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Antonelli, Giovanni Battista (1858–1944)

Antonelli was born near Pisa in 1858. He studied mathematics and then went on to qualify as an engineer. Although his life was devoted to civil engineering, he made an important contribution to early mathematical economics. His *Sulla teoria matematica dell'economia politica* (1886), intended to be the first part of a book, is remarkable, in particular for the conditions he gives for the 'integrability problem'.

This asks under what conditions single valued demand functions are generated by the maximization of a utility function. Antonelli studied the 'local' aspects of this problem. He started from what is now called the indirect demand function:

$$p = M[q]$$

where q is the vector of goods and p the vector of prices. He gave the symmetry of the matrix of the price substitution terms $\partial p_i/\partial q_j$ as a condition for the recoverability of the utility function but should have also required the negative semi-definiteness of this matrix. The importance of this work has been recognized by Samuelson (1950) and later authors, but passed unappreciated if not unnoticed at the time.

In the same work Antonelli derives a condition for a market demand function to be derivable from a market utility function, that is, that individuals have linear parallel Engel curves. This condition was found much later by Gorman (1953) and Eisenberg (1961). Antonelli had an active and productive career in engineering and what would now be called 'operations research' but never came back to theoretical economics. He died in 1944.

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Selected works

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A detailed and careful account of Antonelli's contributions and a translation of his economic paper is given in:

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approximate solutions to dynamic models (linear methods)

Linear methods are often used to compute approximate solutions to dynamic models, as these models often cannot be solved analytically. While a plethora of advanced numerical methods exist, the most popular 'bread-and-butter' method for solving them is linearization. It is described here first with the example of a simple real business cycle model, but is applicable generally to dynamic stochastic general equilibrium (DSGE) models. It is shown how to easily generate the log-linearized equations needed. The linear system is then solved for the recursive law of motion, by using the method of undetermined coefficients. The classic reference for solving linear difference models under rational expectations is Blanchard and Kahn (1980), while Kydland and Prescott (1982) is the origin of the modern approach of calculating numerically approximate solutions to dynamic stochastic models in order to obtain quantitative results. Much of the material here is taken from Uhlig (1999), which builds on the method of undetermined coefficients in King, Plosser and Rebelo (2002).

A basic example

As a basic example, consider a version of the real business cycle model of Hansen (1985). A social planner or representative agent chooses c_t, k_t, y_t, l_t and n_t to maximize the utility function $U = E\left[\sum_{t=0}^{\infty} \beta^t u(c_t, l_t)\right]$ for some