Spectral Correlation Measure for Selecting Intrinsic Mode Functions

Edgar F. Sierra-Alonso*, Oscar Cardona-Morales**,
Carlos D. Acosta-Medina, and Germán Castellanos-Domínguez

Signal Processing and Recognition Group,
Universidad Nacional de Colombia, Manizales, Colombia
{efsierraa,ocardonam,cdacostam,cgcastellanosd}@unal.edu.co

Abstract. Time series analysis implies extracting relevant features from real-world applications to improve pattern recognition tasks. In that sense, representation methods based on time series decomposition and similarity measures are combined to select representative features with physical interpretability. In this work, we introduce two similarity measures based on the cross-power spectral density to select representative intrinsic mode functions (IMF) that characterize the time series. The IMFs are obtained by Ensemble Empirical Mode Decomposition because it deals with non-stationary dynamics present into time series. The proposed similarity measures are an extension of the correlation coefficient and are validate using vibration signals acquired in a test rig under three different machine states (undamaged, unbalance and misalignment). Results show that the proposed measures improve the interpretability in terms of association between an IMF and a fault state, preserving a high classification rate.

Keywords: Time series, similarity measures, cross-power spectral density.

1 Introduction

Time series analysis had turns important due to faster growth of technologic systems that acquired huge volumes of data in distinct scientific fields such as medicine, economy, industry, among others., where it is mandatory extracting relevant information for classification, prediction and clustering tasks. In consequence, several representation methods have been developed in the literature in order to extract features from raw data to decrease drawbacks like processing cost, redundant and irrelevant information, and so on, making useful to develop representation techniques that allow preserving process fundamental characteristics through decomposition methods [10]. Nonetheless, to choose those features that better represent a particular set of time series is needed utilizing similarity measures that provide a proper relation. Thereby, in [4] summarize a set of similarity measures for time series based on whole series matching such as dynamic

* This research is supported by "Programa Jóvenes Investigadores e Innovadores convocatoria 617 de 2013, Convenio Especial de Cooperación No. 0737 de 2013 entre COLCIENCIAS y la Universidad Nacional de Colombia Sede Manizales".

** This research is supported by the project, with code "1101-521-28792", developed between Colciencias and Universidad Nacional de Colombia sede Manizales.

E. Bayro-Corrochano and E. Hancock (Eds.): CIARP 2014, LNCS 8827, pp. 231–238, 2014.

time warping, euclidean distance and its editions, however, those measures have not been used to evaluate their efficiency in a multiple-data framework [8]. Other similarity measures widely known in the state-of-the-art are measures based on the correlation coefficient and the information theory, which are robust to noise and mainly employed in linear and nonlinear time series, respectively [7,1]. Nevertheless, in [8] propose that a classification rate is required to assess the merit of time series similarity measures, which is not sufficient to determine the interpretability of selected characteristics in the real-world application context.

Respecting to representation techniques, there are multiple techniques to time series decomposition, especially, to analyze non-stationary dynamics. Nonetheless, techniques that deal with those non-stationarities like empirical mode decomposition (EMD) are frequently used since it shows overlaps between neighboring components in time and frequency, which other standard iterative techniques do not provide [6,3]. EMD technique decomposes each signal into a set of simpler functions named intrinsic mode functions (IMF), which comprise narrow-band dynamics as filter bank. However, EMD presents a mode mixing problem generating redundant information in each IMF [9], and to solve this issue, an improved version, named ensemble empirical mode decomposition (EEMD), is introduced in [11].

Since EEMD does not have a known structure that allows relating an specific IMF with a typical information about a particular dynamic into time series. The identification task falls on selecting a proper IMF that characterizes different behavior types. Hence, in order to improve interpretation between information provided by the decomposed components and the time series at hand, we propose a methodology for representative IMF selection that decomposes the time series using the EEMD, from which each accomplished IMF is correlated to the input signal to identify those specific spectral dynamics mostly influenced by specific phenomena of the application. To this end, four different similarity measures are considered: *i)* the correlation and *ii* the mutual information coefficients between each mode and the time series; *iii)* the energy and *iv)* cumulative distribution density energy, both from cross-power-spectral density (CPSD) computed for each IMF and the time series. The methodology is validated in a condition monitoring application using mechanical vibration signals that are acquired in a test rig emulating three states under non-stationarity conditions: normal, and two damages (unbalanced and misalignment). Because of introduced CPSD, the selected IMF preserves important spectral properties from the original signal, which is useful to identify failures, whereas the correlation index does not present variability among the observations.

2 Theoretical Background

2.1 Ensemble Empirical Mode Decomposition

This adaptive method decomposes time series into a set of so termed intrinsic mode functions (IMF), which follows: *i)* the amount of local extremes and the zero crossing differs at most by one, *ii*) at any point the mean value between the superior envelope defined by the local maxima and the inferior envelope defined by the local minima is zero. Thus, a time series $x(t)$ can be represented by EMD as follows:

$$x(t) = \sum_{k\in K} \hat{c}_k(t) + r(t), \ \forall t\in T$$

where $\{\hat{c}_k(t)\}$ is the set of IMF, $r(t)$ is the remainder term, and K is the number of the IMF extracted from original data. The first IMF is related to the highest frequency while the last one to the lowest.

However, EMD faces the mode mixing problem because of reached low orthogonality between neighboring IMFs. This issue is overcame by the use of the ensemble empirical mode decomposition (EEMD) that takes advantage of the additive white gaussian noise (AWGN) cancelation property within dyadic filter bank EMD structures [11]. The EEMD is sequentially carried out as follows:

1. An input time series $x(t)$ is contaminated with AWGN as much as J times, i.e., $x_j(t)=x(t)+\eta_j(t)$, $\forall t\in T$, $j=1,\ldots,J$, being $\eta_j(t)$, each j-th trajectory of the randomly generated AWGN,
2. Afterwards, obtained $x_j(t)$ is decomposed, using the conventional EMD, into the corresponding IMF set, $\{\hat{c}_{k,j}(t) : k=1,\ldots,K\}$.
3. At last, an averaged version of $c_k(t)$ is obtained as:

$$c_k(t) = \mathbb{E}\left\{\hat{c}_{k,j} : \forall j \in J\right\}, \ \forall t \in T$$

where notation $\mathbb{E}\{\cdot\}$ stands for expectation operator. Generally, J should be large enough to cancel the AWGN since there is a directly proportional relation between the standard deviation of the AWGN and the amount J.

2.2 Correlation-Based Similarity Measures

Provided both signals, $x(t)\in\mathbb{R}$, and the k-th IMF, $c_k(t)\in\mathbb{R}$, the similarity between them is a measure that quantifies their mutual dependency/independency. The straightforward way to measure the linear relationship of dependency/independency is the Pearson's correlation coefficient, defined as follows:

$$\rho_{1k} = \frac{\mathbb{E}\{(x(t)_1 - \mu_x)(c_k(t) - \mu_{c_k})\}}{\sigma_x\sigma_{c_k}} \tag{1}$$

where notation $\mathbb{E}\{\cdot\}$ stands for expectation operator, and $\mu(\cdot)$, and $\sigma(\cdot)$ are the mean and standard deviation values, respectively.

Although the Pearson correlation coefficient, $\rho_{1k}\in\mathbb{R}[-1,1]$, is a relatively efficient similarity measure, it can not take into account the narrow-band nature of each IMF component, leading to a very distorted estimate. Therefore, there is a need to quantify mutual dependency/independency but in the frequency domain. To this end, we use the cross-correlation function between $\{x(t), c_k(t)\}$, defined as

$$R_{xc_k}(\tau) = \mathbb{E}\{x(t+\tau)c_k(t)\}$$

where $R(0)=\rho_1$ and assuming both correlated signals statistically normalized.

Through the Wiener-Khinchine theorem, we obtain the cross-correlation spectral density (CPSD), $S_{xc_k}(f)\in\mathbb{R}^+$ as frequency domain transformation of $R_{xc_k}(\tau)$, that is, $S_{xc_k}(f)=\mathcal{F}\{R_{xc_k}(\tau)\}$, being $\mathcal{F}\{\cdot\}$ the Fourier transform. However, integration of the

Wiener-Khinchine transform must be carried out within the spectral range of interest, i.e., $f \in [f_1, f_2]$.

Next, we assume the ergodicity of both correlated signals, so that we get a real-valued scalar metric of similarity, $\rho_{2k} \in \mathbb{R}^+$, between the measured signal and each one of the IMF components, as follows:

$$\begin{aligned} \rho_{2k} &= \mathbb{E}\left\{S_{xc_k}(f) : \forall f \in [f_1, f_2]\right\}, \\ &= \frac{1}{f_2 - f_1} \int_{f_1}^{f_2} \mathcal{F}\{R_{xc_k}(\tau)\}(f) df \end{aligned} \quad (2)$$

However, the presence of powerful components, which are locally concentrated in the spectral density, can bias the expectation operator estimator. To overcome this issue, we propose the use of the expectation operator, but over the cumulative spectral density function in the following form:

$$\rho_{3k} = \frac{1}{f_2 - f_1} \int_{f_1}^{f_2} \int_{f_1}^{f} \mathcal{F}\{R_{xc_k}(\tau)\}(\hat{f}) d\hat{f} df, \; \rho_{3k} \in \mathbb{R}^+ \quad (3)$$

2.3 Selection of Intrinsic Mode Function

Although obtained set $\{c_k(t) : k \in K\}$ should encode distinct narrow-band dynamics (from high to low frequencies) [6], in practice, we assume that influence of specific behaviors (like damages, diseases, etc.) differently affects each IMF, and consequently, we must select the IMF that better represents the desired behavior over all observed data. However, due to the self adaptive property, EEMD may reflect similar frequency information, extracted from processed observations, in different $c_k(t)$. Therefore, to correctly select the representative $c_k(t)$, we search for the highest pairwise similarity between the input vibration signal and each $c_k(t)$.

Thus, provided the observed time series set, $\{x^i(t) : i \in I\}$, and the obtained set $\{c_k^i(t) : k \in K\}$, we select the better index mode, k_i^*, as the one having the strongest similarity measure, $\rho_{nk}^i : n = 1, 2, 3$, towards each i-th observed time series, $x^i(t)$, over all IMF set, $\{c_k^i(t) : k \in K\}$, that is:

$$k_i^* = \arg \max_{k \in K} \{\rho_{nk}^i\} \quad (4)$$

For the sake of comparison, we make use of every one of considered metrics, i.e., ρ_{nk}^i introduced in§ 2.2. Lastly, the selection of the index mode, k_i^*, is carried out through the whole observation data, $i \in I$, so that each compared feature vector holds I selected measures, that is: $[\rho_{nk}^i : i \in I]$.

3 Experimental Setup

3.1 Test Rig and Database

The experiment is carried out in the test rig shown in Fig. 1, where we consider, along with the undamaged condition, two other damages: shaft unbalance and coupling misalignment. These faults are common in industry and their presence in machines are the

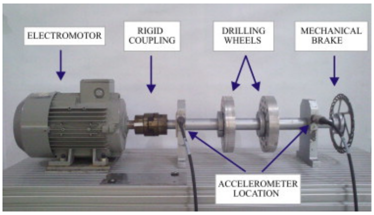
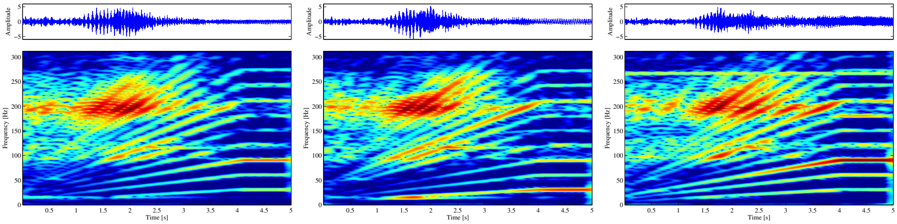


Fig. 1. Test rig scheme [2]



(a) Undamaged (b) Unbalance (c) Misalignment

Fig. 2. Examples of vibration signal with different machine conditions in time domain (top) and its time-frequency representation (bottom)

main source of other damages as rub, defect roller element bearing, etc. The test rig is composed by a DC electromotor of 2HP and two drilling wheels for emulate the shaft unbalance, both connected by rigid coupling for misalignment. The sensor is a standard accelerometer and it is located in horizontal position on the supports. The measuring dynamic range is fixed from 0 to 1800 rpm during 10 seconds (starting from a steady state to maximum speed), and 20kHz of sampling frequency[1].

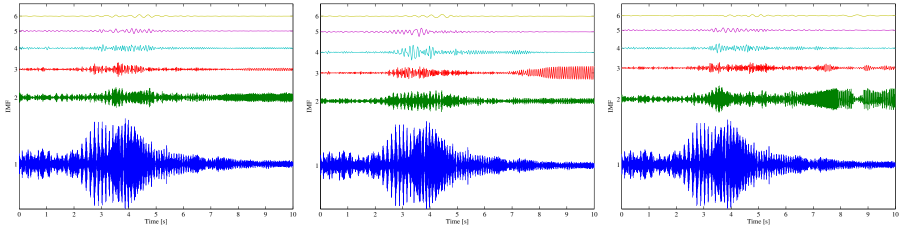
Generated data in the test rig comprises three classes: undamaged (class 1), unbalance (class 2) and misalignment (class 3); each class holds 20 observations. As preprocessing stage, the observed vibration signal $x^i(t)$ where $i = 1, \ldots, 60$, is filtered with a 9-th order low-pass chebyshev filter at a cut frequency of 300Hz, and down-sampled to 625Hz. Carried out preprocessing allows enough orders (shaft speed harmonics) to characterize the studied failures, as shown in Fig. 2.

3.2 Vibration Signal Decomposition Using EEMD

Since vibration signal $x^i(t)$ has mixed orders, EEMD decomposes it into a set of simpler functions $\{c^i_k(t) : k=1, \ldots, K\}$ where each one corresponds to an IMF (see Fig. 3). As suggested in [11], the AWGN standard deviation is fixed to 0.2. . We generate 500 ensembles, which are enough to cancel the added noise, during EEMD computation.

As seen in Fig. 3 showing the IMF set $\{c^i_k(t) : k=1, \ldots, 6\}$ computed for one observation, the first three IMF are noteworthy affected by damage in all considered cases. Yet, the first IMF is rejected since it holds the highest spectral content where the damage is assumed not to influence [5]. In fact, the mode, c_1, has almost the same shape

[1] The database is available by request.



(a) Undamaged (b) Unbalance (c) Misalignment

Fig. 3. Examples of EEMD decomposition into first six intrinsic mode functions

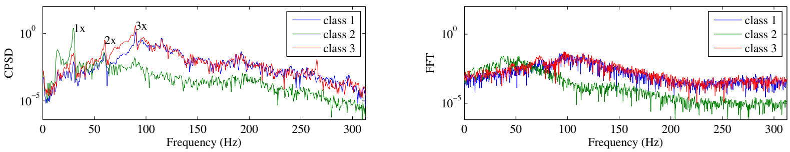


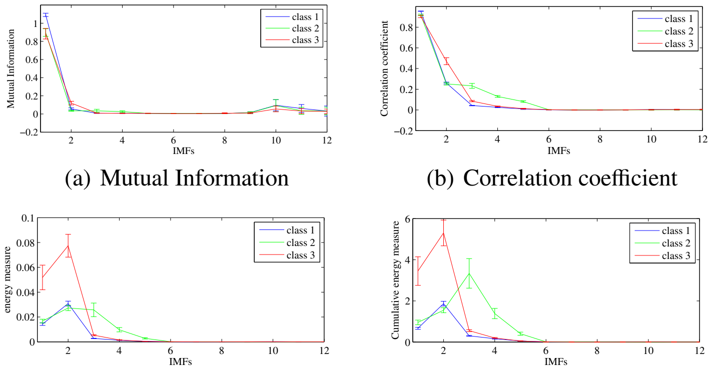
Fig. 4. CPSD (left) and Fourier transform (right) for the first observation per classes, with y axis in logarithmic scale

for all considered conditions, meaning that it has a very low discriminant capability. Generally, sequential application of EMD and order tracking methods makes possible to clarify IMFs in terms of rotational speed and also provides the ability to separate vibrations that modulate order signals (e.g. machine fault vibrations), as discussed in [9]. In consequence, damage conditions are mostly assumed to affect only one IMF due to its narrow-band nature. This fact can be verified after visual examination of Fig. 3(b) showing an example of obtained IMF set for the unbalance condition, where a significant change appears in terms of the cumulative energy for c^i_3. For misalignment, the c^i_2, is the most representative IMF where an strong change becomes visible (see Fig. 3(c)).

3.3 Estimation of Correlation-Based Similarity Measures

To select the most representative IMF better describing the machine fault vibration, we make use of the approaches explained above in Section 2.2, which are compared with the mutual information (MI) measure between the original signal and the modes is used. The analysed frequency bandwidth for the aforementioned approaches is confined within the interval $f \in [f_1, f_2] = [0, 160]$ Hz since shaft speed harmonics associated to the studied failures are usually the 1-th, 2-nd and 3-rd orders ($1x$, $2x$ and $3x$, respectively), i.e. from 30 Hz to 90 Hz. The Fig. 4 shows the computed CPSD, $S_{xc_k}(f)$, for the observation trajectories labeled as $i \in \{1, 21, 41\}$, where the selected IMF set is, $k^*_i \in \{2, 3, 2\}$, respectively. For the sake of comparison, we compute also the Fourier transform of each selected IMF shown in the bottom row, where we can see that the CPSD use improves interpretability about the machine condition because it allows identifying the orders comprised in the corresponding $c^i_{k^*}(t)$.

To determine the behavior of the measures through observations per class, the Fig. 5 shows the mean and standard deviation thought the observations for the first 12 IMFs, which is enough to describe the damage. Moreover, the modes $c^i_k(t) : k{=}5, ..., 13$ could



(a) Mutual Information (b) Correlation coefficient

(c) Spectral energy (d) Cumulative spectral energy

Fig. 5. Mean and standard deviation of the measures between original signals and IMFs through all the observations

be omitted due to those IMF present minimum correlation. For the correlation ρ^i_{1k}, and the MI coefficients (Figs. 5(a) and 5(b)), the index associated to most representative mode is k^*_i=1 for all observations, but this is not an ideal selection because the relevant information is in medium frequency band (around $30 - 90$ Hz), according with considered damages. In contrast, the first proposed energy-based measure, ρ^i_{2k}, omits the first mode as shown in Fig. 5(c), because for this particular application the frequency domain is restricted to $[0, 160]$Hz interval. The selected IMF by the maximum criterion explained in Section 2.3, can clearly identify the most representative mode for the class 3, but the selection fails for the others classes because it is not stable through observations (i.e. there is not a maximum for the all observations). The aforementioned drawback could be due to an error caused for a bias of the expectation estimator, because it does not taking into account powerful locally concentrated components.

Thus, the cumulated energy measure ρ^i_{3k} in Fig. 5(d) exhibits a more robust behaviour, without the drawback of the energy-based measure. Consequently the selection criterion identifies the modes more related with considered faults, which is proved by visual examination of Fig. 3.

3.4 Validation of Proposed Measures to IMF Selection

To validate the measures, a feature set per measure is built through the selected IMF's envelopes per i-th observation, which is computed as the analytic signal module noted by $\boldsymbol{\psi}^i_{k^*} \in \mathbb{R}^{N\times 1}$, being N the number of time instants, therefore, the feature set can be written as $\boldsymbol{Y}=\left[\boldsymbol{\psi}^1_{k^*}\ \boldsymbol{\psi}^2_{k^*}\ \cdots\ \boldsymbol{\psi}^I_{k^*}\right]^T$. A cross-validation classification scheme is made with 100 folds and a k-nearest-neighbors classifier. The feature sets are split into training and testing sets of 60% and 40%, per similarity measure. As result, the obtained classification accuracy is 96.3% ± 3.6%, 96.3% ± 3.6%, 99.5% ± 2.4% and 100% ± 0% for the correlation coefficient, mutual information, spectral energy and cumulative spectral energy measures, respectively. The high classification accuracy is expected due to EEMD presents satisfactory performance in vibrations signals [6]. Nonetheless the proposed cumulative spectral energy measure overcomes the other two measures. Respect interpretability, considering that failures present a varying energy distribution at the shaft speed harmonics, CPSD allows identifying the harmonics contained in the IMFs in comparison with usual PSD, and the proposed measures provide us information about harmonic's energy per mode in a frequency range of interest.

4 Conclusion

A correlation-based similarity measures between two time series are proposed, which is based in the CPSD, allowing to measure relations in terms of spectral energy at frequency interest regions. The time series is decomposed using EEMD to reduce its spectral complexity into narrow-band dynamics, selecting spectral components that preserve the time series relevant information. The proposed measures are validated in a real-world application using vibration signals, where the highest classification performance is achieved by the introduced cumulative spectral energy. Besides, the CPSD provides interpretability about the physical phenomenon, showing the directly relation between representative IMF with respect to considered faults. As future work, the methodology will be tested in other kind of time series like medical and seismic data, among others.

References

1. Boudraa, A.O., Chonavel, T., Cexus, J.C.: -Energy operator and cross-power spectral density. Signal Processing 94, 236–240 (2014)
2. Cardona-Morales, O., Avendaño, L.D., Castellanos-Domínguez, G.: Nonlinear model for condition monitoring of non-stationary vibration signals in ship driveline application. Mechanical Systems and Signal Processing 44(1-2), 134–148 (2014)
3. Feldman, M.: Analytical basics of the EMD: Two harmonics decomposition. Mechanical Systems and Signal Processing 23(7), 2059–2071 (2009)
4. Chung Fu, T.: A review on time series data mining. Engineering Applications of Artificial Intelligence 24(1), 164–181 (2011)
5. Gao, Q., Duan, C., Fan, H., Meng, Q.: Rotating machine fault diagnosis using empirical mode decomposition. Mechanical Systems and Signal Processing 22(5), 1072–1081 (2008)
6. Lei, Y., Lin, J., He, Z., Zuo, M.J.: A review on empirical mode decomposition in fault diagnosis of rotating machinery. Mechanical Systems and Signal Processing 35(1-2), 108–126 (2013)
7. Mehboob, Z., Yin, H.: Information quantification of empirical mode decomposition and applications to field potentials. International Journal of Neural Systems 21(01), 49–63 (2011)
8. Serrá, J., Arcos, J.L.: An empirical evaluation of similarity measures for time series classification. Knowledge-Based Systems (2014)
9. Wang, K., Heyns, P.: Application of computed order tracking, Vold Kalman filtering and EMD in rotating machine vibration. Mechanical Systems and Signal Processing 25(1), 416–430 (2011)
10. Wang, X., Mueen, A., Ding, H., Trajcevski, G., Scheuermann, P., Keogh, E.: Experimental comparison of representation methods and distance measures for time series data. Data Mining and Knowledge Discovery 26(2), 275–309 (2013)
11. Wu, Z., Norden, E.: Ensemble empirical mode decomposition: A noise-assisted data analysis method. Advances in Adaptive Data Analysis 1(1), 1–41 (2009)