
Probabilistic Reasoning for Closed-Room People Monitoring

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Abstract. In this chapter, we present a probabilistic reasoning approach to recognizing people entering and leaving a closed room by exploiting low-level visual features and high-level domain-specific knowledge. Specifically, people in the view of a monitoring camera are first detected and tracked so that their color and facial features can be extracted and analyzed. Then, recognition of people is carried out using a mapped feature similarity measure and exploiting the temporal correlation and constraints among each sequence of observations. The optimality of recognition is achieved in the sense of maximizing the joint posterior probability of the multiple observations. Experimental results of real and synthetic data are reported to show the effectiveness of the proposed approach.

Keywords: people monitoring, probabilistic reasoning, Viterbi algorithm, domain knowledge, HMM

1 Introduction

With the increased concern for physical security in the face of global terrorism and outbreaks of infectious viruses, automated video surveillance for enhanced security in human living and work places has received unprecedented attention from industries, research institutes, and governmental agencies over the past few years [1, 2]. One main task of many video surveillance systems is to associate each person with an identity or to correspond a same person observed at different time instances. The results allow the derivation of such useful information as how long a person has stayed in a site, how many people are in the room during a certain period of concern, and who they are. Potential applications of such a system include, for example, understanding human activities in a monitored work place [8, 9], keeping aware of user identities in an intelligent room [3], and identifying who could possibly be infected with a newly identified victim of Severe Acute Respiratory Syndrome (SARS) [10].

A number of related solutions have been proposed in the literature for people access control and monitoring. For example, biometrics have been increas-

ingly used for identity recognition and obtained satisfactory results. Representative work in biometric-based recognition ranges from fingerprint and iris identification to face and gait recognition [11–14, 16]. However, many of these methods require intrusive data collection, e.g., demanding human proactive action and collaboration during the course of identification, and thus, work mainly in well-controlled environments. Although gait recognition can partly address this limitation by exploring human motion dynamics, gait feature, by itself, has limited discriminating power and only works for people whose motion patterns have been well characterized and pre-stored in a database for matching.

Regardless of the type of features used, most existing approaches accomplish the recognition task based on some maximum likelihood classification rule [15], where a definite decision is made based on features observed at a single time instance/duration. The temporal correlation and constraints among the observations obtained over time, however, are seldom utilized even they exist in some specific contexts; for example, in the case of closed-room monitoring, a person currently inside the room cannot enter the room again without first leaving it, and vice versa.

On the other hand, dynamic Bayesian networks (DBNs) [30, 31] are becoming popular in probabilistic inference due to their ability in incorporating various prior constraints and dealing with uncertainties in a systematic manner. In particular, hidden Markov models (HMMs), a specific form of DBNs, are well suited for modeling and identifying an event, represented as a state sequence, which can best explain a series of observations, for at least two reasons [24–26]. First, the topology among the hidden states, i.e., their interdependencies, can encode prior knowledge about how the event evolves. Second, the forward-backward and Viterbi algorithms [29], which are developed based on HMM's lattice structure, allow one to evaluate the probabilities of different state sequences efficiently, and thus to identify the most likely state sequence.

In this chapter, we present a video-based system using probabilistic reasoning and based on the Viterbi algorithm for monitoring people entering and leaving a closed room (i.e., a room with only a single entrance/exit; e.g., a lab, class room, or meeting room). The system consists of two modules: a feature extraction module to detect/track people entering or leaving the only entrance/exit of the closed room and extract their low-level features for recognition in an unintrusive manner, and a people recognition module to correspond each observed person with a person previously entering the room or to identify him/her as a new person unseen before. Figure 1 depicts the architecture of the proposed system. Rather than using only a single observation, we perform recognition by exploiting the temporal correlation and constraints among multiple people observations acquired at different time instances. Consequently, our method can effectively enhance the limited discriminating power of low-level features, such as color histograms and face features acquired using a camera from a distance. Experimental results demonstrate that the proposed

system can achieve superior recognition accuracy as compared with the existing systems using maximum likelihood approaches.

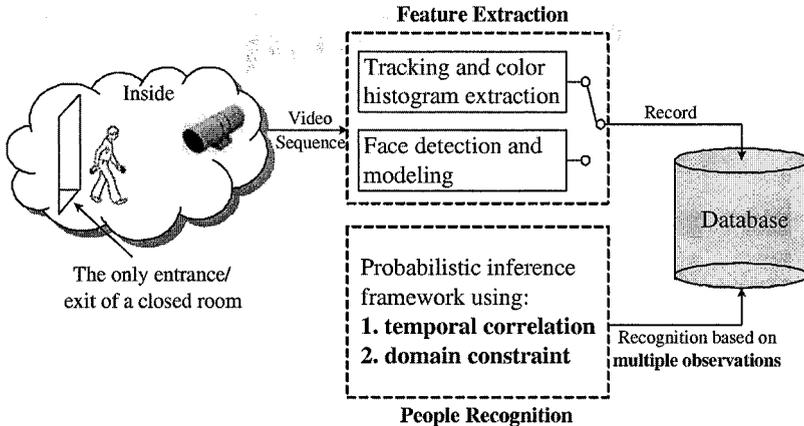


Fig. 1. Overview of the proposed closed-room people monitoring system

2 Viterbi Algorithm

In this section, we briefly review preliminaries of HMM and the Viterbi algorithm, based on which our proposed system is constructed.

In general, an HMM can be characterized by a set of parameters $\Lambda = \{A, B, \pi\}$, where $A = \{a_{ij} | a_{ij} = P[q_{t+1} = S_j | q_t = S_i]\}$ denotes the transition probabilities from state S_i at time t to state S_j at time $t+1$, $B = \{b_j | b_j(O_t) = P([O_t | q_t = S_j])\}$ the observation probabilities of state S_j , and $\pi = \{\pi_i | \pi_i = P[q_1 = S_i]\}$ the initial state probabilities. An example of a three-state HMM is shown in Fig. 2.

With these probabilities, three basic problems can generally be addressed with an HMM [27]: 1) Evaluate $P[O|\Lambda]$, the probability of an observation sequence O given the model Λ ; 2) Find the most likely state sequence given the model and an observation sequence; and 3) Find the model $\Lambda = \{A, B, \pi\}$ that maximizes $P[O|\Lambda]$ for a given observation sequence. The first and third problems are known as model evaluation and training, respectively, and can be solved by forward-backward algorithm and Baum-Welch method [27]. Of relevance to our application is the second problem, in which the Viterbi algorithm plays an important role.

Consider a discrete-time dynamical system which is governed by a Markov chain and generates a sequence of observable outputs (observations) according to a number of hidden (unobservable) states. Our objective is to infer the most probable state sequence from the observation sequence. Straightforwardly, one

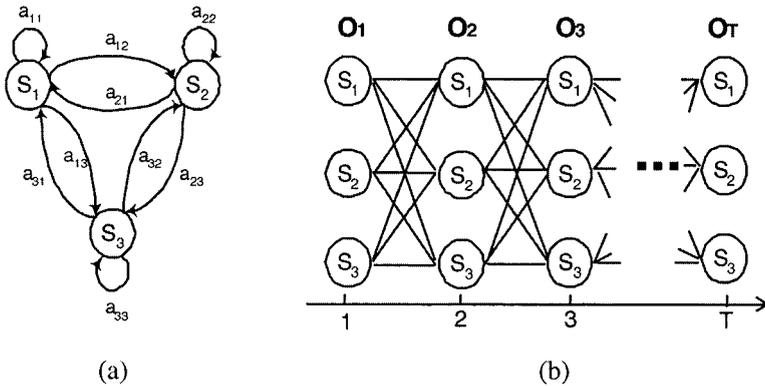


Fig. 2. Structure of a 3-state HMM: (a) transition diagram and (b) temporal view

can find the most probable state sequence by enumerating all possible state sequences and evaluating the probability of the observation sequence due to each possible state sequence. While viable, this exhaustive approach is computationally intensive even for a small number of states and observations. For example, with five observations (i.e. $T = 5$), the 3-state HMM shown in Fig. 2 will have 243 possible state sequences.

Using the Viterbi algorithm, one can exploit the dynamic programming technique to simplify the computation substantially. To see this, let us first define the quantity

$$\delta_t(i) = \max_{q_1, q_2, \dots, q_{t-1}} P[q_1 q_2 \dots q_t = S_i, O_1 O_2 \dots O_t | \Lambda], \quad (1)$$

which is the highest probability of a state sequence which accounts for the first t observations and ends in state S_i at time t . By induction, it is easy to see that

$$\delta_{t+1}(j) = [\max_i \delta_t(i) a_{ij}] \cdot b_j(O_{t+1}). \quad (2)$$

Hence, for each state S_i at time t , one can find one state sequence ending in it and assuming the highest probability $\delta_t(i)$. We shall refer to this state sequence as the partial best state sequence. Once we have determined the hidden state corresponding to the observation obtained at time t , the uncertainties up to t can be resolved. For an HMM with N states, there are N partial best state sequences due to each observation. At the end of the observation (i.e., time T), the Viterbi algorithm can find the most probable state sequence with probability $\max_i \delta_T(i)$. We shall refer to this state sequence as the *best state sequence*.

The procedure for finding the most probable state sequence can be summarized as follows [27]:

1) Initialization:

$$\delta_1(i) = \pi_i b_i(O_1), \quad 1 \leq i \leq N \quad (3a)$$

$$\psi_1(i) = 0. \quad (3b)$$

2) Recursion:

$$\delta_t(j) = \max_{1 \leq i \leq N} [\delta_{t-1}(i) a_{ij}(t-1)] b_j(O_t), \quad 2 \leq t \leq T, 1 \leq j \leq N, \quad (4a)$$

$$\psi_t(j) = \arg \max_{1 \leq i \leq N} [\delta_{t-1}(i) a_{ij}(t-1)], \quad 2 \leq t \leq T, 1 \leq j \leq N. \quad (4b)$$

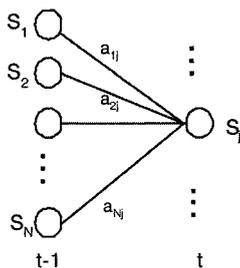


Fig. 3. Illustration of recursive computation of the Viterbi algorithm

3) Termination:

$$p^* = \max_{1 \leq i \leq N} [\delta_T(i)], \quad (5a)$$

$$q_T^* = \arg \max_{1 \leq i \leq N} [\delta_T(i)]. \quad (5b)$$

4) Path (state sequence) backtracking:

$$q_t^* = \psi_{t+1}(q_{t+1}^*), \quad t = T-1, T-2, \dots, 1. \quad (6)$$

The array ψ is introduced to keep track of the argument that maximizes (2) for each t and j , indicating all the preceding states along each partial best state sequence. With ψ , one can retrieve the *best state sequence* of the whole process as well as the partial best state sequence ending at a given state at any time. The latter is particularly useful in our people monitoring application as we shall show later.

From Fig. 3, we can see that the recursion computation of the Viterbi algorithm can be derived based on the temporal view structure of an HMM as shown in Fig. 2(b). The key point of its efficiency is that since there are only N possible states at each observation time, all the possible state sequences will end in these N states no matter how long the observation sequences are.

Apart from its tractability in computation, the Viterbi algorithm bears another important property: it does not make any maximum likelihood decision at each intermediate observation time, but obtains an overall decision

by taking into account the whole sequence of observations. Ambiguity and/or misjudgement based on partial observations can be corrected later when more observations become available. This is well suited for our application, as it can exploit the temporal correlation among the observation sequence and recognize people based on multiple observations, reducing the chance of error due to making hard decision based on features with low discriminating power.

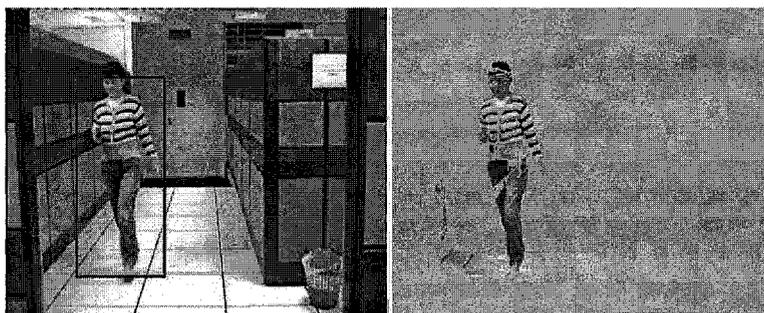
3 Low-Level Features

The feature extraction module of our proposed system currently makes use of two types of low-level features as illustrated in Fig. 4: color histograms and low-resolution human faces. These features are described in detail below.

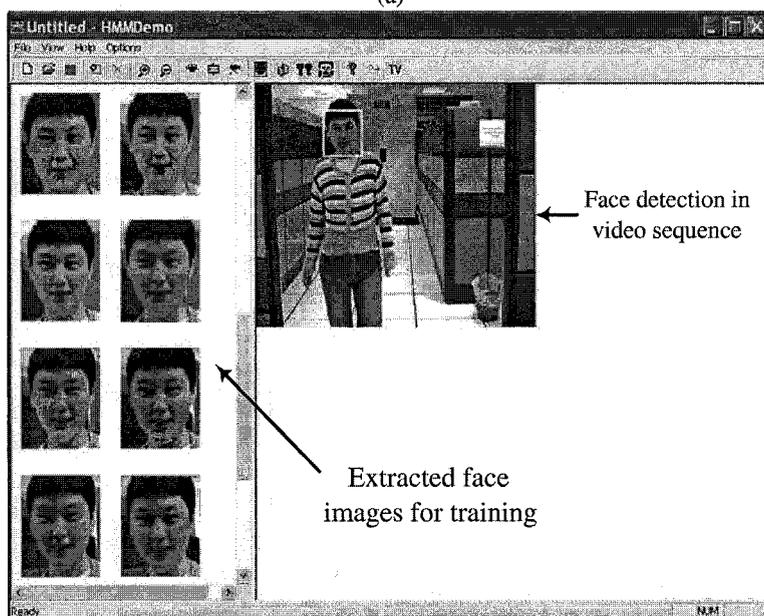
Color histogram is a popular color feature for content-based image and video analysis [20–22]. It is easy to compute and rather invariant to change in shape or size [19]. Our system detects and tracks each moving person as a foreground region and counts the color distribution of pixels within the region (i.e. color histogram) as the appearance feature of the person. The feature similarity of two observed people c_i and c_j can be measured by their color histogram intersection, defined as $\sum_{k=1}^K \min(H_i(k), H_j(k))$, where H_i and H_j are the normalized color histograms of the two people, respectively, and K is the total number of color bins in the histogram. For the details of a color histogram based people tracking and recognition system, see our previous work [17, 18].

For facial feature, we make use of two functions provided in Intel Open Source Computer Vision Library (OpenCV) [4], `HarrFaceDetection` and `HMM-FaceRecognition`, to automatically detect and model human faces in video sequences. The face detector was originally proposed by Viola [5] and further improved by Lienhart [6], while the embedded HMM (EHMM) face recognizer was developed by Nefian *et al.* [7]. It has been shown that the EHMM recognizer can exploit the natural structure of frontal faces and achieve outstanding performance. With a number of face images of a same person, say c_i , we train his/her EHMM feature using a set of observation vectors obtained from the corresponding 2D-DCT coefficients. The likelihood of an unknown face observation c_j with respect to person c_i can be calculated by a doubly embedded Viterbi algorithm [7].

Direct application of the color histogram and face similar measures as defined above poses some potential problems. For example, the color histogram intersection of two different people is generally larger than zero for the reason that some of their appearance features, such as hair and skin, could share similar colors. On the other hand, the same person observed at different times may not have identical color histograms due to difference in lighting conditions, camera view angles, and segmentation results. The low-resolution face features are also subject to similar problems. Moreover, owing to successive multiplications of values less than one, the likelihood of faces calculated by



(a)



(b)

Fig. 4. Two types of low-level features considered: (a) the color histogram of segmented foreground region and (b) the EHMM feature of detected face

the doubly embedded Viterbi algorithm is numerically very small and subject to round-off errors. A function is therefore required to map the value of similarity measure based on color histograms or face features of two observed people c_i and c_j , denoted as $S(c_i, c_j)$, onto a similarity probability $P(c_i \sim c_j)$, which indicates how likely c_i and c_j correspond to the same person.

Conceivably, the mapping function needs to have the following properties: 1) it should be non-decreasing; 2) it should be approaching 0 or 1 as $S(c_i, c_j)$ takes values near its lower or upper limit; and 3) the transition from low to high mapped values should take place at where the value of $S(c_i, c_j)$ becomes

evident to support that the two observed people are likely the same. After some subjective studies and comparisons, we select the *sigmoid* function [23] to perform the mapping, which can be expressed as follows:

$$P[c_i \sim c_j] = \frac{1}{1 + \exp[-\alpha(S(c_i, c_j) - \beta)]}, \quad (7)$$

where α and β are two parameters determining the shape of a sigmoid curve with α controlling the steepness of the transition and β defining the center of transition point. By experiments, we have determined the proper values of these two parameters for the similarity measures based on color histogram and face features, respectively.

It should be noted that many other features/attributes (e.g., fingerprint, iris pattern, voice, gait, etc.) for which a similarity measure is defined can be used in our proposed people monitoring system. To make the system less intrusive, we have only made use of color histogram and face features in the work reported here.

4 Probabilistic Reasoning Framework

4.1 Problem Formulation

Our first attempt is to develop a suitable HMM for the recognition task in a closed room by making use of the Viterbi algorithm. However, the parameters of HMMs need to be pre-learned from a set of representative data based on a fixed number of states. This situation, however, is not applicable to our case as the number of people as well as their activity patterns, i.e., the frequencies of entering and leaving the room, are generally different from place to place or time to time, and hard to be estimated from prior data.

To construct a framework that is well suited for the problem of our concern, we employ the lattice structure and parameter setting of HMMs, and formulate the problem of people recognition as follows. Assume that the closed room is empty when the system is first activated. When a person is entering the room, we append a new state to the state set (database) to represent his/her identity; when a person leaves at time t , he/she will be recorded as a new observation O_t . Thus the states in the state set at time t , denoted as $\mathbf{S}(t) = \{S_1 \cdots S_{N_t}\}$, correspond to the people identities in the database (i.e., people that possibly stay in the room at time t). When an observation sequence $O = \{O_1 \cdots O_T\}$ has been obtained over a period of time, the identities of the people leaving the room can be recognized from the state sets, $\mathbf{S}(1), \mathbf{S}(2), \dots$, and $\mathbf{S}(T)$, recorded over the same period. By characterizing this inference framework with a time-variant parameter set $\Lambda(t) = \{A(t), B(t), \pi(t)\}$, we can use the Viterbi algorithm to find an optimal state sequence $Q = \{q_1 \cdots q_T\}$ associated with the observation sequence to maximize a joint posterior probability $P(Q, O | \Lambda(t))$. In this way, each leaving person can correspond to one of those who are judged still inside the room.

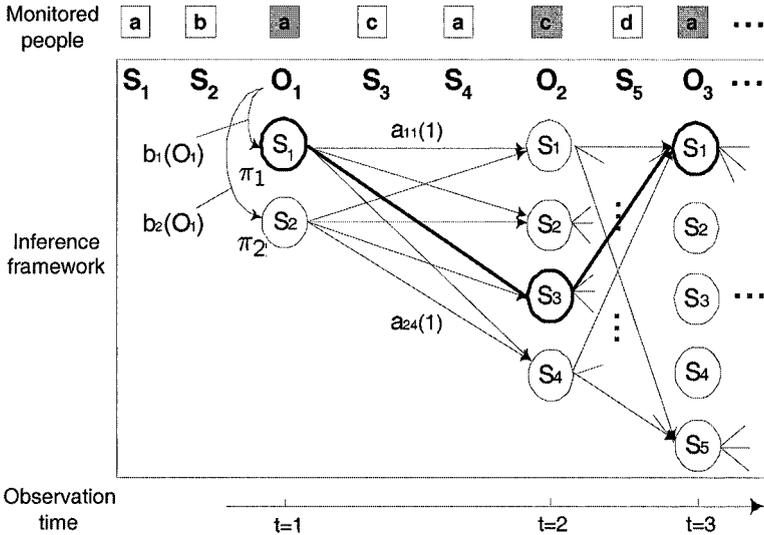


Fig. 5. Recognition example of the proposed probabilistic reasoning framework. The squares in the top row represent the observed people with their real identities manually labeled. The filled and unfilled squares denote the people leaving and entering, respectively

Fig. 5 illustrates a recognition example of the proposed framework, where the optimal state sequence obtained is shown in bold lines. From this sequence, we can identify O_1 as S_1 , O_2 as S_3 , O_3 as S_1 , etc. We can see that the framework has a lattice structure similar to HMMs; however, there are several key differences that distinguish our framework from conventional HMMs. First, the parameter set $\Lambda(t)$ is time variant and needs to be derived at each observation time instance based on all the previous possible states and current observations rather than from some training data. Second, the number of states in our model is not fixed but can increase over time before a decision is made. Third, the states can be indefinite because more than one states could be associated with the same person, e.g., both states S_1 and S_4 in Fig. 5 represent the person ‘a’. It can be seen that when ‘a’ leaves again at $t = 3$, the framework recognizes him/her as S_1 rather than S_4 , which is consistent with his/her identity (state) recognized at $t = 1$ in his/her first exit.

It should be noted that in our framework a state could represent the feature model or the identity of a person. For clarity, we shall use s_i to denote a person’s identity and S_i his/her feature model.

4.2 Framework Construction

From the problem formulation, it can be seen that the main task in constructing the proposed framework lies in the estimation of the time-variant

parameter set $\Lambda(t)$. Once $\Lambda(t)$ is known, we can find the optimal path indicating the recognized people by a “turn-the-crank” procedure given by the Viterbi algorithm [28]. Our solutions are given as follows.

• **Initial state distribution, π :**

According to the definition of the state set $\mathbf{S}(t) = \{S_1 \cdots S_{N_t}\}$, there are N_1 states (people) in the database when the first observation of exit is obtained at $t = 1$. Without any other prior knowledge, we assume that everyone inside the room has an equal probability of leaving the room. Hence,

$$\pi_i = \frac{1}{N_1}, \quad 1 \leq i \leq N_1. \quad (8)$$

• **Output probability of state i at time t , $b_i(O_t)$:**

In analogy with the definition in HMMs, let $b_i(O_t)$ be the probability of observation O_t generated by state i . We regard this probability as how likely an observed O_t is due to person \mathbf{s}_i , and simply approximate it by Eq. (7) using their feature similarity as

$$b_i(O_t) = P[O_t \sim S_i], \quad 1 \leq i \leq N_t. \quad (9)$$

• **State transition probability, $a_{ij}(t)$:**

Before proceeding to the next step, we define a set of probabilities between observation times t and $t + 1$ as shown in Fig. 6. For conciseness, we shall refer to “observation time” as “time” hereafter when there is no confusion. Let $P[\mathbf{s}_{i,t+/-} = 1]$ be the probability of person \mathbf{s}_i staying in the room at time instance $t^{+/-}$, where t^+ and t^- denote the times right after and before the observation O_t being made. (Straightforwardly, we have $P[\mathbf{s}_{i,t+/-} = 0] = 1 - P[\mathbf{s}_{i,t+/-} = 1]$). Let M be a likelihood matrix characterizing the similarities between people entering between times t and $t + 1$ (i.e., $S_{N_{t+1}} \cdots S_{N_{t+1}}$) and the people staying in the room at time t (i.e., $S_1 \cdots S_{N_t}$), defined as

$$M = \{m_{uv} | m_{uv} = P[S_{N_{t+u}} \sim S_u]\}, \quad 1 \leq u \leq N_t, 1 \leq v \leq N_t^{t+1}, \quad (10)$$

where N_t is the number of states (people who possibly stay in the room) at time t and $N_t^{t+1} = N_{t+1} - N_t$ is the number of people entering between times t and $t + 1$. With these auxiliary probabilities, we can now derive the transition probability using the following three steps.

i) *Transition probability, $a_{ij}(t)$:*

The transition probability $a_{ij}(t)$ measures the odds that person \mathbf{s}_i leaves the room at time t and person \mathbf{s}_j leaves at time $t + 1$ (i.e., $q_t = \mathbf{s}_i$ and $q_{t+1} = \mathbf{s}_j$). Lacking other knowledge, it is reasonable to assume that the probability for \mathbf{s}_j to leave at time $t + 1$ is proportional to his/her existence odds in the room at time $t + 1^-$. Conceivably, a person cannot leave a room if he/she is not in the room at all. Hence, we compute the transition probability as

$$a_{ij}(t) = \frac{P[\mathbf{s}_{j,t+1-} = 1 | q_t = \mathbf{s}_i]}{\sum_j P[\mathbf{s}_{j,t+1-} = 1 | q_t = \mathbf{s}_i]}, \quad 1 \leq i \leq N_t, 1 \leq j \leq N_{t+1}, \quad (11)$$

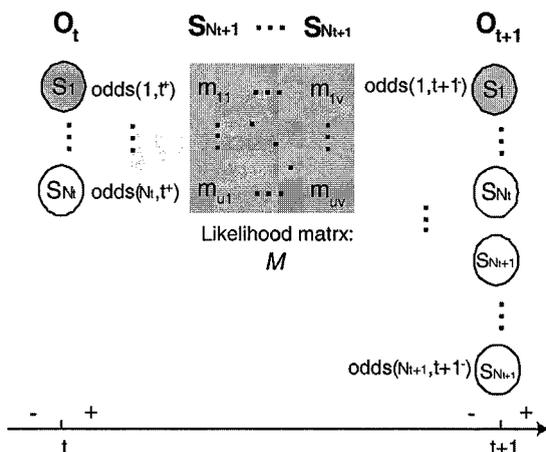


Fig. 6. The likelihood matrix M between times t and $t + 1$ and the existence odds of a person associated with a certain state at times t^+ and $t + 1^-$, where $odds(i, t^+) = P[s_{i,t^+} = 1]$ and $odds(j, t + 1^-) = P[s_{j,t+1^-} = 1]$

where the denominator is a normalization factor so that $\sum_j a_{ij}(t) = 1$ for each i . The numerator in Eq. (11) can be further expanded for each particular state s_i as

$$P[s_{j,t+1^-} = 1 | q_t = s_i] = \sum_{\text{all } cond} P[s_{j,t+1^-} = 1 | cond] \cdot P[cond | q_t = s_i], \quad (12)$$

where $cond = \{s_{1,t^+} = \theta_1 \dots s_{N_t,t^+} = \theta_{N_t}\}$ is one of the possible realizations of $\{s_{1,t^+} \dots s_{N_t,t^+}\}$ over all θ , and $\theta_i = 1$ or 0 designates that the *status* of person s_i is in or out of the room. By assuming that the status of each person is independent of the others, we can express the second term on the right hand side of Eq. (12) as

$$P[cond | q_t = s_i] = \prod_{i=1}^{N_t} P[s_{i,t^+} = \theta_i | q_t = s_i]. \quad (13)$$

From Eqs. (11)–(13), we can see that the transition probability $a_{ij}(t)$ depends on two probabilities: $P[s_{j,t+1^-} = 1 | cond]$ and $P[s_{i,t^+} = \theta_i | q_t = s_i]$; the former is the conditional odds of person s_j existing in the room at time $t + 1^-$ given the status of people observed up to time t^+ , and the latter is the probability of person s_i assuming status θ_i at time t^+ given that person s_i leaves the room at time t . These two probabilities can be calculated as follows.

ii) *Conditional probability*, $P[s_{j,t+1^-} = 1 | cond]$:

We compute this conditional probability with the aid of the likelihood matrix M defined in Eq. (10). The calculation is given by

$$P[\mathbf{s}_{j,t+1^-} = 1 | \text{cond}] = \begin{cases} \theta_j + \sum_v \tilde{m}_{jv} & 1 \leq j \leq N_t \\ 1 - \sum_u \tilde{m}_{u(j-N_t)} & N_t \leq j \leq N_{t+1}, \end{cases} \quad (14)$$

where \tilde{m}_{uv} is the entry of a modified matrix \tilde{M} which is derived from M to indicate how likely person \mathbf{s}_{N_t+v} is in fact person \mathbf{s}_u re-entering the room under a given *cond*. We introduce matrix \tilde{M} in order to incorporate the domain knowledge in the context of the *cond*, making our estimation more accurate. In particular, the value $\sum_v \tilde{m}_{jv}$ can be considered as the gain of the existence odds for person \mathbf{s}_j due to newly entered people between times t and $t+1$. This gain, along with the existence odds of person \mathbf{s}_j at time t^+ , which is the binary value of θ_j , constitutes his/her existence odds at time $t+1^-$. For those newly added states \mathbf{s}_j , where $N_t \leq j \leq N_{t+1}$, the value $1 - \sum_u \tilde{m}_{u(j-N_t)}$ can therefore be considered as the odds for person \mathbf{s}_j being a new person. Note that in this framework we attempt to estimate these odds in an approximate manner, which has been found to be effective for improving the recognition rates.

Specifically, \tilde{m}_{uv} is derived from m_{uv} as

$$\tilde{m}_{uv} = \eta^{2D}[\varepsilon(\theta_u)\rho(S_u)m_{uv}], \quad (15)$$

In (15), η^{2D} is a 2-D normalization operation which will be explained later;

$$\varepsilon(\theta_u) = \begin{cases} 0 & \text{if } \theta_u = 1 \text{ in } \text{cond} \\ 1 & \text{otherwise,} \end{cases} \quad (16)$$

and

$$\rho(S_u) = \begin{cases} 0 & \text{if } S_u \notin PT(S_k, t) \\ 1 & \text{otherwise,} \end{cases} \quad (17)$$

are two indication factors which incorporate the domain knowledge of a particular *cond*. Specifically, $\varepsilon(\theta_u)$ shows that it is impossible for one to enter a closed room if he/she is currently inside, while $\rho(S_u)$ accounts for the fact that an existing person could not enter the room again if he/she has not been observed leaving the room. S_k is the state for which the transition probability is to be calculated at time t and $PT(S_k, t)$ is the partial best state sequence ending in state S_k at time t , i.e., the highest probable state sequence that is retrieved by array ψ and includes all the people who have been observed leaving the room up to time t .

To illustrate the calculation in detail, we provide in the following a numerical example, in which five people enter and three leave a closed room, forming a process of three observations as shown in Fig. 7.

For time pair $\{t, t+1\}$ shown in Fig. 7, suppose that the likelihood matrix obtained by Eq. (10) is equal to

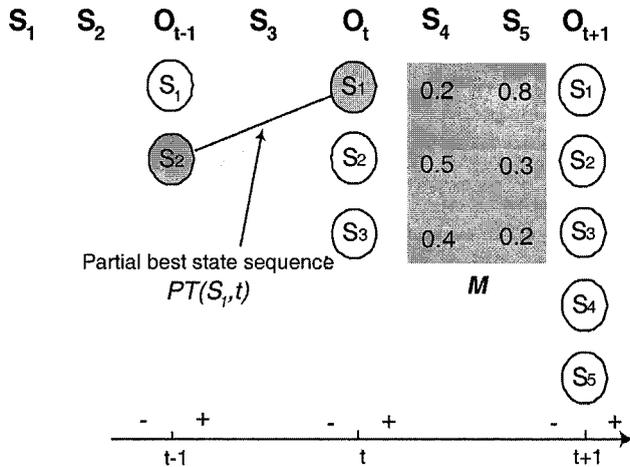


Fig. 7. Estimation of conditional probability

$$M = \begin{matrix} & s_4 & s_5 \\ s_1 & 0.2 & 0.8 \\ s_2 & 0.5 & 0.3 \\ s_3 & 0.4 & 0.2 \end{matrix}$$

Consider one possible realization of people's status at time t^+ : $cond^* = \{s_{1,t^+} = 0, s_{2,t^+} = 0, s_{3,t^+} = 1\}$; that is, $\{\theta_1 = 0, \theta_2 = 0, \theta_3 = 1\}$ and all people are outside the room at time t^+ except for person s_3 . The conditional probabilities that need to be computed are $P[s_{j,t+1-} = 1 | cond^*], 1 \leq j \leq 5$.

First, we incorporate the knowledge of the people status into the likelihood matrix M by using the two indication factors. From Eq. (16), $\varepsilon(\theta_3) = 0$ and the third row of M will be set to zero. This is because if s_3 is known to be inside the room, then neither s_4 nor s_5 can be s_3 regardless of their similarities. The second indication factor relies on the partial best state sequence ending in the state for which the transition probability is to be calculated. Assume that S_1 at time t is the state and the partial best state sequence $PT(S_1, t)$ is as shown in Fig. 7. Since S_3 is not on this path, it means that person s_3 has not left the room since he/she entered. So none of s_4 and s_5 could be s_3 and $\rho(s_3) = 0$ according to Eq. (17). With this domain knowledge, the original likelihood matrix M can be modified as

$$M' = \begin{matrix} & s_4 & s_5 \\ s_1 & 0.2 & 0.8 \\ s_2 & 0.5 & 0.3 \\ s_3 & 0 & 0 \end{matrix}$$

In this example, the two indication factors affect the same row of the likelihood matrix M . In general, the adjustment could be made on different rows

depending on the *cond* as well as the state chosen for evaluating the transition probability.

Next, the likelihood matrix M' is normalized so that the summations of the likelihoods corresponding to all possible circumstances are equal to one. To do this, we introduce a normalization operation, referred to as the correlated normalization operation and denoted as η , which works as follows. Consider a person \mathcal{T} and a group of candidates \mathcal{C} consisting of N people, and define the similarity measure $w_n = P[\mathcal{C}_n \sim \mathcal{T}]$, $\mathcal{C}_n \in \mathcal{C}$. Under the constraint that at most one person (or none) of \mathcal{C} could be \mathcal{T} , we can obtain the following new similarity measures

$$P[\mathcal{C}_n \sim \mathcal{T} | \text{constr}] \propto w_n \cdot \prod_{j \neq n} (1 - w_j), \quad 1 \leq n \leq N, \quad (18)$$

$$P[\mathcal{T} \notin \mathcal{C} | \text{constr}] \propto \prod_{i=1}^N (1 - w_i). \quad (19)$$

To make the probabilities corresponding to all possible circumstances sum up to one, the η operation is defined to normalize the new similarity measures as

$$\eta[w_n] = \frac{w_n \cdot \prod_{j \neq n} (1 - w_j)}{\sum_{i=1}^N w_i \prod_{j \neq i} (1 - w_j) + \prod_{i=1}^N (1 - w_i)}, \quad 1 \leq n \leq N. \quad (20)$$

This normalization operation aims to correlate likelihoods that are measured independently by imposing the above-mentioned constraint. It should be noted that this constraint can be applied to the adjusted likelihood matrix M' both row-wise and column-wise. For instance, at most one of \mathbf{s}_4 and \mathbf{s}_5 (or none of them) could be \mathbf{s}_1 , while \mathbf{s}_4 could be at most one of \mathbf{s}_1 , \mathbf{s}_2 and \mathbf{s}_3 (or none of them, i.e., \mathbf{s}_4 is a new person) in the example shown in Fig. 7. For simplicity, we apply a 2-D η operation (η^{2D}), a row-wise normalization followed by a column-wise normalization, to normalize the likelihood matrix M' and obtain

$$\widetilde{M} = \begin{array}{cc} & \mathbf{s}_4 & \mathbf{s}_5 \\ \mathbf{s}_1 & 0.05 & 0.72 \\ \mathbf{s}_2 & 0.40 & 0.05 \\ \mathbf{s}_3 & 0 & 0 \end{array}$$

From this normalized likelihood matrix \widetilde{M} , we can obtain useful information such as the existence odds of \mathbf{s}_1 is increased by $0.05 + 0.72 = 0.77$, the odds that \mathbf{s}_4 is a new person is $1 - 0.05 - 0.40 - 0 = 0.55$, etc. Consequently, the existence odds at time $t + 1^-$ under *cond** can be obtained using Eq. (14) as

$$\begin{aligned}
P[\mathbf{s}_{1,t+1-} = 1 | \text{cond}^*] &= 0 + 0.05 + 0.72 = 0.77 \\
P[\mathbf{s}_{2,t+1-} = 1 | \text{cond}^*] &= 0 + 0.40 + 0.05 = 0.45 \\
P[\mathbf{s}_{3,t+1-} = 1 | \text{cond}^*] &= 1 + 0 + 0 = 1 \\
P[\mathbf{s}_{4,t+1-} = 1 | \text{cond}^*] &= 1 - 0.05 - 0.40 - 0 = 0.55 \\
P[\mathbf{s}_{5,t+1-} = 1 | \text{cond}^*] &= 1 - 0.72 - 0.05 - 0 = 0.23
\end{aligned}$$

Applying the same procedure to all possible *cond*'s, we can obtain all the conditional probabilities $P[\mathbf{s}_{j,t+1-} = 1 | \text{cond}]$ needed in Eq. (12).

iii) *Probability* $P[\mathbf{s}_{i,t+} = 1 | q_t = \mathbf{s}_i]$:

The remaining unknown for calculating the transition probability $a_{ij}(t)$ is $P[\mathbf{s}_{i,t+} = 1 | q_t = \mathbf{s}_i]$, which is the existence odds of person \mathbf{s}_i at time t^+ given that the person leaving at time t is \mathbf{s}_i . To illustrate how to compute this unknown, we shall extend the example of Fig. 7 to time $t+1$ and show instead how to compute $P[\mathbf{s}_{j,t+1+} = 1 | q_{t+1} = \mathbf{s}_j]$ by making use of the probabilities obtained so far and assuming the full knowledge of $P[\mathbf{s}_{i,t+} = 1 | q_t = \mathbf{s}_{j-}]$, where \mathbf{s}_{j-} is the previous state of \mathbf{S}_j in the partial best state sequence $PT(\mathbf{S}_j, t+1)$. By the same procedure, $P[\mathbf{s}_{i,t+} = 1 | q_t = \mathbf{s}_i]$ can be similarly estimated based on $P[\mathbf{s}_{h,t-1+} = 1 | q_{t-1} = \mathbf{s}_{i-}]$. Note that, h , i , and j are indices of the states at times $t-1$, t and $t+1$, respectively.

As shown in Fig. 8, assume that person \mathbf{s}_1 leaves the room at times t and $t+1$, i.e., $\hat{j}_- = 1$ and $\hat{j} = 1$ (denoted by the shaded circles in Fig. 8). We indicate the values of $P[\mathbf{s}_{i,t+} = 1 | q_t = \mathbf{s}_1]$, which are assumed to be known, on the right hand side of the observation made at time t , and the computed values of $P[\mathbf{s}_{j,t+1-} = 1 | q_t = \mathbf{s}_1]$ on the left hand side of the observation made at time $t+1$. The estimation for the probabilities on the right hand side of observation O_{t+1} and the results are shown in Table 1.

We first examine the gain of existence odds for each person from time t^+ to $t+1^-$: $\gamma(j) = P[\mathbf{s}_{j,t+1-} = 1 | q_t = \mathbf{s}_1] - P[\mathbf{s}_{j,t+} = 1 | q_t = \mathbf{s}_1]$. Clearly, $\sum_j \gamma(j) = 2$ because the gain in existence odds is due to the two newly entered persons. However, if we know for sure that \mathbf{s}_1 leaves the room at time $t+1$, then he/she must be in the room at time $t+1^-$, and thus, $\gamma(1)$ should be equal to 1 rather than 0.77. In other words, the gain for each person needs to be re-adjusted ($\tilde{\gamma}$ in the table) to incorporate the knowledge of this new assumption to ensure that $\tilde{\gamma}(1) = 1$. That $\tilde{\gamma}(1)$ is equal to one can also be explained as follows: since \mathbf{s}_1 leaves at time t and time $t+1$ successively, one of the entered persons \mathbf{s}_4 and \mathbf{s}_5 must be \mathbf{s}_1 . As there is no reason to favor any person other than \mathbf{s}_1 , our approach is to increase \mathbf{s}_1 's existence odds from 0.77 to 1 and decrease the others' proportionally. This is sensible as once we know that the person who leaves the room at time $t+1$ is \mathbf{s}_1 , then \mathbf{s}_4 and \mathbf{s}_5 should be more likely to be \mathbf{s}_1 than what we originally estimate, and consequently, less likely to be other people. At time $t+1^+$, the existence odds of \mathbf{s}_1 becomes zero due to his exit, and the others' can be obtained as the summation of their existence odds at time t^+ and the re-adjusted gain, i.e., $P[\mathbf{s}_{j,t+} = 1 | q_t = \mathbf{s}_1]$ and $\tilde{\gamma}(j)$.

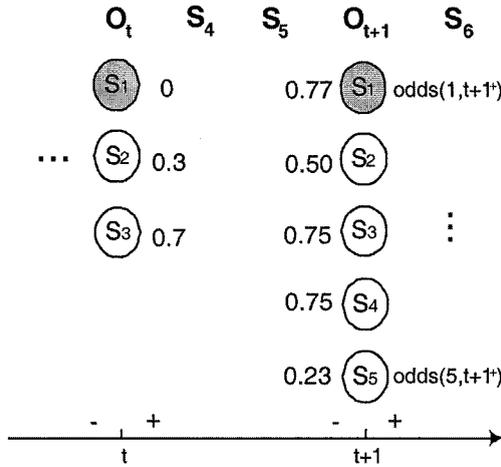


Fig. 8. Estimation of existence odds, where $odds(j, t + 1^+) = P[s_{j,t+1+} = 1|q_{t+1} = s_1]$

Table 1. Estimation of the existence odds

	$P[s_{j,t+} = 1 q_t = s_1]$	$P[s_{j,t+1-} = 1 q_t = s_1]$	γ	$\tilde{\gamma}$	$P[s_{j,t+1+} = 1 q_{t+1} = s_1]$
s_1	0	0.77	0.77	1	0
s_2	0.30	0.50	0.20	0.16	0.46
s_3	0.70	0.75	0.05	0.04	0.74
s_4	n.a.	0.75	0.75	0.61	0.61
s_5	n.a.	0.23	0.23	0.19	0.19

The above analysis and calculation can be summarized into the general formulas below

$$P[S_{j,t+1+} = 1|q_{t+1} = S_j] = \begin{cases} 0 & \text{if } j = \hat{j} \\ P[S_{j,t+} = 1|q_t = S_{\hat{j}_-}] + \tilde{\gamma}(j) & \text{otherwise,} \end{cases} \quad (21)$$

$$\tilde{\gamma}(j) = \begin{cases} 1 & \text{if } j = \hat{j} \\ \gamma(j) \left(1 - \frac{1 - P[S_{\hat{j},t+1-} = 1|q_t = S_{\hat{j}_-}]}{\sum_{j \neq \hat{j}} \gamma(j)} \right) & \text{otherwise,} \end{cases} \quad (22)$$

$$\gamma(j) = P[S_{j,t+1-} = 1|q_t = S_{\hat{j}_-}] - P[S_{j,t+} = 1|q_t = S_{\hat{j}_-}]. \quad (23)$$

It can be seen from these formulas that the estimation of existence odds is recursive; that is, one's existence odds at time $t + 1^+$ depends on that at t^+ . To initialize the estimation, we set $P[s_{i,1+} = 1|q_1 = s_i] = 1$ for all $i \neq \hat{i}$ and $P[s_{i,1+} = 1|q_1 = s_i] = 0$ if s_i is the first person leaving the room (i.e. $q_1 = s_i$), since all the people who have entered the room, except for s_i , should be inside at time 1^+ .

It should also be noted that although the above derivation may appear somewhat complex, it leads to an important underlying property of the proposed framework: the summation of the existence odds of all states at any time instance $t^{+/-}$ (i.e., $\sum_i P[s_{i,t+/-} = 1 | q_t = s_j]$) for any t and j is always equal to the number of people who really stay in the room at that time.

Batch recognition of observations (people who have left the room) can be performed at any time when necessary by retrieving the state sequence with the maximum score of joint posterior probability as the *best state sequence*.

To use the proposed framework for recognizing people re-entering the room is equivalent to finding and merging those states associated with a same person in the *best state sequence*. This can be accomplished by the following local maximum likelihood scheme. Let $q_t = S_i$ and search backward in the *best state sequence* for $q_{t'} = S_i$, where $t' \in \{1, \dots, t-1\}$ and $t - t'$ is minimized. If such $q_{t'}$ exists, person $s_{\hat{j}}$ can be assumed as person s_i re-entering, where \hat{j} is obtained as

$$\hat{j} = \arg \max_{j \in \mathbf{S}(t) \setminus \mathbf{S}(t')} P[S_j \sim S_i], \quad (24)$$

and $\mathbf{S}(t)$ is the state set at time t . Then, $S_{\hat{j}}$ should be merged into S_i and removed from all the state sets containing it thereafter. The backward search is performed from $t = 2$ to the end of the *best state sequence* until all the states possibly representing a same person are identified and merged.

5 Experimental Results

To test the proposed people monitoring system, we have captured two videos in a research laboratory using a low-cost PC camera monitoring the lab's only entrance. During an one-hour monitoring period, video-I recorded eleven people who were unaware of the experiment, of which four entered and left twice, another four entered and left only once, and three entered without leaving. Video-II simulated the process of people entering and leaving with the help of eight students, among whom seven entered and left the lab for three times and another one for two times. In video-II, the volunteers were asked to approach the camera so that their faces could be recorded by the camera from a reasonable distance.

In our experiments described below, color histogram is tested on Video-I and Video-II while face is tested on Video-II and the face database of Olivetti Research (400 images of 40 individuals, 10 images per individual at the resolution of 92×112 pixels). A synthetic process generator is also designed to randomly re-arrange the entries and exits from the two videos and synthesize processes from the face database according to the rule that one cannot enter unless he/she is outside the lab, and vice versa. This generator allows us to simulate a large combinations of entries and exits over time from the same group of people.

For comparison, we also implement a recognition approach based on maximum likelihood (ML) classification [32, 33] as follows. When a person is detected entering a closed room, he/she is compared with people who are in the system’s database and currently labeled as ‘out’. If the maximum likelihood with respect to an ‘out’ person is larger than a threshold T_m , they are considered the same person. Then, the observed person is labeled as ‘in’ and his/her corresponding feature template in the database is updated with the latest one. Otherwise, the person is assumed to be new and then labeled as ‘in’ with a new identity label. When there are multiple exits without entries among them, the leaving people are recognized from the people with label ‘in’ by maximizing a joint likelihood.

We now use an example of the synthesized entry/exit process to illustrate the superiority of our approach against the maximum likelihood approach. The process is obtained based on the eight people ($P1-P8$) recorded in video-II as shown in Table 2, where ‘I’ and ‘O’ represent in (entry) and out (exit), respectively. A total of 44 entries and exits are observed, among them 22 are entering while the other 22 are exiting. Note that in this example we take color histogram as the low-level feature .

Table 2. A sequence of entries and exits obtained from the synthetic generator using the eight people observed in Video-II

Person	P6	P2	P6	P6	P1	P1	P3	P2	P1	P8	P2	P1	P7	P5	P3
In/Out	I	I	O	I	I	O	I	O	I	I	I	O	I	I	O
Person	P5	P1	P8	P7	P3	P5	P2	P6	P4	P6	P8	P5	P8	P7	P2
Cont’d	O	I	O	O	I	I	O	O	I	I	I	O	O	I	I
Person	P7	P2	P6	P5	P7	P4	P4	P4	P7	P1	P3	P8	P8	P5	
Cont’d	O	O	O	I	I	O	I	O	O	O	O	I	O	O	

Table 3. Comparison of the recognition results obtained by the maximum likelihood (ML) approach and our proposed approach

Obs. time	1	2	3	4	5	6	7	8	9	10	11
Ground truth	S_1	S_4	S_2	S_4	S_5	S_{10}	S_7	S_9	S_2	S_1	S_{10}
Proposed approach	S_1	S_4	S_2	S_4	S_5	S_{10}	S_7	S_9	S_2	S_1	S_{10}
ML approach	S_1	S_4	S_2	S_4	S_5	S_{10}	S_4	S_9	S_2	S_3	S_{10}
cont’d	12	13	14	15	16	17	18	19	20	21	22
Ground truth	S_7	S_9	S_2	S_1	S_{14}	S_{14}	S_9	S_4	S_5	S_7	S_{10}
Proposed approach	S_7	S_9	S_2	S_1	S_{14}	S_{14}	S_9	S_4	S_5	S_7	S_{10}
ML approach	S_7	S_{17}	S_2	S_3	S_{14}	S_{14}	S_{12}	S_{16}	S_9	S_7	S_{10}

The recognition results of the eight people (at observation time) are given in Table 3. The results obtained by our proposed approach and the ML approach are compared against the ground truth. In this particular example, our approach achieves 100% recognition rate, outperforming the ML approach. On the other hand, the ML approach wrongly recognizes $P8$ as $P1$ at time 7 and the other way around at time 19. Furthermore, $P7$ is incorrectly identified as $P3$ at time 18 and reversely at time 20. These recognition errors are mainly due to that the similarity measures between these different people exceed the preset threshold T_m . On the contrary, the errors at times 10, 13 and 15, where $P6$ and $P7$ are wrongly identified as new people when they are just re-entering, are due to that the similarities of their features observed at different times are lower than T_m . In other words, the ML approach is rather sensitive to the threshold T_m , inappropriate selection of which often results in false recognitions. In comparison, our proposed approach benefits from the threshold-free scheme; therefore, it is more robust to variations in feature extractions as well as changes in lighting conditions or view angles.

Recall that at each observation time there is a partial best state sequence ending in each individual state with the probability score of $\delta_t(i)$. To make a recognition decision at time t is thus to choose from all the partial best state sequence the one with the largest value of $\delta_t(i)$. Consequently, a confidence index can be defined in the range $[0, 1]$ to evaluate the reliability of a decision that is made at each time

$$Conf(t) = \frac{\max(\delta_t(i))}{\sum_i \delta_t(i)}. \quad (25)$$

Fig. 9 shows the variation of the confidence index over time for the above example.

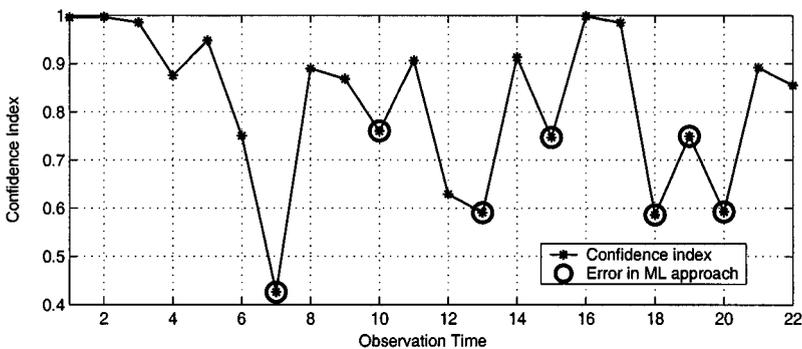


Fig. 9. The confidence index over observation time

In the early stage of monitoring, the system has only few choices for making decision (few people are observed); therefore, the confidence index is usually high. With the increase of the state number, i.e., more possible paths

to choose from, the reliability of a decision may decrease. However, the advantage of our approach lies in that it is capable of maintaining the decision reliability at a later observation time by collectively considering all the available observations—a merit of the Viterbi algorithm as described in Sec. 2. In Fig. 9, when the confidence index is lower than 0.8, the ML approach is likely to make a wrong decision at the corresponding time (as indicated by the circles). Meanwhile, the proposed approach can still make the right choice since the probabilities (or scores) of the other paths are even lower.

It should be noted that the computational complexity of our framework increases rapidly when more states are generated. However, when the confidence index is sufficiently high, the total number of states can be reduced by making a definite decision to merge the states associated with the same person (e.g., at time 16 in Fig. 9). We consider this a promising topic for future investigation.

Table 4 summarizes the recognition performances of the experimental results, where 20 synthesized processes of entries and exits are generated for each test data set and for each feature type considered. It is evident from the table that our proposed approach can notably improve the recognition accuracies as compared with that of the ML approach.

Table 4. Recognition accuracies obtained by the maximum likelihood (ML) approach and our proposed approach for two types of low-level features

Data	Color histogram		Face feature	
	ML	Proposed	ML	Proposed
Video-I	83.3%	99.6%	—	—
Video-II	75.0%	97.5%	82.3%	98.5%
ORL Database	—	—	85.0%	100%

6 Conclusion

We have presented in this chapter a novel probabilistic reasoning framework for monitoring people in a closed room. Rather than identifying each single observation from a database, the framework is devised to recognize people based on multiple observations by exploiting the temporal correlations and constraints imposed by the application domain. In addition, the proposed framework permits its parameters to be estimated and updated at each observation instance by combing low-level features and domain-specific knowledge. Experimental results demonstrate that the proposed approach outperforms the existing maximum-likelihood approach when using the same features and being tested with the same test videos.

It should be noted that the proposed system can be readily extended to monitor multiple entrances or adjacent areas with the use of an array of cameras, all being interconnected and sharing the results obtained by each own analysis unit.

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