Forced Vibration of Eight Mistuned Bladed Disks on a Solid Shaft—Excitation of the First Compressor Bladed Disc

Romuald Rzadkowski and Artur Maurin

Abstract Considered here is the effect of multistage coupling on the dynamics of an aircraft engine rotor with eight mistuned bladed discs on a drum-disc shaft. Each disc had a different number of rotor blades. Free and forced vibrations were examined using finite element models of single rotating blades, bladed discs, and an entire rotor. Calculations of the global rotating mode shapes of flexible mistuned bladed discs-shaft assemblies took into account the excitation of the first compressor bladed disc with 0EO, 1EO and 2EO forces. The obtained maximal stress values of all of the rotor blades were carefully examined and compared with a tuned system to discover resonance conditions and coupling effects. Mistuning changes the stress distribution in individual rotor blades and the level of maximum stress increases or decreases as compared to bladed discs which are analyzed without the shaft.

Keywords Multistage coupling · Mistuning · Blades · Bladed disc

1 Introduction

The effects of multistage coupling on mistuned bladed disc dynamics were analysed by Bladh et al. [1], Shahab and Thomas [2], Sharma et al. [3], Sinha [4], Laxalde et al. [5]. Generally, these papers considered a small number of mistuned bladed discs, so the analysis of multistage coupling was insufficient.
Rzadkowski and Drewczynski [6] analyzed the forced vibration results for rotors with 8 tuned bladed discs, each with different numbers of blades. This showed that

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taking into account the differences in blade numbers is essential in the analysis of multistage coupling.

Rzadkowski and Maurin [7] analyzed the dynamic behavior of a rotor consisting of eight mistuned bladed discs on a shaft. Mistuning distorted the nodal diameters of mode shapes primarily in unshrouded bladed discs and caused multistage coupling between various nodal diameters. In the mistuned bladed system particular discs vibrated in modes with diverse nodal diameters. For example, there were modes where one bladed disc vibrated with two nodal diameters and at the same time another disc vibrated with one and zero nodal diameter.

The free vibration of mistuned aircraft engine rotor bladed discs were next analysed by Rzadkowski et al. [8].

In [9] we have considered how forced vibrations of a turbine bladed disc influenced the vibration of compressor stages in order to simulate how combustion chamber excitation might influence the level of compressor stage vibrations trough the shaft.

To better understand this problem, in this paper we present a forced vibration analysis on the same rotor, where the first bladed disc is excited by the fluid flow forces.

The responses of tuned bladed discs will be compared with those of mistuned bladed discs (henceforth referred to as case 1 and case 2, respectively).

More multistage coupling was revealed in the forced vibration analysis than in the free vibration analysis.

2 Model Description

A finite element model of aircraft engine bladed discs on a shaft is presented in Fig. 1. The main dimensions are as follows: the outer diameter of the largest—turbine disc is 0.512 m and the shaft length is 1.166 m. The number of blades on each disc corresponds to that of a real system. Therefore there were 28 rotor blades in compressor stage I, 41 in stage II, 41 in stage III, 47 in stage IV, 57 in stage V, 47 in stage VI, 49 in stage VII and 83 in the turbine stage. The final FE model had over 1.5 million DOFs.

Two axis-symmetric bearing types were modelled as springs with $k_{xx} = k_{yy} = k_{xy} = k_{yx} = 1{,}400$ N/μm stiffness for two roller bearings and $k_{xx} = k_{yy} = k_{xy} = = k_{yx} = k_{zz} = 1{,}000$ N/μm for the ball bearing. The material damping is assumed to be 0.005 %.

Two rotor models—tuned (case 1) and mistuned (case 2), were developed for the numerical analyses. The blade mistuning was modelled using the modified Young modulus for every blade in each stage, according to blade frequencies measured in real aircraft engine (see Table 1). Free vibration analyses were carried out in both models at 15,000 rpm.

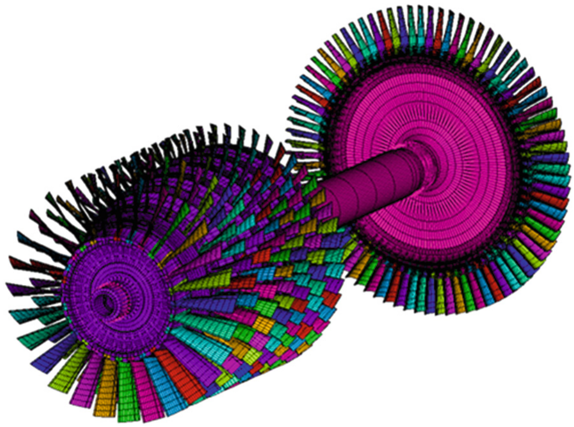

Fig. 1 FEM of aircraft engine rotor

Table 1 Natural frequencies of the blades on the discs

Bladed disc (1F Mode)	Tuned (Hz)	Mistuning (Hz)	Mistuning (%)
1	499	498–515	3.2
2	670	619–660	6.11
3	1,210	1,187–1,341	12.7
4	1,130	1,099–1,247	13.1
5	1,387	1,228–1,366	9.9
6	1,597	1,585–1,623	2.4
7	1,635	1,604–1,645	2.5
T	1,278	1,101–1,245	11.3

3 Free Vibration Analysis

In both the tuned and mistuned models the natural frequencies for all the bladed discs were computed in [9] using the ANSYS code. The modes of each mistuned bladed disc were classified in a similar way to those of the tuned bladed disc but only with a small number of mode diameters. However, this was not possible in the case of larger nodal diameter modes as the mistuned blades distorted the nodal lines of the mode shapes. The natural frequencies of tuned and mistuned bladed discs on the shaft were divided into the modes dominated by the bladed discs and the modes dominated by the shaft with the discs. The number of frequencies of the tuned and mistuned systems is the same.

The natural frequencies of the mistuned cantilever blades, single mistuned bladed discs and the complete flexible shaft with eight mistuned discs were carefully examined to find resonance conditions and coupling effects.

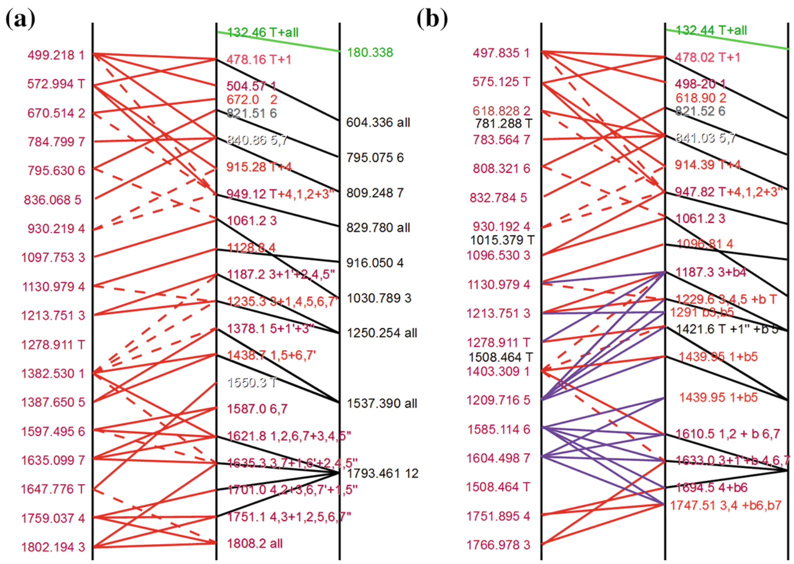

Fig. 2 Natural frequencies of **a** tuned and **b** mistuned bladed discs on the shaft corresponding to zero-nodal diameter modes

Figures 2, 3 and 4 present the zero-, one- and two-nodal diameter natural frequencies of tuned (left) and mistuned systems (right). The right axes indicate the natural frequencies of the eight discs (without blades) on the shaft, the middle axes show the natural frequencies of the mistuned bladed discs on the shaft, while the left axes present the uncoupled natural frequencies of single cantilever mistuned bladed discs corresponding to a given nodal diameter. The uncoupled modes of single blades and bladed discs were calculated separately. The frequencies of the turbine bladed disc are marked with the letter T, whereas those of the compressor are given the number (1–7) of their particular stage. The letter 'b' indicates single blade vibrations in a given stage.

The longitudinal modes are presented in black, the torsional modes in green, the bending bladed disc modes in red and the bending shaft dominated modes in blue. The frequencies with corresponding mode shapes are connected between axes with lines. Purple lines indicate couplings between single blades and entire stages. The lines connecting the natural frequencies, are divided into two types: continuous lines indicating strong coupling and dashed lines for weaker coupling (marked with single and double quotation marks next to the bladed disc symbol). Strong coupling occurs when the amplitudes of particular blades are very visible, whereas weak coupling occurs when the amplitude is relatively small. In the latter case we observe the vibrations of a shaft and disc without blades. The frequencies and mode shapes of mistuned bladed discs on a shaft were analyzed, starting from zero-nodal diameter modes.

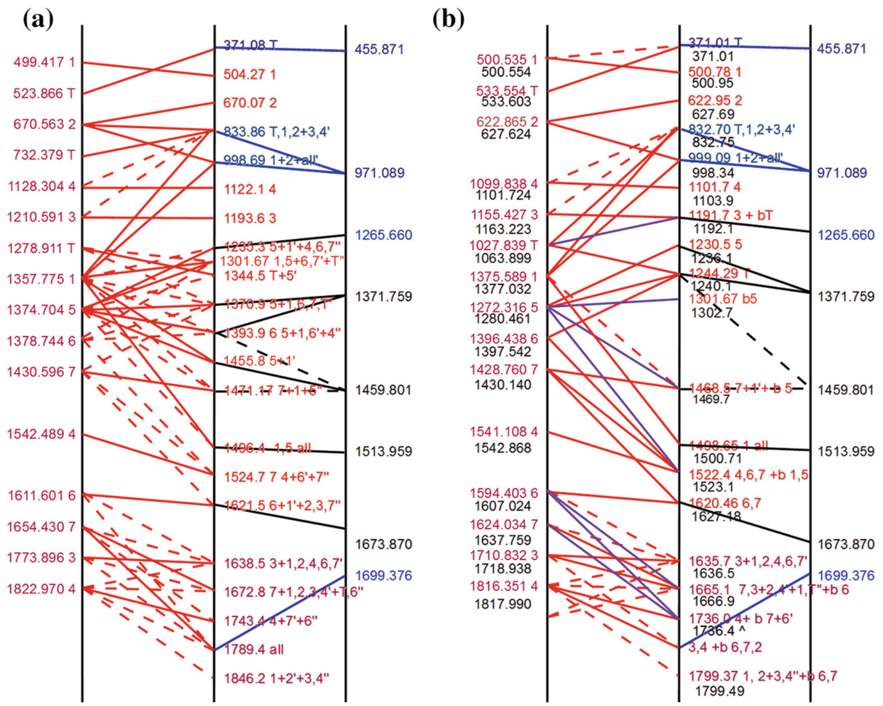

Fig. 3 Natural frequencies of (**a**) tuned and (**b**) mistuned bladed discs on the shaft corresponding to one-nodal diameter modes

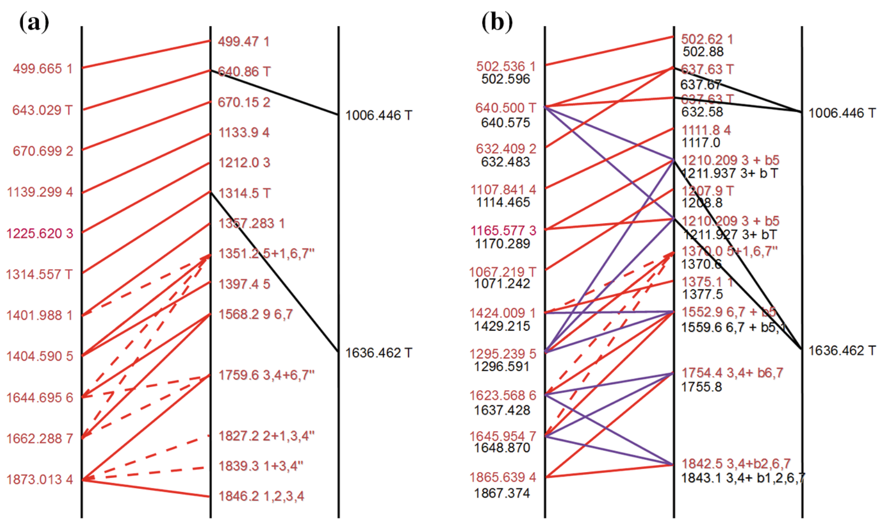

Fig. 4 Natural frequencies of (**a**) tuned and (**b**) mistuned bladed discs on a shaft with two-nodal diameter modes

3.1 Zero-Nodal Diameter Modes

As shown in Fig. 2, the first frequency mode corresponding to the zero-nodal diameter (132.46 Hz in case 1 and 132.44 Hz in case 2) was connected with a torsional rotor mode at 180.338 Hz, in both cases causing all the bladed discs to vibrate. The next frequency mode (478.16 Hz in case 1 and 478.02 Hz in case 2) produce coupling between the compressor first stage bladed disc and the turbine disc. These vibrations were also visible on the unbladed rotor at 604.336 Hz, thus the line connecting the two frequencies in Fig. 2.

In the frequency mode, 915.28 Hz (case 1) and 914.39 Hz (case 2), the turbine bladed disc and 4th compressor bladed disc vibrate predominately. At 949.12 Hz (case 1) and 947.82 Hz (case 2) the turbine bladed disc and 4th bladed disc are coupled along with smaller vibrations occurring in the 1st, 2nd and 3rd bladed disc.

Similar mode couplings and distortions also occurred in subsequent zero-nodal diameter modes, up to 1,187 Hz (see Fig. 2). It is too difficult to group frequencies on the basis of a free vibration analysis (see [9]), and therefore one has to refer a forced vibration analysis. Mistuning alters the vibration amplitudes in particular blades. For example each blade of the 1st bladed disc has a slightly different amplitude in an almost zero-nodal diameter mode.

3.2 One-Nodal Diameter Modes

Figure 3 presents the natural frequencies of tuned and mistuned bladed discs on the shaft, vibrating with a one-nodal diameter. The splitting of one-nodal diameter double frequencies occurs as an effect of blade mistuning. The bending shaft dominated modes are presented in blue. The 371.08 Hz mode and the 371.01 Hz mode are dominated by the bending motion of the shaft caused by the one-nodal diameter mode shape of the turbine disc at 455.871 Hz. The cantilevered turbine disc vibrates at 523.866 Hz (case 1) and at 533.554 and 533.603 Hz (case 2). Next, only the 1st bladed disc vibrates, with modes of 504.27 Hz in case 1, while in case 2 with 500.78 Hz. Similar mode couplings and distortions can be observed in subsequent one-nodal diameter modes, up to 1,370 Hz (Fig. 3).

One-nodal diameter vibrations of a turbine bladed disc at 1,344.5 Hz are coupled with the 5th bladed disc in case 1, whereas in case 2, the corresponding mode shape is approximately 100 Hz lower at 1,244.29 Hz, therefore vibrations only appear on the turbine disc. Here, the one-nodal diameter mode is greatly distorted.

3.3 Two-Nodal Diameter Modes

Figure 4 presents the natural frequencies of tuned and mistuned bladed discs on a shaft with two-nodal diameter mode shapes. The splitting into two two-nodal diameter frequencies appears in the mistuned system on account of the mistuning.

At 499.47 Hz only the 1st bladed disc of the tuned system vibrates with a two-nodal diameter mode. An equivalent mode shape appears in case 2 at 502.62 Hz. Next the turbine bladed disc vibrates at 640.86 Hz in the tuned system and at 632.58(637.63) Hz in the mistuned system.

Similar mode couplings can be observed in subsequent two-nodal diameter modes, up to 1,370 Hz (Fig. 4). The two-nodal diameter vibration mode only appears in the 5th bladed disc and to a lesser extent in the 1st, 6th and 7th bladed discs of the tuned system at 1,351.2 Hz, whereas in the mistuned system coupling with this mode is observed at 1,370.0 Hz. In the latter case, however, only some of the blades in the 5th stage vibrate, while in the 1st stage all the blades vibrate with an almost two-nodal diameter.

4 Excitation by 0EO

This paper aims to explain how the excitation of a first mistuned bladed disc influences mistuned turbine bladed disc (case 2) in comparison with a tuned compressor (case 1). For such an analysis modes had to be selected in which the first bladed disc and turbine bladed disc vibrate simultaneously. In case of 0EO excitation those modes are as follows (see Fig. 2): 478 Hz, 949 Hz—in the tuned system, and 478 Hz, 948 Hz, 1,421 Hz—in the mistuned system.

The first compressor bladed disc in case 1 and case 2 was excited by 0EO with frequencies ranging from 400 to 1,800 Hz (Fig. 5) and the maximal blades stress (von Misses stress) amplitudes measured in the blade roots were next analysed. The axial and circumferential amplitude forces obtained from CFD calculations were applied to the gravity centers of 10 profiles, evenly dividing blade length, for every blade of first compressor stage.

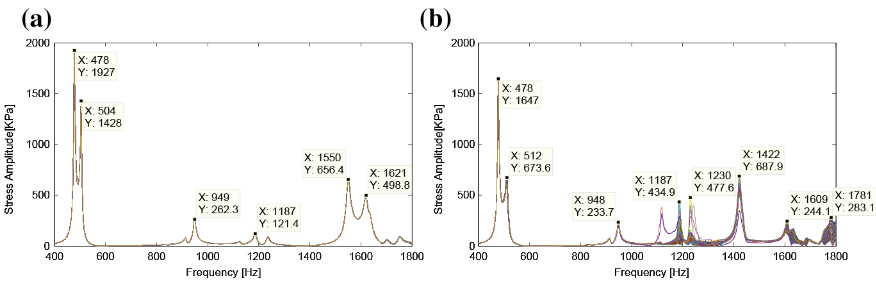

Fig. 5 The response of **a** tuned and **b** mistuned turbine bladed discs when the first compressor bladed disc is excited by 0EO at 400–1,800 Hz

4.1 Response of the First Bladed Disc to 0EO Excitation

0EO excitation (case 1) causes the first tuned bladed compressor disc to vibrate with a zero nodal diameter at 477 Hz and 505 Hz with the maximum von Misses stress values of 26 MPa and 173 MPa respectively (further in text those amplitude stress values will be given in parentheses next to resonance frequency value). The 477 Hz frequency corresponds to the mode in which the turbine bladed disc and first compressor bladed disc vibrate simultaneously (see Fig. 2). The 505 Hz frequency corresponds to the mode in which the 1st compressor bladed disc vibrates with a zero nodal diameter. In the free vibration analysis the vibration of the turbine bladed disc in this mode is not visible. Therefore 0EO of a first compressor bladed disc causes a zero nodal diameter mode in the turbine compressor bladed disc. The forced analysis provides more information than the free vibration analysis.

0EO excitation of the mistuned first compressor bladed disc (case 2) causes vibrations with a zero nodal diameter at 477 Hz (27 MPa) and 512 Hz (321 MPa).

While in the tuned system (case 1) 505 Hz corresponds to the mode in which the 1st compressor bladed disc vibrates with a zero nodal diameter, in the mistuned system a group of separate blade stress peaks appears in the region of 498–515 Hz (up to 321 MPa, which is almost two times higher than in case 1 (173 MPa)). These peaks originate from mistuned blade modes of the 1st compressor stage.

4.2 Response of the Turbine Bladed Disc to 0EO Excitation of the First Bladed Disk

In Fig. 5 we see that 0EO excitation of the first compressor bladed disc causes the turbine bladed disc to vibrate with a zero nodal diameter at 478 Hz with an amplitude of 1,927 kPa, 504 Hz (1,428 kPa), 949 Hz (262 kPa), 1,187 Hz (121 kPa), 1,550 Hz (656 kPa) and 1,621 Hz (498 kPa) in case 1 and at 478 Hz with an amplitude of 1,647 kPa, 512 Hz (673 kPa), 948 Hz (233 kPa), 1,187 Hz (434 kPa), 1,230 Hz (477 kPa) and 1,422 Hz (687 kPa) in case 2. The two remaining peak values: 1,609 and 1,781 Hz, are caused by strong blade coupling between the 1st and 7th mistuned bladed disks. Corresponding frequencies are presented in Fig. 2 on the middle axes for both diagrams (of the tuned and mistuned systems) with frequencies bearing the letter “T”. In these modes the turbine bladed disc vibration is a result of coupling through the shaft. The new result for the mistuned system is that an additional group of modes with blade stress peaks appears in the region of 1,101–1,245 Hz (with maximum values of 434 and 477 kPa). These modes originate from mistuned blade modes of the turbine stage (1,101–1,245 Hz, see Table 1). We may conclude that overall stress level in case 2 region of 1,101–1,245 Hz is higher. The maximum blade stress level in the mistuned system reaches 1,647 kPa, whereas in the tuned system it is significantly higher: 1,927 kPa.

5 Excitation by 1EO

Having examined 0EO excitation, we shall now look at the influence of 1EO excitation of the first compressor bladed disc on the turbine bladed disc in an aircraft rotor. For such an analysis modes had to be selected in which the first bladed disc and turbine bladed disc vibrate simultaneously. In case of 1EO excitation those modes are as follows (see Fig. 3): 833 Hz, 999 Hz, 1,302 Hz, 1,371 Hz, 1,789 Hz—in the tuned system, and 833 Hz, 999 Hz—in the mistuned system.

The first compressor bladed disc in tuned (case 1) and mistuned systems (case 2) was excited by 1EO at frequencies ranging from 400 to 1,800 Hz (Fig. 3) and the stress response of turbine bladed disc was analysed. The axial and circumferential amplitude forces obtained from CFD calculations were applied to the gravity centers of 10 profiles, evenly dividing blade length, for every blade of first compressor stage.

5.1 Response of the First Bladed Disc to 1EO Excitation

In case 1, the 1EO excitation of the first compressor bladed disc causes it to vibrate with a one nodal diameter at: 504 Hz (see Fig. 3) (188 MPa). The 504 Hz frequency corresponds to the mode in which the 1st compressor bladed disc vibrates with a one nodal diameter.

The 1EO excitation of the mistuned first compressor bladed disc (case 2) causes the first mistuned bladed disc to vibrate. An additional group of modes with separate blade stress peaks appears in the region of 498–515 Hz (with a maximum blade stress of 323 MPa, 42 % higher than in case 1). These modes originate from mistuned blade modes of the 1st compressor stage.

5.2 Response of the Turbine Bladed Disc to 1EO Excitation of the First Bladed Disk

In case 1 (Fig. 6, left) 1EO excitation of the first compressor bladed disc caused the turbine bladed disc to vibrate with a one nodal diameter at 504 Hz (392 kPa), 834 Hz (315 kPa), 999 Hz (476 kPa), 1,302 Hz (828 kPa), 1,371 (484 kPa), and at 1,789 (627 kPa).

These frequencies are presented in Fig. 3 on the middle axes of both the case 1 and case 2 diagrams. Those of the turbine disc blades are marked 'T', whereas those of the compressor blades are given their relevant disc number. In these modes the turbine bladed disc and some of the compressor bladed discs vibrate as the result of coupling through the shaft as in the case of 0 nodal diameter modes (Fig. 2).

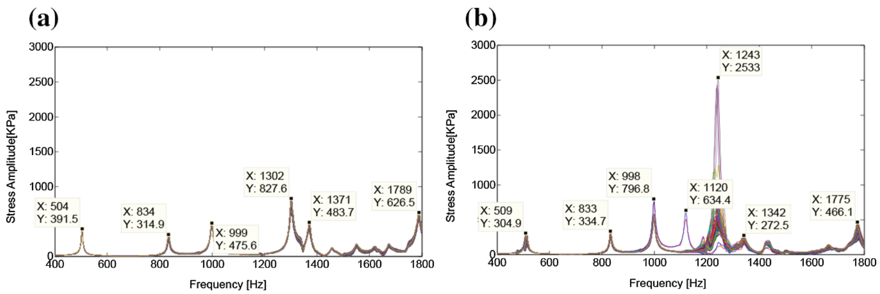

Fig. 6 The response of (**a**) tuned and (**b**) mistuned turbine bladed discs when first compressor bladed disc is excited by 1EO at 400–1,800 Hz

In case 2 (Fig. 6, right) 1EO excitation occurs at 509 Hz with an amplitude of 305 kPa, 833 Hz (335 kPa), 998 Hz (797 kPa) and at 1,775 (466 kPa) similar to case 1. The frequency of 1,371 Hz in case 1 is shifted down to 1,342 Hz (273 kPa).

In the mistuned system an additional group of modes with blade stress peaks up to 2,533 kPa appears in the region of 1,101–1,245 Hz, which is a new result. These peaks originate from mistuned blade modes (at 1,101–1,245 Hz) of the turbine stage (Table 1). The blade stress level in the mistuned system reaches 2,533 kPa, whereas in the tuned system it is no more than 828 kPa, which is over 3 times lower. Such difference occurs due to the fact that the frequency of one of the mistuned blades is covering the one nodal-diameter turbine disc mode at 1,244 Hz (refer to Fig. 3), causing the disc-blade resonance.

6 Excitation by 2EO

So far we have examined how 0EO and 1EO excitation of the first compressor bladed discs on a shaft influences the turbine bladed disc. Now we shall look at the effects of 2EO excitation. In free vibration analysis the two nodal-diameter modes that coupled the first compressor and turbine bladed disc were undetectable by the means of modal analysis for the tuned as well as for mistuned system.

6.1 Response of the First Compressor Disc to 2EO Excitation

In case 1 2EO excitation of the first compressor bladed disc causes it to vibrate with a one nodal diameter at: 499 Hz (see Fig. 4) (193 MPa). The 499 Hz frequency corresponds to the mode in which the 1st compressor bladed disc vibrates with a two nodal diameters.

Blade stress at 1,353 Hz have smaller maximal stress values than at 499 Hz. At 1,353 Hz 2EO excitation shows that the 1st bladed disc vibrates, whereas in free vibration analysis at this frequency the turbine disc and the compressor 5th bladed disc vibrate. Therefore the forced vibration analysis provides more information about multistage coupling.

2EO excitation of the mistuned first compressor bladed disc (case 2) causes the first mistuned bladed disc to vibrate. An additional group of modes with separate blade stress peaks appears in the region of 498–515 Hz (with a maximum blade stress of 288 MPa, 33 % higher than in case 1).

6.2 Response of the Turbine Bladed Disc to 2EO Excitation of the First Bladed Disk

The first compressor bladed disc in case 1 and case 2 was excited by 2EO with frequencies ranging from 400 to 1,800 Hz (refer Fig. 4) and the stress response of turbine bladed disc was analysed. The axial and circumferential amplitude forces obtained from CFD calculations were applied to the gravity centers of 10 profiles, evenly dividing blade length, for every blade of first compressor stage.

In case 1 (Fig. 7, left) 2EO excitation of the first compressor bladed disc causes the turbine bladed disc to vibrate with a two-nodal diameters at 504 Hz, 950 Hz, 1,186 Hz, 1,354 Hz, 1,551 Hz, 1,620 Hz and 1,753 Hz (refer to Fig. 4), all with stress amplitudes level under 200 kPa. In case 2 (Fig. 7, right) at 508 Hz the amplitude stress peak rises to 191 kPa—34 times higher than it case 1 it is still under level of 200 kPa. The overall response level of the turbine blades is low, due to the fact that the shaft construction between compressor and turbine disc disallows the two and higher nodal-diameters mode vibrations to transfer.

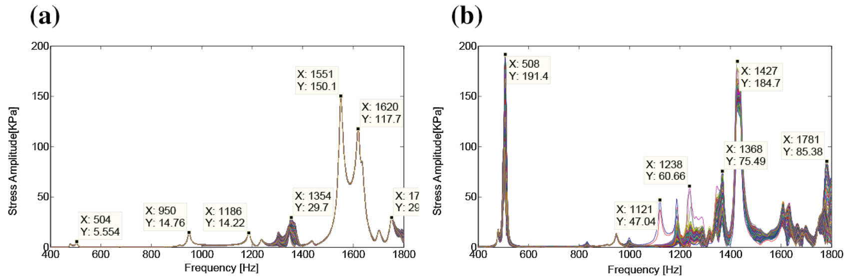

Fig. 7 The response of (**a**) tuned and (**b**) mistuned turbine bladed discs when first compressor bladed disc is excited by 2EO at 400–1,800 Hz

7 Conclusions

In this paper we consider the influence of the low engine order excitation of a tuned and mistuned first compressor bladed disc on the vibrations of tuned and mistuned turbine bladed disc.

The free vibration analysis is an insufficient tool to explain multistage coupling in mistuned system. The forced vibration analysis of an aircraft rotor showed that mistuning considerably increases multistage coupling when compared with a tuned rotor system. An additional modes may appear due to resonances caused by mistuned disc blade frequency shift.

The occurring resonances can cause a significant rise of blade stress levels and are potentially dangerous from the blade-life point of view.

The maximum blade stress response level measured in the mistuned system may be lower than in the tuned system. This occurs when the stress amplitudes of the exited stage are higher in the mistuned system than in the tuned system.

In an aircraft engine with a disc-drum shaft multistage coupling between the compressor stages and the turbine bladed disc primarily occurs in 0E0 and 1EO excitation of the turbine bladed disc. The effect appears in this particular system due to specific shaft connection between the turbine and compressor stages.

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