Introduction

This chapter contains five papers on a variety of topics including random walks in locally compact groups, approximation theory, stochastic analysis, learning theory and Bayesian statistics. This shows the breadth of Dudley's interests ranging from very pure areas of probability and analysis to much more applied areas including statistics and even computer science.

The first paper deals with the problem of existence of a recurrent random walk in discrete locally compact abelian groups. A random walk in a countable abelian group G is a Markov chain whose transition function is invariant with respect to translations. It is called recurrent if it visits any open set $U \subset G$ infinitely often with probability 1. Dudley proved that such a random walk exists in a locally compact abelian group if and only if the factor group G/H is countable for any open subgroup H of G and there is no discrete subgroup that is free abelian on three generators. The study of this problem started in an earlier paper by Dudley (Proc. Amer. Math. Soc. 13, 1962, 447–450) and it continues a classical line of research on random walks in discrete additive subgroups of finite dimensional vector spaces (e.g, Chung, K.L. and Fuchs, W.H.J., Mem. Amer. Math. Soc., 1951, n. 6).

The subject of the second paper is bounding metric entropies of some classes of sets in $\mathbf{R}^k$. The notion of metric entropy of compact sets was studied by Kolmogorov and Tihomirov in the 50s (Uspehi Mat. Nauk, 1959, 14, 2, 3–86). In part, they were motivated by Hilbert's 13th problem. They proved a number of bounds on the ε-entropy of various classes of smooth functions equipped with classical metrics. In the 60s, Dudley and Sudakov developed a metric entropy method of bounding sup-norms of Gaussian processes. Dudley's interest in studying entropies of classes of sets was apparently related to the development of empirical processes theory where such bounds have become one of the main tools. The paper deals with classes of subsets of $\mathbf{R}^k$ with boundaries of given "smoothness" $\alpha > 0$ and also of closed convex subsets of the unit ball in $\mathbf{R}^k$. The metrics used in computation of their ε-entropy are the Hausdorff metric and the Lebesgue measure of symmetric difference of two sets. In particular, it is shown that for the classes of sets with boundary of smoothness α the entropy is of the order $O(\varepsilon^{-(k-1)/\alpha})$ as $\varepsilon \to 0$ and for the convex sets it is of the order $O(\varepsilon^{-(k-1)/2})$ as $\varepsilon \to 0$.

A short note on Wiener functionals as Itô integrals deals with a basic question in stochastic analysis: given a standard Wiener process $W(t)$, $0 \le t \le 1$ and the corresponding filtration $\mathcal{F}_t^W := \sigma\{W(s) : 0 \le s \le t\}$, is it possible to represent an $\mathcal{F}_t^W$-measurable random variable X as an Itô stochastic integral of an adapted process? It is proved in the paper that it is possible under the assumption that X is finite a.s., strengthening the previously known result for integrable X.

The fourth paper in this chapter is joint with Kulkarni, Richardson and Zeitouni. It deals with a problem in learning theory posed by Benedek and Itai as to whether uniform bounds on the metric entropy of a class of sets ("concepts") would suffice for the probably approximately correct (PAC) learnability of a concept from the class. It happens that the metric entropy is not sufficient for this, which is proved in this paper by constructing a rather subtle counterexample.

The last paper is joint with Haughton and it deals with classical problems in Bayesian statistics about asymptotic normality of posterior probabilities of half-spaces. Namely, let $\mathcal{P}$ be a statistical model with parameter space Θ that is an open subset of $\mathbf{R}^d$ and let π be a prior law on Θ with a continuous, strictly positive density. Let A be a half-space of $\mathbf{R}^d$ that does not contain the maximum likelihood estimator of parameter θ (otherwise A is replaced by its complement). Finally, let 2Δ denote the likelihood ratio statistic. It is shown, under some conditions, including a Glivenko-Cantelli condition on the second partial derivatives of the log likelihood function with respect to θ, that uniformly over all half-spaces A, either the posterior probability $\pi_n(A)$ is asymptotic to $\Phi(-\sqrt{2\Delta})$, or both $\pi_n(A)$ and $\Phi(-\sqrt{2\Delta})$ converge to zero exponentially fast.

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