gains energy but looses momentum bringing the electron in the energy-momentum space to an intermediate point, P_I, from where it can reach its final state on the "particle"-line by emitting a photon as shown in Fig. 23.3. This is the process involved in the generation of synchrotron radiation. Static magnetic fields in the laboratory system appear as electromagnetic fields like an incoming (virtual) photon in the electron system with which the electron can collide. Energy-momentum conservation give us the fundamental and necessary conditions under

Fig. 23.3 Energy and momentum conservation for Compton scattering process



which a free charged particle can emit or absorb a photon. We turn our attention now to the actual interaction of charged particles with an electromagnetic field.

23.3 Electromagnetic Radiation

Phenomenologically, synchrotron radiation is the consequence of a finite value for the velocity of light. Electric fields extend infinitely into space from charged particles in uniform motion. When charged particles become accelerated, however, parts of these fields cannot catch up with the particle anymore and give rise to synchrotron radiation. This happens more so as the particle velocity approaches the velocity of light.

The emission of light can be described by applying Maxwell's equations to moving charged particles. The mathematical derivation of the theory of radiation from Maxwell's equations is straightforward although mathematically elaborate and we will postpone this to Chap. 25. Here we follow a more intuitive discussion[1] which displays visually the physics of synchrotron radiation from basic physical principles.

Electromagnetic radiation occurs wherever electric and magnetic fields exist with components orthogonal to each other such that the Poynting vector

$$\boldsymbol{S} = \frac{1}{\mu_0}[\boldsymbol{E} \times \boldsymbol{B}] \neq 0\,. \tag{23.3}$$

[1]The author would like to thank Prof. M. Eriksson, Lund, Sweden for introducing him to this approach into the theory of synchrotron radiation.

It is interesting to ask what happens if we have a static electric and magnetic field such that $[\boldsymbol{E}\times\boldsymbol{B}] \neq 0$. We know there is no radiation but the Poynting vector is nonzero. Applying energy conservation (1.87) we find the first two terms to be zero which renders the third term zero as well. For a static electric and magnetic field the integral defining the radiation loss or absorption is equal to zero and therefore no radiation or energy transport occurs.

Similarly, in case of a stationary electrostatic charge, we note that the electrostatic fields extend radially from the charge to infinity which violates the requirement that the field be orthogonal to the direction of observation or energy flow. Furthermore, the charge is stationary and therefore there is no magnetic field.

23.3.1 Coulomb Regime

Next, we consider a charge in uniform motion. In the rest frame of the moving charge we have no radiation since the charge is at rest as just discussed. In the laboratory system, however, the field components are different. Since the charge is moving, it constitutes an electric current which generates a magnetic field. Formulating the Poynting vector in the laboratory system we express the fields by the pure electric field in the particle rest frame $\mathcal{L}^*$. We accomplish that by an inverse Lorentz transformations to (1.9), where the laboratory system $\mathcal{L}$ now moves with the velocity $-\beta_z$ with respect to $\mathcal{L}^*$ and β_z in (1.9) must be replaced by $-\beta_z$ for

$$\begin{pmatrix} E_x \\ E_y \\ E_z \\ cB_x \\ cB_y \\ cB_z \end{pmatrix} = \begin{pmatrix} \gamma & 0 & 0 & 0 & \beta_z\gamma & 0 \\ 0 & \gamma & 0 & -\beta_z\gamma & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -\beta_z\gamma & 0 & \gamma & 0 & 0 \\ \beta_z\gamma & 0 & 0 & 0 & \gamma & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} E_x^* \\ E_y^* \\ E_z^* \\ cB_x^* \\ cB_y^* \\ cB_z^* \end{pmatrix} . \tag{23.4}$$

In the laboratory system $\mathcal{L}$ the components of the Poynting vector (23.3) become then with $\boldsymbol{B}^* = 0$

$$\begin{aligned} c\mu_0 S_x &= -\gamma\beta_z E_x^* E_z^* , \\ c\mu_0 S_y &= -\gamma\beta_z E_y^* E_z^* , \\ c\mu_0 S_z &= +\gamma^2\beta_z \left(E_x^{*2} + E_y^{*2}\right) , \end{aligned} \tag{23.5}$$

where * indicates quantities in the moving system $\mathcal{L}^*$ and $\beta_z = v_z/c$. The Poynting vector is nonzero and describes the flow of field energy in the environment of a moving charged particle. The fields drop off rapidly with distance from the particle and the "radiation" is therefore confined close to the location of the particle. Specifically, the fields are attached to the charge and travel in the vicinity and with the charge. This part of electromagnetic radiation is called the Coulomb regime in contrast to the radiation regime and is, for example, responsible for the transport of electric energy along electrical wires and transmission lines.

We will ignore this regime in our further discussion of synchrotron radiation. It should be noted, however, that measurements of radiation parameters close to radiating charges may be affected by the presence of the Coulomb radiation regime. Such situations occur, for example, when radiation is observed close to the source point. Related theories deal with this mixing by specifying a formation length defining the minimum distance from the source required to sufficiently separate the Coulomb regime from the radiation regime.

23.3.2 Radiation Regime

In this text we are only interested in the radiation regime and therefore ignore from now on the Coulomb regime. To describe the physics of emission of radiation, we consider a coordinate system moving with a constant velocity equal to that of the charged particle and associated electric fields. The charge is at rest in the moving reference system, the electric field lines extend radially out to infinity, and there is no radiation as discussed before. Acceleration of the charge causes it to move with respect to this reference system generating a distortion of the purely radial electric fields of a uniformly moving charge (Fig. 23.4). This distortion, resulting in a rearrangement of field lines to the new charge position, travels outward at the velocity of light giving rise to what we call radiation.

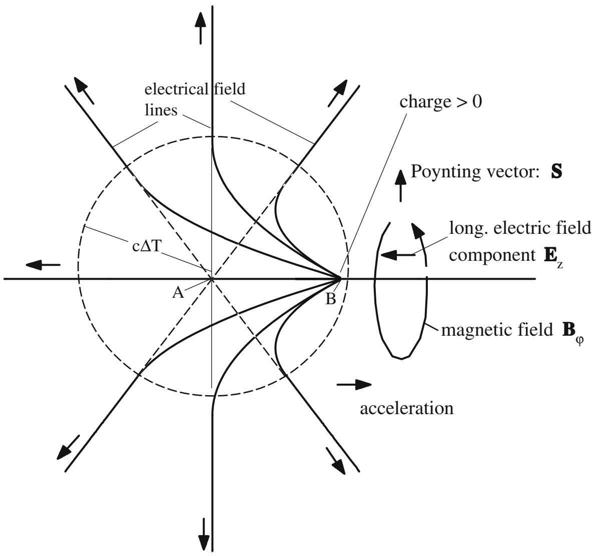


Fig. 23.4 Distortion of fields due to longitudinal acceleration

To be more specific, we consider a positive charge in uniform motion for $t \leqslant 0$, the we apply an accelerating force at time $t = 0$ for a time ΔT and observe the charged particle and its fields in the uniformly moving frame of reference. Due to acceleration the charge moves in this reference system during the time ΔT from point A to point B and as a consequence the field lines become distorted within a radius $c\Delta T$ from the original location A of the particle. It is this distortion, travelling away from the source at the speed of light, that we call radiation.

The effects on the fields are shown schematically in Fig. 23.4 for an acceleration of a positive charge along its direction of motion. At time $t = 0$ all electric field lines extend radially from the charge located at point A to infinity. During acceleration fieldlines emerge from the charge now at locations between A and B. The new field lines must join the old field lines which, due to the finite velocity of light, are still unperturbed at distances larger than $c\Delta T$. As long as the acceleration lasts, a nonradial field component, parallel and opposite to the acceleration, is created. Furthermore, the moving charge creates an azimuthal magnetic field $B^*_\varphi(t)$ and the Poynting vector becomes nonzero causing the emission of radiation from an accelerated electrical charge.

Obviously, acceleration would not result in any radiation if the velocity of propagation for electromagnetic fields were infinite ($c \to \infty$). In this case the radial fields at all distances from the charge would instantly move in synchrony with the movement of the charge. Only the Coulomb regime would exist.

The electrical field perturbation is proportional to the electrical charge q and the acceleration a^*. Acceleration along the z-axis generates an electric field $\boldsymbol{E}^*_z \neq 0$ and its component normal to the direction of observation scales like $\sin\Theta^*$, where Θ^* is the angle between the line of observation and the direction of particle acceleration. During the acceleration a fixed amount of field energy is created which propagates radially outward from the source. Since the total radiation energy must stay constant and the volume of the expanding spherical sheath of field perturbation increases like R^2, the field strength decays linear with distance R. With this, we make the ansatz

$$\boldsymbol{E}^*_\parallel = -\frac{1}{4\pi\epsilon_0}\frac{e\boldsymbol{a}^*}{c^2R}\sin\Theta^* \tag{23.6}$$

for the electric field, where we have added a factor c^2 in the denominator to be dimensionally correct. For an electron ($e < 0$) the field perturbation would be positive pointing in the direction of the acceleration. As expected from the definition of the Poynting vector, the radiation is emitted predominantly orthogonal to the direction of acceleration and is highly polarized in the direction of acceleration. From (1.89)

$$\boldsymbol{S} = \frac{1}{c\mu_0}E^{*2}_\parallel\boldsymbol{n}^*\,, \tag{23.7}$$

where $\mathbf{n}^*$ is the unit vector in the direction of observation from the observer toward the radiation source. The result is consistent with our earlier finding that no free

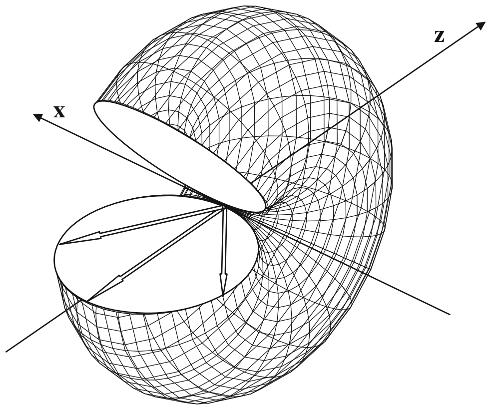


Fig. 23.5 Spatial radiation distribution in the rest frame of the radiating charge

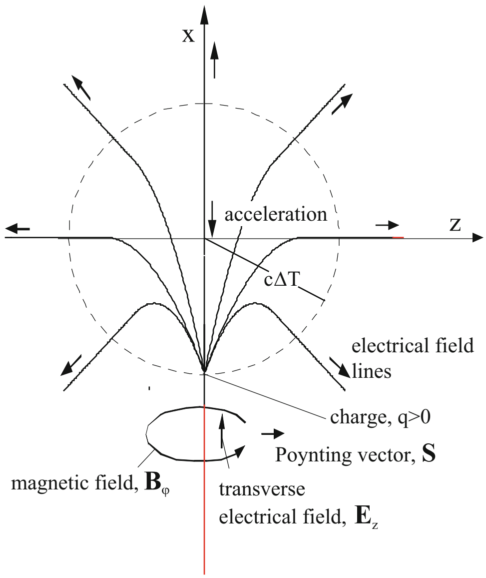


Fig. 23.6 Distortion of field lines due to transverse acceleration

radiation is emitted from a charge at rest or uniform motion ($\boldsymbol{a}^* \rightarrow 0$). The spatial radiation distribution is from (23.6) and (23.7) characterized by a $\sin^2 \Theta^*$-distribution resembling the shape of a doughnut as shown in Fig. 23.5, where the acceleration occurs along the x-axis.

Acceleration may not only occur in the longitudinal direction but also in the direction transverse to the velocity of the particle as shown in Fig. 23.6. The distortion of field lines in this case creates primarily transverse or radial field components. The radiation field component transverse to the direction of observation is

$$\boldsymbol{E}_\perp^* = -\frac{\mu_0}{4\pi}\frac{e\boldsymbol{a}^*}{R}\cos\Theta^*. \tag{23.8}$$

This case of transverse acceleration describes the appearance of synchrotron radiation created by charged particles being deflected in magnetic fields. Similar to (23.7) the Poynting vector for transverse acceleration is

$$\boldsymbol{S} = \frac{1}{c\mu_0} E_{\perp}^{*2} \boldsymbol{n}^{*}. \tag{23.9}$$

Problems

23.1 (S). Consider a relativistic electron traveling along the z-axis. In its own system, the electrical field lines extend radially from the charge. Considering only the xz-plane, derive an expressions for the electrical field lines in the laboratory frame of reference. Sketch the field pattern in the electron rest frame and in the laboratory system of reference.

23.2 (S). Use a 10 MeV electron beam passing through atmospheric air. Can you observe Cherenkov radiation and if so at what angle? Answer the same questions also for a 50 MeV electron beam. Describe and explain with Fig. 23.2 the fundamental difference of your results ($n_{\mathrm{air}} = 1.0002769$ for $\lambda = 5,600$ Å).

23.3 (S). A 10 MeV electron beam passes with normal incidence through a plate of polystyren scintilator ($n = 1.58$). Is there any Cherenkov radiation and if so at what angle? Where does this radiation escape the plate?

23.4 (S). An electron beam orbits in a circular accelerator with a circumference of 300 m at an average current of 250 mA and the beam consists of 500 equally spaced bunches each 1 cm long. How many particles are orbiting? How many particles are in each bunch? Assuming the time structure of synchrotron radiation is the same as the particle beam time-structure specify and plot the radiation time-structure in the photon beam line.

23.5 (S). From Heisenberg's uncertainty relation construct a "characteristic volume" of a photon with energy $\epsilon_{\mathrm{ph}} = \hbar\omega$. What is the average electric field in this volume for a 1 eV photon and an X-ray photon of 10 keV?

23.6 (S). Derive from (1.38) the formula for the classical Doppler effect valid for sound waves emitted at a frequency f_{s} from a source moving with velocity v and received at an angle ϑ.

23.7 (S). Consider an electron storage ring at an energy of 800 MeV, a circulating current of 1 amp and a bending radius of $\rho = 1.784$ m. Calculate the energy loss per turn, and the total synchrotron radiation power from all bending magnets. What would the radiation power be if the particles were 800 MeV muons.

23.8 (S). For the electron beam of exercise 23.7 calculate the critical energy and plot the radiation spectrum. What is the useful frequency range for experimentation assuming that the spectral intensity should be within 1 % of the maximum value? Express the maximum useful photon energy in terms of the critical photon energy (only one significant digit!).

23.9 (S). What beam energy would be required to produce x-rays at a critical photon energy of 10 keV from the storage ring of exercise 23.7? Is that energy feasible from a conventional magnet point of view or would the ring have to be larger? What would the new bending radius and energy have to be?

23.10. Verify that for a 10 MeV electron colliding head-on with a Ti-Saphire laser ($\lambda = 0.8\,\mu$m) the wavelength in it's own system is $\lambda^* = 40.88$ nm. Also show that the wavelength of the backscattered photon in the laboratory system is $\lambda_\gamma = 10.4$ Å. What electron beam energy do you need to produce 1 Å radiation? What is the maximum acceptance angle allowable to still get a photon beam with a band width of 10 % or less? Show that the acceptance angle is ± 18.15 mrad.

23.11. Consider a ray of 123.8 meV and 10 keV photons, both at a power density of 100 Watt/mm^2. How many photons occupy their respective "characteristic volumes"? Show that the photon flux density is 1.875×10^{10} photons (100 meV)/mm^3 and 1.875×10^5 photons(10 keV)/mm^3. Verify that, 61.07 photons (123.8 meV) and 1.44×10^{-18} photons (10 keV) occupy, on average, its own characteristic volume in a 100 W/mm^2 beam. The X-ray photon distribution is indeed sparse among it's characteristic volume. What are the respective characteristic volumes?

23.12. Show that Eqs. (1.88) and (1.89) are the same for electromagnetic waves.

References

1. A. Liènard, L'Eclaire Electrique **16**(5) (1898)
2. E. Wiechert, Arch. Neerl. **546**(5) (1900)
3. G.A. Schott, Ann. Physik **24**, 635 (1907)
4. G.A. Schott, Philos. Mag. [6] **13**, 194 (1907)
5. G.A. Schott, *Electromagnetic Radiation* (Cambridge University Press, Cambridge, 1912)
6. A.S. Skrinski, N.A. Vinokurov, Technical Report Preprint INP 77–59, Institute of Nuclear Physics, Novosibirsk, USSR (1977)