Macro grammars, Lindenmayer systems and other copying devices

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1. Introduction

We consider and compare macro grammars, L systems, stack automata and topdown tree transducers: extensions of context-free grammars which allow certain kinds of copying. Macro grammars are context-free grammars in which the non-terminals have parameters; in particular in 'basic' macro grammars the actual parameters of a nonterminal are terminal strings (i.e. nonterminals are not nested). We also consider 'extended' macro grammars in which the actual parameters may be finite sets of terminal strings. ETOL systems are like context-free grammars, but the rewriting is done in parallel and several independent sets of productions are allowed. Stack automata are pushdown automata that may also read in the stack. Topdown tree transducers transform the set of derivation trees of a context-free grammar. The language of yields of the resulting set of trees is called a tree transformation language.

Several relationships between these devices are known: in both macro grammars and L-systems the operation of iterated substitution plays a role, macro languages can be recognized by 'nested stack automata', nonerasing stack languages are special ETOL languages, ETOL languages are both special tree transformation languages and special macro languages, and finally macro grammars generate the yields of context-free tree languages.

In this paper we continue the comparison of these devices. We show that the additional facilities present in the basic macro grammars, the ETOL systems and the stack automata are independent in the sense that the corresponding classes of languages are incomparable. In particular we present a language which is both in Basic and Stack but is not even a tree transformation language (this also shows that the context-free tree grammars are independent from the topdown tree transducers as string language generating systems). We then prove that Basic, ETOL and Stack are contained in the class EB of extended basic macro languages. Results analogous to those for Stack are given on the operation of substitution in EB. It follows that EB is a full AFL which is not substitution closed. We finally show that the smallest full hyper-AFL (i.e. full AFL closed under iterated substitution) containing EB lies properly between EB and the class of all (OI) macro languages. This shows that OI (= the class of indexed languages) cannot be reached from Basic or Stack by full hyper-AFL operations.

Proofs of these results will only be sketched; full proofs will appear elsewhere.

2. Terminology and facts

We assume the reader to be familiar with macro grammars [16], and more or less with iterated substitution [26], stack automata [18, 19] and tree transducers [25]. What follows is meant to fix some notation and mention some facts.

The length of a string is denoted by $|w|$; $|\lambda| = 0$.

An (OI or IO) macro grammar G consists of an alphabet Σ of terminals, an alphabet N of nonterminals each of which has a specified rank (i.e. a nonnegative number of arguments), an initial nonterminal S of rank 0, and a finite set R of rules of the form $A(x_1,\ldots,x_n) \rightarrow t$ where A is a nonterminal of rank n, $x_1,\ldots,x_n$ are special symbols called variables and t is a term formed from $\{x_1,\ldots,x_n\} \cup \Sigma \cup \{\lambda\}$ by concatenation and the use of the nonterminals as formal operation symbols. The rules are applied in the obvious (outside-in or inside-out respectively) way and L(G) denotes the generated language. For formal definitions see [16]. In a basic macro grammar the terms in right-hand sides of rules do not have nested nonterminals and in a linear basic macro grammar they have at most one nonterminal. The classes of OI, IO, basic and linear basic macro languages are denoted by IO, OI, Basic and LB respectively. An extended macro grammar (cf. [9]) is a macro grammar in which the operation of union, denoted by +, and the empty set, denoted by $\emptyset$, may also used in terms. Thus, during derivation, (representations of) finite sets of terms are stored in the arguments and, at the end of the derivation, (the representation of) a finite set of terminal strings is produced; the union of these sets is the language generated. The classes of extended basic and linear basic languages are denoted by EB and ELB respectively. Clearly each extended basic macro grammar can be simulated by an ordinary OI macro grammar which uses additional nonterminals + and $\emptyset$ (with rules $+(x,y) \rightarrow x$ and $+(x,y) \rightarrow y$ for +, and no rules for $\emptyset$). Thus EB $\subseteq$ OI. As an example, the EB (even ELB) grammar G with rules $S \rightarrow A(\emptyset)$, $A(x) \rightarrow B(x,\lambda)$, $A(x) \rightarrow x$, $B(x,y) \rightarrow aB(x,ya)$ for all $a \in \Sigma$, and $B(x,y) \rightarrow \#A(x+y)$ generates $L(G) = \{w_1\# w_2\#\ldots\# w_n\# w \mid n \geq 1,\ w \in \{w_1,\ldots,w_n\}\}$.

For a finite set U of substitutions and a language L we define $U^*(L) = \cup\{f_n \ldots f_1(L) \mid n \geq 0,\ f_i \in U\}$. U^* is called an iterated substitution. For a family K of languages we define $H(K) = \{U^*(L) \cap \Sigma^* \mid L \in K$, U is a finite set of K-substitutions and Σ is an alphabet$\}$, where a K-substitution is a substitution that maps symbols into languages of K. A construct (V,Σ,U,L) with $L \subseteq V^*$ is called a K-iteration grammar: a generalization of ETOL system (see for instance [26]). A family K is called a full hyper-AFL if it is a full AFL closed under iterated substitution (i.e. $H(K) \subseteq K$). The families H(FIN) and H(ONE), where FIN and ONE are the finite and the singleton languages, are denoted by ETOL and EDTOL respectively.

For the definition of (one-way nondeterministic) stack automaton, nonerasing stack automaton and checking stack automaton we refer to [18, 19]. The corresponding classes of languages will be denoted by Stack, NEStack and CStack respectively.

For the definition of a topdown tree transducer we refer to [25, 11, 7]. The family of tree transformation languages (i.e. yields of images of the recognizable tree languages under topdown tree transducers) is denoted by yD_1, and the subfamily of deterministic tree transformation languages by $ydetD_1$. We note that $ydetD_1$ equals the class of ranges of generalized syntax directed translations [3].

We now list a few facts taken from the literature, which establish some connections between the above mentioned concepts.

Known facts

(1) ETOL and OI are full hyper-AFLs [8, 9, 26].

(2) ETOL = ELB and EDTOL = LB [9], hence ETOL $\subseteq$ OI.

(3) Stack $\subseteq$ OI [2, 16] and NEStack $\subseteq$ ETOL [22].

(4) ETOL $\subseteq$ yD_1 and EDTOL $\subseteq$ $ydetD_1$ [12].

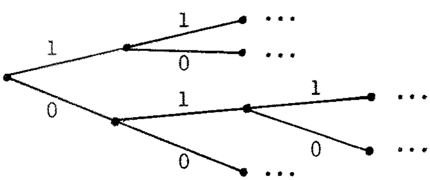
(5) All inclusions in (1)-(4) are proper [10, 19, 15].

We finally refer to [15] for the properties P2 and P3 of a language L. Intuitively P2 says that in a string from L one cannot find two nonoverlapping substrings which may be changed into other substrings independently (without leaving L). P3 says that one cannot find two different nonoverlapping substrings that may be used in place of each other (whithout leaving L). Thus P2 implies P3.

3. The language of cuts

In this section we present a language L_o which is both in Basic and Stack but not in yD_1 (and hence not in ETOL, cf. section 2). It follows that OI and yD_1 are incomparable. At the end of the section we put several language families into an inclusion diagram.

L_o will represent the set of all cuts through the infinite binary tree



A cut is a finite nonempty sequence of words over {0, 1} defined recursively as follows: (i) $\langle\lambda\rangle$ is a cut, (ii) if $\langle v_1,\dots,v_k\rangle$ and $\langle w_1,\dots,w_n\rangle$ are cuts, then so is $\langle 0v_1,\dots,0v_k, 1w_1,\dots,1w_n\rangle$. The strings w_i in a cut $\langle w_1,\dots,w_n\rangle$ are called nodes. An example of a cut, corresponding to the above picture, is <00, 010, 011, 10, 11>. A cut is also called a complete binary code.

Definition. Let a and b be symbols different from 0 and 1.
$L_o = \{aw_1 0bw_1 1aw_2 0bw_2 1\dots aw_n 0bw_n 1 \mid \langle w_1,\dots,w_n\rangle \text{ is a cut}\}$.

Note that if $\langle w_1,\dots,w_n\rangle$ is a cut, then so is $\langle w_1 0, w_1 1,\dots,w_n 0, w_n 1\rangle$.

L_o is generated by the basic macro grammar with rules $S \to A(\lambda)$, $A(x) \to A(x0)A(x1)$,

$A(x) \to ax0bx1$.

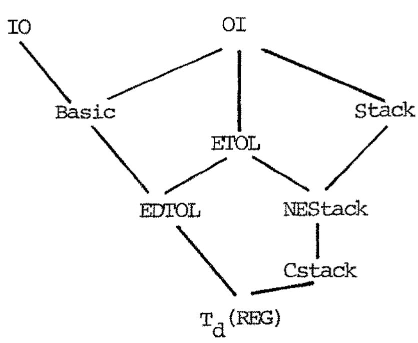
L_o can easily be recognized by a stack automaton that stores the consecutive nodes of a cut corresponding to a word of L_o in its stack (one at a time).

L_o has property P3 (to see this one needs, apart from the special form of L_o, that, for any cut $<w_1,\ldots,w_n>$, the nodes w_i are all different and $\sum_{i=1}^{n} 2^{-|w_i|} = 1$; moreover one needs that for given integers $k_1,\ldots,k_n$ there is at most one cut $<w_1,\ldots,w_n>$ such that $|w_i| = k_i$ for $1 \le i \le n$). Theorem 5 of [15] says that any language in yD_1 with property P3 is in $ydetD_1$. Thus it now suffices to show that $L_o \notin ydetD_1$. In [24] an intercalation lemma for tree transducer languages is proved that in a straightforward way gives rise to the following intercalation lemma for $ydetD_1$: for each L in $ydetD_1$ there is an integer p such that every z in L longer than p can be written as $z = z_1 \ldots z_k$ and (i) $|z_i| \le p$ for all $1 \le i \le k$, and (ii) for every N there are strings $v_1,\ldots,v_k$ such that $v_1 \ldots v_k \in L$, $|v_1 \ldots v_k| > N$ and $\min(v_i) = \min(z_i)$ for all $1 \le i \le k$ (where, for a string w, min(w) denotes the set of symbols occuring in w). Thus, assuming that L_o is in $ydetD_1$, every long string of L_o can be divided into small substrings which can be pumped up keeping the same min alphabet. Take $z = aw_1 0bw_1 1 \ldots aw_n 0bw_n 1$ in L_o with $|w_i| \ge p$ for all $1 \le i \le n$. Then pumping up $z_1,\ldots,z_k$ can only influence the 0's and 1's. This would give arbitrary long cuts with the same number (2n) of nodes. This is clearly a contradiction. Hence $L_o \notin yD_1$.

We note that the reader interested only in ETOL can use Theorem 1 of [15] instead, and give the (easy) proof for the above intercalation lemma for EDTOL.

The existence of L_o solves the problem left open in [15] whether $OI \subseteq yD_1$. Hence OI and yD_1 are incomparable (cf. [10, 15]); in other words, the classes of context-free tree languages and ranges of topdown finite state tree transducers are incomparable even when yields are taken. We conjecture that L_o is not in any yD_n (i.e. cannot be obtained by the application of any sequence of tree transducers, cf. [7]).

We can now draw the following inclusion diagram, in which T_d(REG) denotes the class of images of the regular languages under deterministic 2-way gsm's (see [21], where it is shown that $T_d(REG) \subseteq$ CStack).



The inclusions are clear apart from the inclusion $T_d(REG) \subseteq EDT0L$ which follows from the fact that $T_d(REG)$ is closed under copying [21] and a copying theorem for ET0L (Theorem 1 of [15]). Incomparabilities and proper inclusions follow from

(1) $L_o \in$ (Basic $\cap$ Stack) - ET0L (proved above).

(2) $\{a^n b^{n^2} c^n \mid n \geq 1\} \in$ EDT0L - Stack (see [23]).

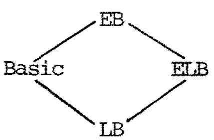
(3) $\{w \in \{a,b\}^* \mid$ the number of b's in w is not prime$\}$ is in CStack [19], but not in IO; the latter follows by observing that the proof in [16] of the existence of a language in OI - IO proves in fact that if $L \subseteq b^*$ and $h^{-1}(L) \in$ IO (where $h(a) = \lambda$ and $h(b) = b$), then L is regular.

(4) the existence of a language in IO - OI [16].

(5) $\{a^{n^2} \mid n \geq 1\} \in$ NEStack - CStack [19].

4. Extended basic macro languages

In this section we show several properties of EB, in particular the inclusion of Stack in EB and a result on substitution of EB languages. We first note that by the previous section the following diagram is correct (cf. section 2):



When macro grammars are viewed as nondeterministic recursive program schemas (see [14]), the notion of "extendendness" corresponds to allowing choices (tests) in the parameters of a procedure call. Thus the diagram shows that for nonnested recursive program schemes this feature extends their computational power (independent of linearity).

We extend EB grammars still more as follows. Let RB denote the class of languages generated by basic macro grammars in which union, concatenation and moreover Kleene star are used as operations. Thus regular languages are stored in the arguments of a nonterminal rather than finite ones as in EB.

We now list some facts about EB together with sketches of proof.

(1) RB = EB.

Proof. Any finite approximation of a regular language can be computed in some additional arguments of a nonterminal.

(2) EB is a full AFL.

Proof. $\cup$, $\cdot$, $*$ as for context-free grammars; regular substitution using (1); $\cap R$ by a standard proof (cf [16]).

(3) Stack $\subseteq$ EB.

Proof. By (1) and (2) it suffices to show that a full AFL-generator of Stack is in RB. The full generator of Stack given in [17, Example 5.3.2] is generated by the following RB grammar which remembers the possible sequences of stack-reading instructions in the argument of T:

$S \to T(\lambda)$, $\quad T(x) \to \lambda$,

$T(x) \to a(a^L xa^R)^* T((a^L xa^R)^*)a^E xT(x)$ and a similar rule for b.

(4) Let, for languages L_1 and L_2 over disjoint alphabets, $\tau_{L_2}(L_1)$ denote the result of substituting aL_2 for each symbol a in L_1. If $\tau_{L_2}(L_1) \in$ EB, then L_1 is context-free or $L_2 \in$ ELB.

Proof (cf. [19]). Consider the ELB grammar G' obtained by replacing each rule $A(\ldots) \to B_1(\ldots)B_2(\ldots)\ldots B_n(\ldots)$ of the EB grammar G generating $\tau_{L_2}(L_1)$ by the n rules $A(\ldots) \to B_i(\ldots)$. Either L_2 can be obtained from L(G') by a finite state transducer producing all words of L_2 occuring between symbols of L_1 (and ELB = ETOL is a full AFL); or G can be changed into a context-free grammar generating L_1, since it only has to remember a finite amount of information in its arguments.

(5) EB is not substitution closed.

Proof. Let $L_1 = \{a^{2^n} \mid n \geq 1\} \in$ EB − CF and $L_2 = L_o$ from section 2 which is in EB − ELB. Then (4) implies that $\tau_{L_2}(L_1) \notin$ EB.

(6) The converse of (4) also holds. In fact EB is closed under substitution into context-free languages and under substitution by ELB languages.

Proof. By straightforward grammatical constructions.

We note that results (4, 5, 6) are similar to those in [19] concerning Stack and CStack.

5. A full hyper-AFL between EB and OI

Consider the family H(EB). Since EB is a full AFL, H(EB) is the smallest full hyper-AFL containing EB [4]. Since each full hyper-AFL is substitution closed, it follows that EB $\subsetneq$ H(EB). We shall show that H(EB) $\subsetneq$ OI. The main technique is that of copying as used in [27, 10, 15].

(1) If L has property P2 and L $\in$ H(EB), then L $\in$ LB.

Proof. Property P2 forces each EB grammar the language of which is used in the substitutions whose iteration gives L, to be linear. Hence L $\in$ H(ELB). Since ELB is a full hyper-AFL (cf. section 2), L $\in$ ELB (= ETOL). Hence, by Theorem 1 of [15], L $\in$ EDTOL = LB.

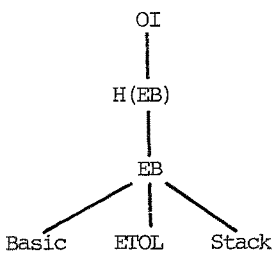
(2) If L $\in$ Basic, then $\{w\#w^R \mid w \in L\} \in$ OI $\cap$ IO, where w^R is the reverse of w.

Proof. by a grammatical construction in which the sentential form $A_1(s_1)A_2(s_2)\ldots A_n(s_n)$ of the basic macro grammar, where s_i is the sequence of arguments of A_i, is represented as $A_1(s_1,s_1',A_2(s_2,s_2',A_3(\ldots,A_n(s_n,s_n')\ldots)))$ in the new grammar, where s_i' contains the reverses of the elements of s_i.

(3) H(EB) $\subsetneq$ OI.

Proof. Let $L_1 = \{w\#w^R \mid w \in L_o\}$ where $L_o \in$ Basic − ETOL is the one of section 3. By (2), $L_1 \in$ OI. Since L_1 has property P2, $L_1 \in$ H(EB) would imply $L_1 \in$ ETOL by (1).

We thus obtain the following diagram.



This diagram solves the question in [16] whether OI is the smallest full AFL containing Basic, it shows the existence of a full hyper-AFL between the full hyper-AFLs ETOL and OI, and it improves the result in [20] that OI cannot be reached from Stack by nested iterated substitution (in fact the proof in [20] shows that the smallest super-AFL containing Stack is properly contained in H(EB)).

Remark 1. An indexed grammar is restricted if after consumption of a flag no new flags can be created (see [1], last page, for a formal definition). It can be shown that the class of restricted indexed languages is equal to EB. The inclusion of Stack in this class was shown in [2]. The result of this section shows that not all indexed languages can be obtained from the restricted indexed languages by hyper-AFL operations.

Remark 2. A reasoning similar to the one in this section shows that IO cannot be reached from Basic by iterated 'deterministic' substitution (cf. [5]; in the notation of that paper: $\eta(\mathrm{CF}) \subsetneq \eta(\mathrm{Basic}) \subsetneq \mathrm{IO}$, where L_0 and L_1 are the respective counterexamples.

6. Future work

(1) In [22] the cs-pd machine is defined which recognizes precisely ETOL (= ELB). It is a checking stack automaton with a restricted facility of writing on its stack. A good machine for EB is the s-pd machine, which is a stack automaton with the same writing facility (the machine may write on a second track of its stack in a pushdown fashion: the reading head points at the top of pushdown track whereas its bottom is at the top of the stack). This explains the similarity of the results in [19] and those in section 4. The above s-pd machines are the subject of [13].

(2) For any family K of languages one can define Basic(K)-grammars similar to extended basic grammars but with languages from K rather than finite ones. Thus EB = Basic(FIN) = RB = Basic(REG). Similarly LB(K)-grammars can be defined. Generalizing the proof in [9] that ETOL = ELB, it follows that under weak restrictions on K, LB(K) = H(K). Let us call a full AFL K such that $\mathrm{Basic}(K) \subseteq K$, a full basic-AFL (rather than hyphyper-AFL). It can be shown that the smallest full basic-AFL is properly contained in OI (L_1 of section 5 being the counterexample). It is conjectured that it is the union of a proper hierarchy of full hyper-AFLs. These "basic extensions" are the subject of [6].

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