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Reliability and Performance Analyses of Balanced Gamma Network for use in Broadband Communication Switch Fabrics

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Abstract

Balanced gamma network, a multipath, multistage interconnection network, which features 4×4 switching elements may be considered as a potential candidate for use in broadband communication switch fabrics. This paper studies the fault tolerance and reliability of the balanced gamma network. The three main reliability measures -- terminal reliability, broadcast reliability, and network reliability -- are computed for the balanced gamma network in this paper. The input-output and network mean times to failure of the balanced gamma network are also calculated.

The performance of a multistage interconnection network in the presence of faults gives a better understanding of the dependability of the network than the reliability measures normally used. The performance analysis of the balanced gamma network with faults, under the uniform traffic type is presented. It is shown that the balanced gamma network is single fault-tolerant and robust under multiple faults. Simulation results of the throughput performance of the balanced gamma network with and without faults, under uniform traffic patterns, are also compared in this paper.

Keywords

Balanced gamma network, dependability, faults, fault tolerance, multistage interconnection networks, performance, reliability, switching elements, throughput.

1. INTRODUCTION

ATM (Asynchronous transfer mode) has been identified by ITU-T as the switching system capable of meeting the requirements of B-ISDN (broadband integrated services digital network) such as very high throughput, a low switching delay, a low probability of packet loss, expandability, testability and fault-tolerance, low cost and ability to achieve broadcasting as well as multicasting. ATM is a packet-oriented transfer mode using statistical time division multiplexing techniques [3,13,22]. An ATM switch requires very high speed switch fabrics in

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order to meet the needs of high throughput. Several switch fabrics have been investigated by researchers and organizations [3].

A multipath, single fault-tolerant MIN called the BG network [24] is discussed in this paper. The BG network has improved throughput performance under several different traffic patterns [19,20,24,25]. In this paper it is shown that the BG network exhibits excellent fault tolerance properties, high reliability and increased throughput performance even in the presence of SE faults.

In Section 2, the BG network is discussed in detail. In Section 3, the fault tolerance properties of the BG network have been highlighted and compared with other fault-tolerant MINs in [21]. The reliability analysis of the BG network is reported in Section 4. In Section 5, the performance of BG network under SE faults has been discussed, followed by the conclusion in Section 6.

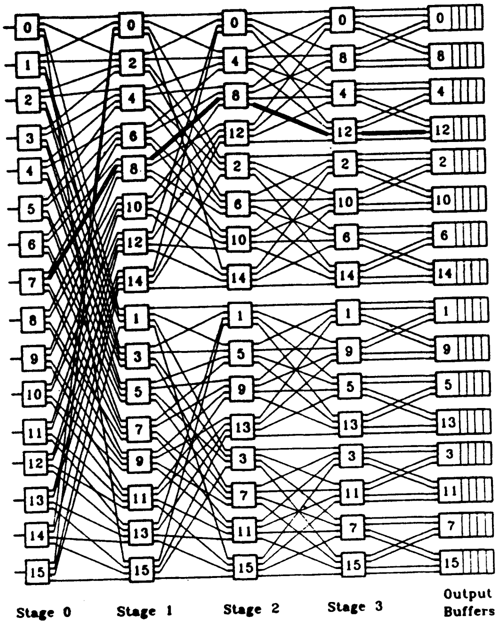

Figure 1 A 16×16 Balanced Gamma network showing routing of a packet from input port 7 to output port 12.

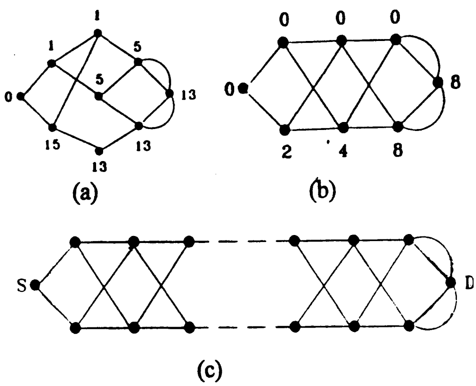

Figure 2 Terminal Reliability R-graph of the BG network.

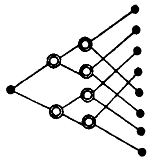

Figure 3 Reduced Broadcast Reliability R-graph of an 8x8 BG network.

2. THE BALANCED GAMMA NETWORK

The BG network [24] features 4×4 SEs and is derived by enhancing the gamma network. An $N \times N$ gamma network [15] consists of one stage of 1×3 SEs, followed by $log_2N\text{-}1$ stages of 3×3 SEs and finally one stage of 3×1 SEs. Each stage of the gamma network consists of N SEs. The ith SE in the jth stage is connected to SEs i, $(i+2^j)$ mod N, and $(i-2^j)$ mod N, in the $(j+1)$th stage.

An additional link is added to an ith SE of jth stage of an NxN gamma network and is connected to SE $(i+2^{j+1})$ mod N in the $(j+1)^{th}$ stage to form the BG network.

Thus an $N \times N$ BG network uses an input stage of 1×4 SEs, log_2N-1 stages of 4×4 SEs and an output stage of buffers, each of which is designed to accept up to 4 packets in each cycle. The buffers are used to store the packets from the network and to feed them to their respective destinations. Each stage comprises of N SEs, numbered 0 to $N-1$. The stages are numbered 0 to $n-1$, where $n = log_2N$, as shown in Figure 1. When the output buffers in the BG network are replaced by concentrators, then this network is identical to the kappa network [12]. Each SE in the BG network is addressed as $SE_{i,j}$ $(0 \leq i < N-1,\ 0 \leq j < n-1)$ where i indicates the SE number within a stage and j indicates the stage number.

The BG network uses a reverse destination tag routing scheme [20,21]. The routing tag bits for a cell to be routed from source S to destination D is D in binary. The SEs interpret the tag bits in the reverse order, i.e., the SE in the first stage (stage 0) switches based on the least significant bit (bit 0). In any SE, if the corresponding tag bit is 0 (1), then the packet is routed through either of the two top (bottom) output links. The general routing algorithm employed in the BG network is explained in [21]. An example of a *packet* (referred to as *cell* in ATM nomenclature) being routed from input port 12 to output port 7 in a $16x16$ BG network is shown in Figure 1. There are 2^n paths available between any input-output pair in an NxN BG network [21].

3. FAULT TOLERANCE IN THE BALANCED GAMMA NETWORK

A *fault-tolerant* MIN is one that is able to route packets from input ports to the requested output ports, in at least some cases, even when some of its network components (SEs or links or buffers) are faulty. The *fault model* characterizes all faults assumed to occur, stating the failure modes (if any) for each network component. We choose the following *fault model* to study the *fault tolerance* properties of the BG network :

1. Faults can occur in any network component except the first stage SEs, links connecting the last stage and the output buffers.
2. All faults are random and independent.
3. All faults are permanent.
4. Each faulty component is totally unusable.
5. All faults are detected with success, and the faulty component identified correctly.

The *fault tolerance criterion* is the condition that must be met for the network to be said to have tolerated a given fault or faults. The *fault tolerance criterion* chosen here for the BG network is fulfillment of the condition that there exists at least one path between any input-output pair of the network, which is called as the full access property [2]. The BG network is considered failed when the number and locations of the faulty components prevent the connection between at least one input-output pair of the network. The feature of the BG network that enables *fault tolerance* is the existence of alternate route(s) between every input-output pair in case of network component failures. In order to determine SE failures, each SE continually tests itself, also tests the SEs which are connected to its output links. Using this scheme, failure of an SE and of all the output links connected to it can be combined, which means that we need to consider only SE failures.

The failure of certain pairs of SEs in the BG network will result in loss of full access property for the BG network and so these SEs are called *critical pairs*. The different critical pairs of SEs of a *16×16* BG network are shown enclosed within boxes in Figure 2. The failure of a critical pair in ith stage of a BG network results in packet not being routed to the (*j*+*1*)th stage.

In the BG network each SE, $SE_{i,j}$, forms critical pair with exactly two other SEs, $SE_{i+2^j,j}$ and $SE_{i-2^j,j}$ (i ∈ {*0,..N-1*} ,j∈{*0*,..,*n*-1, ($i+2^j$) and ($i-2^j$) are mod *N* operations), which are within the same stage of the BG network. There are exactly *N* critical pairs of SEs in each stage of the BG network except in the last stage where the total number of critical pair of SEs is equal to *N/2*. Therefore the total number of critical pairs in an *N×N* BG network is equal to *N×(n-3/2)*. It can be shown that the BG network can tolerate up to *N/2×(n-2)* SE failures without losing full access so long as at least one of the SEs in each critical pair is not faulty [21].

Theorem 1 : *The BG network will lose full access property if and only if any of the critical pairs of SEs fails.*

Proof : Given in [21].

The fault model, fault tolerance criterion and the fault tolerance method used for analyzing the fault tolerance properties of the BG network are the same as those given in [1]. To sum up, the BG network is a *single fault-tolerant* MIN and is robust under multiple faults.

4. RELIABILITY ANALYSIS FOR THE BALANCED GAMMA NETWORK

MINs being considered for use in broadband communication switch fabrics, apart from having high throughput performance, must also be highly reliable as they would be used as real-time communication networks. For real-time systems the principal concerns are time-dependant reliability and the mean time to failure (MTTF) [10].

The reliability of a component (system) is the conditional probability that the component (system) operates correctly throughout the interval [t_0, t] given that it was operating correctly at time t_0 [18]. Exact reliability analysis of a complex system is complicated even under simplifying assumptions of perfect coverage and no repair [5]. The reliability analyses of several MINs have been reported in [5,6,7,14,16,17,23]. There are three basic forms of connections, and consequently three main reliability measures are computed for MINs [18]. These are the terminal reliability, the broadcast reliability and the network reliability. The MTTF which specifies the quality of a system is the expected time that a system will operate before the first failure occurs. The two figures which are usually calculated for MINs are the input-output MTTF and network MTTF. For the reliability analysis the following assumptions are made.

1. The SEs in the first stage and the output buffers are highly reliable.
2. All SEs in the subsequent stages have an identical and constant failure rate of λ, and SE failures are statistically independent.
3. SE faults are permanent and each faulty SE is totally unusable.

4.1 Terminal Reliability

Terminal reliability (TR) is the probability that at least one path exists from a particular input port to a particular output port. TR is always associated with a terminal path (TP) which is a one-to-one connection between an input port (the source) and an output port (the destination). A network is considered failed if it is not able to establish a connection from a given source to a given destination. TR is normally used as a measure of the robustness of a communication network [23].

In the BG network, a packet has an option of being routed through either the regular or the alternate link at each stage. For the evaluation of the terminal reliability of the BG network, the set of paths in the network between the given input-output pair is represented as a directed graph, sometimes referred to as the redundancy graph (or R-graph) [2], with its vertices representing the SEs and the edges representing the connecting links. The R-graph of the BG network is not same for each input-output pair and is dependent on the destination. Certain input-output pairs use only a pair of SEs in each stage in their TP and hence have a lower TR than the other pairs which use more than two SEs at certain stages. The TR of an input-output pair is the lowest when the least significant $\lceil \frac{n}{2} \rceil$ bits are identical.

While Figure 2(a) shows the R-graph corresponding to a destination in a *16x16* BG network having the best TR, Figure 2(b) shows an example of the worst case. Figure 2(a) shows the best case TR because it can tolerate upto 2 SE failures in stage 2, while Figure 2(b) shows the worst case TR because it can tolerate only 1 SE failure in stage 2. Figure 2(c) shows the R-graph of an *NxN* BG network having the worst case TR. The worst case TR of the BG network is

$$TR(t) = (1-(1-p)^2)^{(n-1)}, \tag{1}$$

where p = reliability of each SE in stages 1 to n-1 = $e^{-\lambda t}$.

Thanks to rapid advancements in very large scale integrated circuits, component (SE) failure rates of 10^{-6} failures/hour (or better) are quite realistic. The terminal reliability of the BG network for 10 years is given in Table 2. For this and subsequent calculations in this paper, a failure rate of $\lambda = 10^{-6}$ failures/hour is assumed.

One has to remember that the exact terminal reliability is obtained by multiplying the figure in the table by the reliabilities of the components which are assumed to be fault-free, viz., the SE in the first stage, the destination buffers and the links connecting the last stage SEs to the buffers.

Table 1 Terminal reliability of the BG networks.

Network Size(N)	Terminal Reliability
4	0.992956
8	0.985962
16	0.979017
32	0.972121
64	0.965273
128	0.958474
256	0.951723
512	0.945019
1024	0.938362

Table 2 Broadcast reliability of the BG networks.

Network Size(N)	Broadcast Reliability
4	0.985962
8	0.958474
16	0.905776
32	0.808913
64	0.645155
128	0.410382
256	0.166049
512	0.027185
1024	0.000729

4.2 Broadcast Reliability

Broadcast reliability (BR) is the probability that at least one path exists from a particular input port to all the output ports. BR is always associated with a broadcast path (BP) which is a connection from one source to all destinations in the network. BR is usually referred to as the source-to-multiple terminal (SMT) reliability [23. Under this criterion, the network is considered failed when a connection cannot be made from a given input port to at least one of the output ports.

The BR for each input port is identical in the BG network and so the BG network has a uniform broadcast reliability. The equivalent reduced broadcast reliability R-graph for an 8x8 BG network is given in Figure 3 [21]. Each of the composite nodes in Figure 3, shown by a double circle, represents a pair of SEs in the network; the edge between two composite nodes indicates the set of edges between any one of the SEs in the first composite node and any one of the SEs in the second. From the broadcast reliability redundancy graph shown in Figure 3, the broadcast reliability of an $N{\times}N$ BG network is

$$BR(t) = \prod_{i=1}^{n-1}(1-(1-p)^2)^{2^i}\ . \tag{2}$$

Table 3 gives the broadcast reliability of the BG network for 10 years.

4.3 Network Reliability

Network reliability (NR) is the probability of maintaining full access capability throughout the network. This measure considers the tolerable average number of switch failures [7]. NR is associated with network path (NP) which is a many-to-many connection, linking sources to many destinations.

The BG network loses full access property only when one or more critical pairs fail. Considering all the possibile combinations of critical pair failures, we arrive at the NR of the BG network as

$$NR(t) = 1 - \sum_{i=2}^{TN} F_i \times (1-p)^i \times p^{TN-i}\ , \tag{3}$$

where
$TN = N{\times}(n\text{-}1)$ - Total number of simultaneous SE faults.
F_i - Total possible combinations of failure of i SEs causing the BG network to lose full access property where $2 \le i \le TN$.

As N increases the calculation of F_i becomes a very tedious process. The evaluation of NR of a network is known to be an NP-hard problem. Calculation of NR for the BG network for $N{=}8$ using (3) is done in [21] and this is equal to 0.922639. One alternative approach is to develop approximate methods as the one given in [16]. The lower and upper bounds for NR of the BG network using approximations is given in this paper.

For the BG network to retain full access none of the critical pairs can be faulty. Both SEs in any critical pair are always located within a stage of the network. Therefore, each stage of the BG network has to be reliable for the BG network to be reliable. Stages *1* to *n-2* are identical as they have N critical pairs and so each of them has the same reliability; stage *n-1* has only *N/2* critical pairs and so has a different reliability. Since the reliability of each stage of the BG network is independent of other stages, the NR of the BG network is the product of reliabilities of the each stage (SR) in the BG network. The relability of any stage i ($i \in \{1,2,\ldots,n\text{-}2\}$) is given by

$$SR(t) = \sum_{i=0}^{N} INF_i\, p^{N-i}(1-p)^i \tag{4}$$

and the reliability of the last stage (LSR) is given by

$$LSR(t) = \sum_{i=0}^{N} LNF_i\, p^{N-i}(1-p)^i \tag{5}$$

where INF_i and LNF_i indicate all possible combinations of i SE failures in the intermediate stages and in the last stage respectively, which do not make the BG network lose full access. The NR of the BG network is given by

$$NR_1(t) = SR(t)^{(n-2)} \times LSR(t). \tag{6}$$

The NR of the BG network given by $NR_1(t)$ is slightly less than the actual NR of the BG network because (6) does not take into account the different possible combinations of SE failures in the overall network. The difference in NR given by (3) and (6) is 8.13574×10^{-4} for an *8x8* BG network.

The computation of INF_i and LNF_i is cumbersome for larger values of *N*. Moreover, the values of INF_i and LNF_i for i=0 and 1 have a major contribution to the values of SR and LSR respectively. Therefore we truncate the values of LR and SR at i = 4 and get the NR lower bound of the BG network as

$$NR_{LOW}(t) = \left(\sum_{i=0}^{3} INF_i p^{N-i}(1-p)^i\right)^{n-2}\left(\sum_{i=0}^{3} LNF_i p^{N-i}(1-p)^i\right) \tag{7}$$

The computation of F_i is even more complex than INF_i and LNF_i. The values of F_2 and F_3 constitute the major factor in (3). Therefore, the upper bound of NR obtained by approximating NR given in (3) is

$$NR_{UP}(t) = 1 - \sum_{i=2}^{3} F_i \times (1-p)^i \times p^{TN-i} \tag{8}$$

General expressions for INF_i, LNF_i and F_i used in $NR_{LOW}(t)$ and $NR_{LOW}(t)$ are given in [21]. For an 8×8 BG network the values of NR(t), $NR_{LOW}(t)$, and $NR_{UP}(t)$ are 0.92263944, 0.92182587 and 0.94496154 respectively. It can be clearly seen from these values that the NR of an 8×8 BG network is much closer to the lower bound than to the upper bound.

The lower and upper bounds of NR for BG networks of different sizes for 10 years are given in Table 4. It can be seen from the table the lower bound $NR_{LOW}(t)$ of the BG network drops down as the size of the BG network increases. Since NR has been shown to be closer to the lower bound, it can be concluded that the NR of the BG network reduces to quite low values with increase in network size. Since the values of lower bound are nearly zero and those of the higher bound are one for N>128, these values have not been included in the above table. It should also be noted that the actual value of NR would be much lower if we take into account the reliabilities of the first stage SEs and of the last stage buffers. However, NR gives the probability that full access is never lost over a period of ten years assuming no repair is possible.

Table 3 Network reliability bounds of the BG networks.

Network Size(N)	$NR_{LOW}(t)$	$NR_{UP}(t)$
4	0.985962	0.986012
8	0.921826	0.944962
16	0.727123	0.971385
32	0.199360	0.999811
64	1.637E-6	1.000000
128	7.27E-19	1.000000

4.4 Input-Output MTTF

The input-output MTTF is the expected time a network will be functional before the failure of at least one of its terminal paths. The input-output MTTF for the BG network is given as

$$MTTF_T = \int_0^\infty TR(t)dt. \tag{9}$$

Using the TR expression in (1) we have

$$MTTF_T = \int_0^{\infty} (1-(1-e^{-\lambda t})^2)^{n-1} dt. \tag{10}$$

Table 4 shows the $MTTF_T$ of the BG Network for different values of N.

4.5 Network MTTF

The network MTTF is the expected time a network will be functional before it loses the full access property. The network MTTF for the BG network is given as

$$MTTF_N = \int_0^{\infty} NR(t)dt \tag{11}$$

Since the exact NR of the BG network is difficult to be computed we use the lower bound given in (7) as this closer to the exact NR than the upper bound given in (8). Subsituting the lower bound experession (7) in (11) we have

$$MTTF_{N-LOW} = \int_0^{\infty} \left(\sum_{i=0}^{3} INF_i p^{N-i}(1-p)^i\right)^{n-2} \left(\sum_{i=0}^{3} LNF_i p^{N-i}(1-p)^i dt\right) \tag{12}$$

Table 5 shows the MTTF lower bound of the BG Network for different values of N. It can be seen from Table 5, that for $N=1024$, the network loses full access in around 2 months. Therefore, if repair is not possible, each SE of the large sized BG networks should have very low failure rates.

Table 4 Input-Output MTTF (in 10^6 hours) for different sizes of the BG network.

Network Size(N)	Input-Output MTTF
4	1.500000
8	0.916667
16	0.700000
32	0.582143
64	0.506349
128	0.452742
256	0.412421
512	0.380760
1024	0.355094

Table 5 Lower bound Network MTTF (in 10^6 hours) for different sizes of the BG etwork.

Network Size(N)	$MTTF_{N-LOW}$
4	0.916667
8	0.315664
16	0.136992
32	0.063382
64	0.029908
128	0.014223
256	0.006795
512	0.003258
1024	0.001568

5. PERFORMANCE OF THE BG NETWORK IN THE PRESENCE OF SE FAULTS

The various reliability measures explained in the previous section do not provide a clear indication of the dependability of the network. On the other hand, the performance of a network under faulty network components gives us a better insight in this regard.

Performance analysis and simulation results of the BG network under uniform random traffic pattern have been reported in [24] and [20] respectively. Here we present the performance analysis of the BG network in the presence of an SE fault under the uniform random traffic pattern, where one packet is assumed to arrive at each input line during each cycle. The random traffic implies that each output line is equally probable to be requested by the arriving packets.

Let T(N) be the throughput performance of an NxN BG network without faults under uniform random traffic. Let the failed SE be $SE_{i,j}$ $(j\neq 0)$. The performance of the BG network due to the failure of $SE_{i,j}$ is lower than that of the BG network without the failure of $SE_{i,j}$ because the traffic which normally goes through $SE_{i,j}$ would be routed through the SEs which form critical pairs with $SE_{i,j}$. Due to increased contention at the SEs which form critical pairs with $SE_{i,j}$ the throughput of the BG network drops down. The degradation of performance depends upon the stage in which the SE fault occurs.

5.1 SE fault at stage 1

There is no cell loss at stage 1 of an $N\times N$ BG network if there are no SE faults at stage 1 because each SE in stage 1 receives no more than two cells during each cycle. Due to the presence of an SE fault at stage 1 some cells could be lost at stage 1. Without loss of generality let us consider an SE fault at $SE_{4,1}$ in an $N\times N$ BG network where $N>4$. Due to the SE fault at $SE_{4,1}$, cells which would have normally been routed through it (under no fault condition) from $SE_{4,0}$ and $SE_{3,0}$ would now be routed through $SE_{6,1}$ and $SE_{2,1}$.. $SE_{6,1}$ and $SE_{2,1}$ form critical pairs with the faulty $SE_{4,1}$. Now $SE_{6,1}$ and $SE_{2,1}$ have a possibility of receiving three cells each from stage 0. If all these three cells have the same switching bit for stage 1, then one packet would be dropped. There are four possible combinations in which packets can be lost -- each of $SE_{6,1}$ and $SE_{2,1}$ receiving three packets, where all these three packets have the same switching bit (0 or 1) in each of the SEs. Under full load, 4 out of N incoming packets are lost. The probability $P_{3\text{-}stage1}$ that each of the SEs $SE_{6,1}$ and $SE_{2,1}$ receive three packets and all these three packets having switching bit for stage 1 is given by the expression

$$P_{3-stage1} = \left(\frac{1}{2}\right)^3 \times \left(\frac{1}{2}\right)^3. \tag{13}$$

Therefore the probability of cell loss at stage 1, $P_{cell\ loss\text{-}stage\ 1}$, is

$$P_{cellloss-stage1} = \frac{4}{N} \times \left(\frac{1}{2}\right)^3 \times \left(\frac{1}{2}\right)^3. \tag{14}$$

The throughput of an $N\times N$ BG network in the presence of a fault can be approximated as

$$Throughput = T(N) - P_{cellloss-stage1}. \tag{15}$$

The 4×4 BG network constitutes a special case where the stage 1 is the last stage in the network. Due to this fact, throughput of the 4×4 BG network is further degraded by a factor, which can be shown to be 2^{-7} [21]. The throughput of an $N\times N$ BG network in the presence of an SE fault, computed using (15) is given in Table 7. The throughput of the BG network without fault, $T(N)$, under uniform random traffic is taken from [20]. Simulation results of the throughput of the BG network under an SE fault are also provided in Table 6.

It can be clearly seen that the throughput of the BG network in the presence of an SE fault at stage 1 is comparable to that obtained by simulation. It is to be noted that the presence of a single SE fault at stage 1 does not affect the throughput performance of the BG network under uniform random traffic in large networks. This is due to the fact that the probability of cell loss is dependent on the size of the network, as given by (15).

5.2 SE fault at stage j ($j \in \{2,3, ...,n-1\}$)

The throughput analysis of an $N \times N$ BG network ($N>4$) in the presence of single SE fault in stage j ($j>1$) is presented in this section. When $SE_{i,j}$ is failed, the packets which are normally routed through $SE_{i,j}$ would now be routed through SEs which form critical pairs with $SE_{i,j}$, which receives packets from four SEs in stage j-1. Of these four SEs, regular links of two SEs are connected to $SE_{i,j}$ and alternate links of the other two SEs are connected to $SE_{i,j}$. Let these SEs be SE1, SE2, SE3 and SE4 respectively. SE1 and SE2 can reroute the packets to be switched to $SE_{i,j}$ through their alternate links. Out of the k packets which arrive at SE3 or SE4 having the same switching bit as that of a packet to be switched to $SE_{i,j}$, k-1 packets are dropped at these respective SEs causing packet loss. This is due to the failure of $SE_{i,j}$. Similarly if SE3 and SE4 have packets to be switched to $SE_{i,j}$ then these packets are dropped at SE3 and SE4 respectively. Therefore, the probability of packet loss at stage j-1 due to failure of $SE_{i,j}$ is

$$P_{CL-STG:J-1} = \frac{4}{N} \times al_{j-1} \tag{16}$$

where al_{j-1} is the probability that the alternate links of stage j-1 carry a packet.

Due to the failure of $SE_{i,j}$ packets are lost even at stage j. If SE1 or SE2 has a packet p_k, which is to be routed to $SE_{i,j}$ then this packet would be routed through the alternate links of SE1 or SE2 causing p_k to go to SEs which form critical pair with $SE_{i,j}$. Let these SEs which form critical pairs with $SE_{i,j}$ be called SEc1, SEc2. If SEc1 and SEc2 already have a minimum of two packets, having the same switching bit as that of p_k for the stage j, then the packet p_k is dropped at stage j. The probability that packet loss occurs at stage j due to the failure of a $SE_{i,j}$ is

$$P_{CL-STG-J} = \frac{4}{N} \times al_{j-1} \times al_j \tag{17}$$

Table 6 Throughput of BG network in the presence of a single SE fault at stage 1.

Networ Size(N)	Throughput without faults T(N)	Throughput with faults by analysis	by simulation
4	1.000000	0.976563	0.978250
8	0.993250	0.985438	0.987125
16	0.985625	0.981718	0.983000
32	0.976250	0.974297	0.974875
64	0.968750	0.967773	0.966391
128	0.959656	0.959168	0.959031
256	0.951191	0.950946	0.950348
512	0.942209	0.942087	0.942387
1024	0.934344	0.934283	0.934273

Table 7 Throughput Analysis of BG network in the presence of a single SE fault at different stages under uniform random traffic.

Networ Size	Analysis results of the performance of the BG network in which SE has failed in stage							
N	2	3	4	5	6	7	8	9
8	0.959	NA	NA	NA	NA	NA	NA	NA
16	0.968	0.967	NA	NA	NA	NA	NA	NA
32	0.967	0.966	0.966	NA	NA	NA	NA	NA
64	0.964	0.964	0.964	0.964	NA	NA	NA	NA
128	0.957	0.957	0.957	0.957	0.957	NA	NA	NA
256	0.950	0.950	0.950	0.950	0.950	0.950	NA	NA
512	0.941	0.941	0.941	0.941	0.941	0.941	09417	NA
1024	0.934	0.934	0.934	0.934	0.934	0.933	0.934	09341

Therefore the overall probability of packet loss due to a failure of $SE_{i,j}$ is the sum of the right hand sides of (16) and (17). Thus the throughput of the BG network under single SE fault at a stage other than it first two stages is

$$Throughput = T(N) - P_{CL-STG-J-1} - P_{CL-STG-J} \tag{18}$$

The throughput performance of the BG network without SE failure, under uniform random traffic *T(N)* is taken from [20]. The values of al_j and al_{j-1} are calculated by the recursive expression given in [24]. The throughput performance of an $N \times N$ BG network using (18) is presented in Table 7. The simulation results closely agree (within 0.2%) with those in Table 7. It can be noted that the throughput of large size BG network is very minimally affected by the presence of single SE fault.

6. CONCLUSION

The fault tolerance properties of the BG network under the chosen fault model and fault tolerance criterion have been studied in this paper. The BG network has been shown to be single fault-tolerant and robust under multiple faults. It has been shown that the BG network fails to satisfy the full access property only due to the failure of certain critical pairs SEs. The reliability analysis of the BG network has been carried out and the different reliability metrics have been computed. From the reliability metrics it can be concluded that either repair should be possible or components used in the BG network should have very low failure rates in order to obtain high network reliability.

It has been shown that the performance of the BG network in the presence of faults gives a better understanding of the dependability of the network than the reliability metrics which have been reported for most of the MINs studied so far. Performance analysis of the BG network in the presence of single SE fault at any stage, under uniform random traffic has been presented in this paper. Results from analysis and simulation exhibit a close match. Performance of the BG network is not significantly degraded when a single SE fault occurs. The throughput performance of the basic BG network is not high enough to meet the specifications of broadband communications. Buffered and enhanced forms of the BG network -- enlargement, dilation and replication. [19,25] -- provide increased throughput and high reliability, thus making them worthy of consideration as potential candidates for use in broadband communication switch fabrics.

7. REFERENCES

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