GROUPS PRESENTED BY CERTAIN CLASSES OF FINITE LENGTH-REDUCING STRING-REWRITING SYSTEMS

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1. Introduction

A string-rewriting system R on alphabet Σ defines a monoid M_R, which is the factor monoid of the free monoid Σ^* generated by Σ modulo the Thue congruence $\overset{*}{\longleftrightarrow}_R$ generated by R. One aspect of the various classes of string-rewriting systems considered in the literature is their **descriptive power**: which monoids can be defined by which classes of string-rewriting systems. To capture the descriptive power of string-rewriting systems one would like to establish a one-to-one correspondence between the various syntactic restrictions of string-rewriting systems and certain algebraic properties of the monoids presented.

So far only a few results of this flavor have been obtained most of them dealing with groups rather than with monoids in general. Here we shall also restrict our attention to the case of groups. So we ask which classes of groups can be presented by which classes of string-rewriting systems.

In this paper we give an overview over the results concerning this question that have been obtained so far, including some new ones. Specifically, we are interested in the following classes of finite length-reducing string-rewriting systems:

- C_{sp}, the class of finite special systems R that are confluent,
- $C_{sp,1}$, the class of finite special systems R that are confluent on $[1]_R$ and that provide an inverse of length 1 for each of the generators,

- C_{2m}, the class of finite 2-monadic systems R that are confluent,
- C_m, the class of finite monadic systems R that are confluent
- $C_{m,1}$, the class of finite monadic systems R that are confluent on $[1]_R$ and that provide inverses of length 1 for the generators,
- C_{lr}, the class of finite length-reducing systems R that are confluent, and
- $C_{lr,1}$, the class of finite length-reducing systems R that are confluent on $[1]_R$ and that provide inverses of length 1 the generators.

Here 1 denotes the **empty word**, and $[w]_R$ denotes the **congruence class** of $w \in \Sigma^*$ modulo the congruence $\overset{*}{\longleftrightarrow}_R$. A string-rewriting system R on Σ is called **length-reducing**, if $|l| > |r|$ for each rule $(l,r) \in R$, where $|.|: \Sigma^* \to \mathbf{N}$ denotes the usual length function. It is called **monadic**, if it is length-reducing and $|r| \leq 1$ for each rule $(l,r) \in R$, and it is called **2-monadic**, if it is monadic and $|l| \leq 2$ for each rule $(l,r) \in R$. Finally, R is called **special**, if it is length-reducing and the right-hand side r of each rule $(l,r) \in R$ is the empty word 1. A string-rewriting system R on Σ is **confluent**, if each of its congruence classes contains a unique irreducible word, which can then be taken as the **normal form** of its class. Since the process of reduction in a finite length-reducing system can be performed in linear time, the **word problem** for each finite length-reducing and confluent system is decidable in linear time [3]. Finally, a string-rewriting system R on Σ is said to be **confluent on** $[1]_R$, if $w \overset{*}{\to}_R 1$ for each word $w \in [1]_R$. Since we are dealing with groups, confluence on $[1]_R$ is sufficient to guarantee that the word problem of a finite length-reducing R is decidable in linear time. For more details on the concepts introduced so far, see for example Book's excellent overview paper [4].

The following overview summarizes the results that we shall present here.

O V E R V I E W

Finite Presentations		Groups
special confluent	$=$	cyclic * free
special confluent on $[1]_R$	$=$	
2-monadic confluent	$=$	finite * free
monadic confluent	$\supseteq$	
length-reducing confluent	$\subsetneq$	context-free
monadic confluent on $[1]_R$	$=$	$=$
2-monadic confluent on $[1]_R$	$=$	finitely generated
length-reducing confluent on $[1]_R$	$\supsetneq$	virtually free
length-reducing confluent on $[1]_R$	$\supseteq$	small cancellation

These results show very nicely that by relaxing the requirement that a finite string-rewriting system R be confluent everywhere to the requirement that it only be confluent on $[1]_R$ the descriptive power is strictly increased. As long as we are dealing with monadic systems that are at least confluent on $[1]_R$, we can only present context-free groups, which by results of Dunwoody [8] and Muller and Schupp [14] are exactly the finitely generated virtually free groups. Even finite length-reducing and confluent systems only present a proper subclass of this class; however, finite length-reducing systems that are only confluent on $[1]_R$ already present strictly more than the context-free groups.

2. Special and monadic string-rewriting systems

The first characterization theorem is due to Cochet.

Theorem 2.1 [6]. A group G has a presentation of the form $(\Sigma;R)$, where R is a finite special confluent system on Σ, if and only if G is the free product of finitely many cyclic groups.

Here the factors of G may be finite or infinite cyclic groups. This result takes care of the class C_{sp}. It is easily seen that every group G that is the free product of finitely many cyclic groups can be presented by a finite special string rewriting system R on alphabet Σ such that R is confluent on $[1]_R$ and such that each letter $a \in \Sigma$ has an inverse $b \in \Sigma$. Hence, the descriptive power of the class $C_{sp,1}$ (with respect to the presentation of groups) is at least as large as the descriptive power of the class C_{sp}. In fact, it is even larger, as shown by the following lemma.

Lemma 2.2. Let G be a free product of a free group of finite rank and a finite number of finite groups. Then G has a presentation $(\Sigma;R)$ for some finite string-rewriting system R from the class $C_{sp,1}$.

Proof. If G_1 and G_2 have presentations of this form, say $(\Sigma_1;R_1)$ and $(\Sigma_2;R_2)$ with $\Sigma_1 \cap \Sigma_2 = \emptyset$, then $(\Sigma_1 \cup \Sigma_2; R_1 \cup R_2)$ is a presentation of this form for the free product $G_1 * G_2$. If G is a free group, then the standard presentation $(\Sigma \cup \bar{\Sigma};\ a\bar{a} \to 1, \bar{a}a \to 1 \mid a \in \Sigma\})$ is of this form. Thus, it only remains to consider the case that G is an arbitrary finite group.

So let $G = \{1_G, g_1, \ldots, g_{n-1}\}$ be a finite group, where 1_G denotes the identity of G and $\cdot$ denotes the operation of G. Let $\Sigma = \{a_1, \ldots, a_{n-1}\}$, and let $f\colon \Sigma^* \to G$ denote the monoid-homomorphism induced by $a_i \to g_i$ $(i = 1,2,\ldots,n-1)$. Further, let $R = \{(w,1) \mid w \in \Sigma^*,\ 1 < |w| \leq n,\ f(w) =_G 1_G\}$, where $=_G$ denotes the equality in G.

Claim 1. Let $w \in \Sigma^*$, $|w| > n$, such that $f(w) =_G 1_G$. Then w can be factored as $w = xyz$ with $|y| \leq n$ such that $f(y) =_G 1_G$.

Proof. Let $w = a_1 a_2 \ldots a_m$, $m > n$, with $a_1, a_2, \ldots, a_m \in \Sigma$, and let $e_{i_j} = f(a_1 a_2 \ldots a_j)$ for $j = 1,2,\ldots,m$. Since $m > n$, there exist indices $1 \leq j < k \leq j+n$ such that $e_{i_j} = e_{i_k}$. Thus, $f(a_1 \ldots a_j) =_G e_{i_j} = e_{i_k} =_G f(a_1 \ldots a_j a_{j+1} \ldots a_k)$ implying that $f(a_{j+1} \ldots a_k) =_G 1_G$, i.e., $y = a_{j+1} \ldots a_k$ satisfies the requirements. □

In particular, this means that $w \xrightarrow{*}_R 1$ for all $w \in \Sigma^*$ satisfying $f(w) =_G 1_G$. Further $(\Sigma;R)$ is a presentation of G as shown by the following claim.

Claim 2. $\forall u,v \in \Sigma^*$: $u \overset{*}{\longleftrightarrow}_R v$ if and only if $f(u) =_G f(v)$.
Proof. Let $u,v \in \Sigma^*$. If $u \overset{*}{\longleftrightarrow}_R v$, then $u \longleftrightarrow_R u_1 \longleftrightarrow_R \cdots \longleftrightarrow_R u_r = v$. Since $(w,1) \in R$ implies $f(w) =_G 1_G = f(1)$, it can be verified easily by induction on r that $f(u) =_G f(v)$.

On the other hand, if $f(u) =_G f(v)$, then $f(u)(f(v))^{-1} =_G 1_G$ Since G is a group, there exists an element $g_i^{-1} \in \{g_1,g_2,\ldots, g_{n-1}\}$ for each element g_i, and hence a function $(.)^{-1}\colon \Sigma^* \to \Sigma^*$ can be defined. Now $f(v^{-1}) =_G (f(v))^{-1}$, and so $f(u)(f(v))^{-1} =_G f(uv^{-1})$. Thus, by Claim 1 $uv^{-1} \overset{*}{\to}_R 1$. Hower, the trivial relations $(\bar{a}a,1)$ are in R for all $a \in \Sigma$, and so $u \overset{*}{\longleftrightarrow}_R uv^{-1}v \overset{*}{\longleftrightarrow}_R v$. □

Since $R \in C_{sp,1}$, this completes the proof of Lemma 2.2. □

If a group G has a presentation of the form (Σ,R) with $R \in C_{sp,1}$, then because of the inverses of length 1 for the generators $a \in \Sigma$, $(\Sigma;R)$ is in fact a **group presentation** in the sense of [13]. Further, the set $W(R) = \{w \in \Sigma^* \mid w \overset{*}{\longleftrightarrow}_R 1$, but no proper factor u of w satisfies $u \overset{*}{\longleftrightarrow}_R 1\}$ is a subset of the set of left-hand sides of rules of R, and hence it is finite. Thus, the following lemma applies.

Lemma 2.3 [10]. Let $(\Sigma;R)$ be a group presentation such that the set W(R) is finite. Then the group G presented by $(\Sigma;R)$ is the free product of a finitely generated free group and finitely many finite groups.

Hence, we have the following characterization theorem.

Theorem 2.4. A group G has a presentation of the form $(\Sigma;R)$, where R is a finite special string-rewriting system on Σ such that R is confluent on $[1]_R$ and each generator $a \in \Sigma$ has an inverse $b \in \Sigma$, if and only if G is the free product of a free group of finite rank and a finite number of finite groups.

These are exactly those groups that have a presentation with a simple reduced word problem [10]. Another characterization of this class of groups has been obtained by Avenhaus, Madlener, and Otto.

Theorem 2.5 [2]. A group G has a presentation of the form $(\Sigma;R)$, where R is a finite 2-monadic and confluent string-rewriting system on Σ, if and only if G is the free product of a free group of finite rank and a finite number of finite groups.

It has been conjectured [9] that when we take presentations involving finite monadic confluent systems, we will still have the same class of groups. Up to now this conjecture is still open. On the other hand, whenever R is a finite monadic confluent string-rewriting system on Σ, then $[1]_R$ is a context-free language [3]. Thus, if the monoid M_R presented by $(\Sigma;R)$ is a group, then it is a **context-free group.** Muller and Schupp have characterized the class of context-free groups as follows.

Theorem 2.6 [14]. A finitely generated group G is context-free if and only if it is virtually free.

A group G is called **virtually free**, if it contains a free subgroup of finite index. Actually, Muller and Schupp showed that a finitely generated group G is virtually free if and only if it is context-free and accessible. However, since every finitely generated context-free group is finitely presented [1], and since every finitely presented group is accessible [8], the condition of accessibility can be dropped from the Muller-Schupp Theorem. Thus, we know at least that evey group that is presented by a finite monadic confluent string-rewriting system is virtually free.

When R is a finite monadic string-rewriting system that is confluent on $[1]_R$, then $[1]_R$ is still a context-free language. Thus, whenever a group G is presented by a system of this form, then G is a virtually free group. On the other hand, each finitely generated virtually free group can be presented by a quadratic NTS g-grammar, and hence the group language of G is an NTS language [16]. Since a quadratic NTS g-grammar yields a finite 2-monadic string-rewriting system R on alphabet Σ such that R is confluent on $[1]_R$ and such that each generator $a \in \Sigma$ has an inverse $b \in \Sigma$, this shows the following.

Theorem 2.7 [16]. A group G has a presentation $(\Sigma;R)$, where R is a finite monadic string-rewriting system on Σ that is confluent on $[1]_R$ and that provided inverses of length 1 for all the generators $a \in \Sigma$, if and only if G is a finitely generated virtually free group.

3. Length-reducing string-rewriting systems

Let R be a finite length-reducing and confluent string-rewriting system on Σ. Then for each word $w \in \Sigma^*$, the congruence class $[w]_R$ is a context-sensitive language [3], but in general $[w]_R$ is not context-free [15]. However, if the monoid M_R presented by $(\Sigma;R)$ is a group, then the situation is different.

Theorem 3.1. Let R be a finite length-reducing and confluent string-rewriting system on Σ, and let $L \subset IRR(R)$ be a regular set of irreducible words. If the monoid M_R is a group, then the set $[L]_R := \{u \in \Sigma^* \mid \exists w \in L: u \overset{*}{\longleftrightarrow}_R w\}$ is a deterministic context-free language.

Proof. Since M_R is a group, there exists a word $u_a \in \Sigma^*$ such that $au_a \overset{*}{\rightarrow}_R 1$ and $u_a a \overset{*}{\rightarrow}_R 1$ for each letter $a \in \Sigma$. Let $\lambda := \max\{|l| \mid \exists r \in \Sigma^*: (l,r) \in R\}$, and $\mu := (\max\{|u_a| \mid a \in \Sigma\}+1)(\lambda-1)$. Now for $u \in IRR(R)$, if $|u| \geq \mu$ and $ua \overset{*}{\rightarrow}_R v \in IRR(R)$ for some $a \in \Sigma$, then $u \overset{*}{\leftarrow}_R uau_a \overset{*}{\rightarrow}_R vu_a$ implying that $vu_a \overset{*}{\rightarrow}_R u$. Thus, $|v| \leq |u|+1$ and $|u| \leq |v|+|u_a|$, i.e., $0 \leq |ua|-|v| \leq |u_a|+1$. Since R is length-reducing, this means that $ua \overset{i}{\rightarrow}_R v$ for some $i \leq |u_a|+1$, and since u is irreducible, $u = u_1u_2$, $|u_2| \leq (|u_1|+1)(\lambda-1) \leq \mu$ such that $u_2a \overset{*}{\rightarrow}_R v_2$ and $v = u_1v_2$. Thus, in order to reduce ua only the suffix u_2 of u of length μ must be considered.

Now we construct a deterministic pushdown automaton (dpda) C that will accept the language $[L]_R = \{u \in \Sigma^* \mid \exists w \in L: u \overset{*}{\rightarrow}_R w\}$.

First we construct a dpda A as follows. As **input alphabet** we take Σ, and as **stack alphabet** we choose $\Sigma \cup \{\#\}$, where # serves as the **start symbol** marking the bottom of the pushdown store. A can store a word of length $\leq \mu$ in its finite control, i.e., the **set of finite states** is $Q = (\bigcup_{i=0}^{\mu} \Sigma^i) \cap IRR(R)$. Finally, the **transition function** $\delta: Q\times(\Sigma \cup \{\#\})\times(\Sigma \cup \{1\}) \rightarrow Q\times(\Sigma \cup \{\#\})^*$ is defined as follows:

$$\delta(w,\#,a) := \begin{cases} (u,\#) & \text{if } wa \overset{*}{\rightarrow}_R u \in IRR(R) \text{ and } |u| \leq \mu, \\ (va,\#b) & \text{if } w = bv,\ |w| = \mu, \text{ and } wa \in IRR(R), \end{cases}$$

$$\delta(w,b,1) := (bw,1) \qquad \text{if } |w| < \mu,$$

$$\delta(w,b,a) := \begin{cases} (va,bc) & \text{if } w = cv,\ |w| = \mu, \text{ and } wa \in \mathrm{IRR}(R), \\ (u,b) & \text{if } |w| = \mu,\ wa \overset{*}{\rightarrow}_R u \in \mathrm{IRR}(R) \text{ and} \\ & |u| \leq \mu. \end{cases}$$

Starting with the configuration (1,w,#), where 1 is the **initial state**, w is the **input**, and # is the **stack contents**, A finally reaches a configuration of the form (v,1,#u) such that $w \overset{*}{\rightarrow}_R uv$ and $uv \in \mathrm{IRR}(R)$. Thus, $w \in [L]_R$ if and only if $uv \in L$.

Since L is regular, there exists a deterministic finite state acceptor B for L. Now A and B can be conbined such that for each configuration (u,w,#v) of A, B is in its state $\delta_B(vu)$ This gives a dpda C for $[L]_R$. □

In particular, each congruence class $[w]_R$ of a finite length-reducing confluent string-rewriting system R presenting a group is context-free, which implies the following result.

Corollary 3.2. Let G be a group that can be presented by a finite length-reducing and confluent string-rewriting system. Then G is a context-free group.

This result was independently obtained by V. Diekert [7], who also proved the following.

Theorem 3.3 [7]. Let G be an infinite group that has an abelian subgroup of finite index. Then G has a presentation by some finite length-reducing and confluent string-rewriting system if and only if G is isomorphic to Z or to the free product $Z_2 * Z_2$.

Concerning commuting elements in groups presented by finite length-reducing and confluent systems the following result can be shown.

Lemma 3.4 [12]. Let R be a finite length-reducing and confluent string-rewriting system on Σ such that the monoid M_R is a group, and let $u \in \Sigma^*$ be a word of infinite order. Then, for all $v \in \Sigma^*$, the following two statements are equivalent:

(i) u and v commute modulo R;

(ii) the subgroup of M_R generated by {u,v} is cyclic.

As a consequence of this lemma we obtain the following results.

Theorem 3.5 [12]. Let G be a group that can be presented by a finite length-reducing and confluent string-rewriting system. Then for each element g of G, if g has infinite order, then the centralizer $C_G(g)$ of g in G is isomorphic to Z.

Corollary 3.6. Let G be a group that can be presented by a finite length-reducing and confluent string-rewriting system.
(a) Every abelian subgroup of G that contains an element of infinite order is isomorphic to Z.
(b) Every finitely generated abelian subgroup of G is either finite or isomoprhic to Z.
(c) If the center of G is non-trivial, then G is either finite or isomorphic to Z.
(d) If G contains a non-trivial normal subgroup that is finite then G itself is finite.

This corollary is very important in that it provides ways of proving that a certain group cannot be presented by a finite length-reducing and confluent string-rewriting system. For example, consider $G = F_2 x Z_2$, i.e., G is the direct product of the free group F_2 of rank 2 and the cyclic group Z_2 of order 2. Then G is an infinite context-free group, but Z_2 is a finite normal subgroup of G. Hence, G cannot be presented by a finite length-reducing and confluent system. Thus, we have the following.

Corollary 3.7. The class of groups that can be presented by finite length-reducing and confluent string-rewriting systems is a proper subclass of the class of finitely generated virtually free groups.

By definition a virtually free group G contains a free subgroup H of finite index. If $H \cong Z$, then G has a **linear growth function**, otherwise, G has an **exponential growth function**. Now if $H \cong Z$, then H is an abelian subgroup of finite index of G. Thus, if G has a presentation by some finite length-reducing and confluent string-rewriting system, then G is isomorphic to Z or to $Z_2 * Z_2$.

Corollary 3.8. Let G be an infinite group that is presented by a finite length-reducing and confluent string-rewriting system Then either G is isomorphic to Z, or to Z_2*Z_2, in which cases G has a linear growth function, or G has an exponential growth function.

Since a finitely generated nilpotent group has a polynomial growth function [17], Z is the only infinite nilpotent group that can be presented by a finite length-reducing and confluent string-rewriting system.

Finally, we turn to string-rewriting systems R that are length-reducing, confluent on $[1]_R$, and that provide inverses of length 1 for the generators.

From the result in [16], which says that each context-free group can be presented by a quadratic NTS g-grammar, it follows that each context-free group has a presentation of the form $(\Sigma;R)$, where R is a finite monadic string-rewriting system that is confluent on $[1]_R$ and that provides inverses of length 1 for the generators $a \in \Sigma$. Thus, we do in particular have the following weaker result.

Theorem 3.9. Each finitely generated virtually free group G has a presentation of the form $(\Sigma;R)$, where R is a finite length-reducing string-rewriting system on Σ that is confluent on $[1]_R$ and that provides inverses of length 1 for the generators $a \in \Sigma$.

However, this result can be derived immediately from the fact that a finitely generated virtually free group G is context-free using an idea of [1].

Proof of Theorem 3.9. Let $<\Gamma;L>$ be a finite group-presentation of G, where $L \subset (\Gamma \cup \Gamma^{-})^*$. Since G is context-free, $[1]_L$ is a context-free language over $\Sigma := \Gamma \cup \Gamma^{-}$. Let $n_1 := \max(\{|w| \mid w \in L\} \cup \{2\})$, and let n_2 be the constant of the pumping lemma belonging to $[1]_L$. Finally, let $n := \max\{n_1,n_2\}$, and $R := \{(u,1) \mid u =_G 1 \text{ and } |u| \leq n\} \cup \{(xyz,y) \mid xyz =_G y,\ |xyz| \leq n, \text{ and } xz \neq 1\}$.

Then $(w,1) \in R$ for all words $w \in L$, and $(\bar{a}a,1),(a\bar{a},1) \in R$ for all $a \in \Gamma$, i.e., $G \cong M_R = \Sigma^*/\overset{*}{\longleftrightarrow}_R$. Obviously, R is a finite length-reducing string-rewriting system on Σ providing inverses of length 1 for the generators.

Now let $w \in [1]_L = [1]_R$. If $|w| \leq n$, then $(w,1) \in R$, i.e., $w \to_R 1$. If $|w| > n$, then $w = uxyzv$, where $|xyz| \leq n$, $xz \neq 1$, and $\{ux^kyz^kv \mid k \geq 0\} \subset [1]_R$ by the pumping lemma for context-free languages. In particular, $uyv =_G uxyzv$ implying $xyz =_G y$. Thus, $(xyz,y) \in R$, and so $w = uxyzv \to_R uyv$. Inductively, we obtain $w \overset{*}{\to}_R 1$, i.e., R is confluent on $[1]_R$. □

Dehn's algorithm [11] for solving the word problem actually amounts to applying a finite length-reducing string-rewriting system R that is confluent on $[1]_R$. Thus, all classical small cancellation groups have presentations by rewriting systems R from the class $G_{lr,1}$. This fact has been first observed by Buecken [5] who investigates the class of small cancellation groups from the standpoint of rewriting systems. On the other hand, the Fuchsian group $G_2 = \langle a,b,c,d;\ ab\bar{a}\bar{b} = dc\bar{d}\bar{c} \rangle$ satisfies the small cancellation condition $C'(1/7)$, i.e., G_2 has a presentation by some system R from $C_{lr,1}$. G_2 is torsion-free, since it is the free product of the free group F_2 with itself with amalgamated cyclic subgroups, but G_2 is not a free group. Hence, G_2 is not a context-free group, and so it does not have a presentation by some finite length-reducing and confluent string-rewriting system. This gives the following.

Theorem 3.10. The class of finitely generated virtually free groups is a proper subclass of the class of groups that can be presented by finite length-reducing string-rewriting systems R that are confluent on $[1]_R$ and that provide inverses of length 1 for the generators.

It is conjectured that the classes C_{2m}, C_m, and C_{lr} are all equal to the class of free products of a free group of finite rank and a finite number of finite groups. The proof of this fact would give a complete algebraic characterization of groups presented by length-reducing systems.

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