Feature Based Defuzzification in $\mathbb{Z}^2$ and $\mathbb{Z}^3$ Using a Scale Space Approach

Joakim Lindblad[1], Nataša Sladoje[2,⋆], and Tibor Lukić[2]

[1] Centre for Image Analysis, Swedish University of Agricultural Sciences, Uppsala, Sweden
joakim@cb.uu.se
[2] Faculty of Engineering, University of Novi Sad, Novi Sad, Serbia
{sladoje, tibor}@uns.ns.ac.yu

Abstract. A defuzzification method based on feature distance minimization is further improved by incorporating into the distance function feature values measured on object representations at different scales. It is noticed that such an approach can improve defuzzification results by better preserving the properties of a fuzzy set; area preservation at scales in-between local (pixel-size) and global (the whole object) provides that characteristics of the fuzzy object are more appropriately exhibited in the defuzzification. For the purpose of comparing sets of different resolution, we propose a feature vector representation of a (fuzzy and crisp) set, utilizing a resolution pyramid. The distance measure is accordingly adjusted. The defuzzification method is extended to the 3D case. Illustrative examples are given.

1 Introduction

The advantages of representing objects in images as fuzzy spatial sets are numerous and have lead to increased interest for fuzzy approaches in image analysis [1]. Preservation of fuzziness by utilizing fuzzy segmented images implies preservation of important information about objects. However, a crisp representation of objects in the images may still be needed. Reasons for that are, e.g., to facilitate easier visualization and interpretation. Even though it contains less information, a crisp representation is often easier to interpret and understand, especially if the spatial dimensionality of the image is higher than two. Moreover, analogues for many tools available for the analysis of binary images are still not developed for fuzzy images. This may force us to perform at least some steps in the analysis process by using a crisp representation of the objects.

In our previous work related to defuzzification, i.e., the process of generating a crisp representation of a fuzzy digital object, we introduced a distance

⋆ The author is financially supported by the Ministry of Science of the Republic of Serbia through the Project ON144029 of the Mathematical Institute of the Serbian Academy of Science and Arts.

A. Kuba, L.G. Nyúl, and K. Palágyi (Eds.): DGCI 2006, LNCS 4245, pp. 379–390, 2006.
© Springer-Verlag Berlin Heidelberg 2006

measure between fuzzy sets incorporating a number of quantitative global and local features of the sets [2]. We have suggested to perform defuzzification by choosing the crisp representation that is closest to the given fuzzy set in terms of the proposed distance measure. The resulting crisp set can be generated at higher spatial resolution, compared to the spatial resolution of the fuzzy object, as shown in [3]. In that way, a (crisp) segmentation technique that provides crisp objects represented at a higher spatial resolution than the given image resolution has been proposed.

In this paper, we consider matching additional features in the defuzzification. We have noticed that the lack of requirement for feature preservation at meso-scale, i.e., scales in-between the local (pixel-size) and the global (the whole object) scale, may lead to rather inappropriate defuzzification solutions in spite of successful matching of both local and global features in the optimization algorithm. Therefore, we introduce meso-scale area components in the feature representation of a set to be a subject of optimization, in addition to already existing membership values of pixels, seen as local area components, and the global area of a set. Global features, perimeter and centroid, are considered in defuzzification, as it is suggested in [2]. Global features are important for successful defuzzification at increased spatial resolution, [3]. To facilitate the use of meso-scale area components (their calculation and updating during the optimization process), we utilize representations of the sets at a range of spatial resolution, i.e., we generate a resolution pyramid representations. We implement the defuzzification method in both the 2D and the 3D case.

The paper is organized as follows: Section 2 gives an overview of existing results related to the proposed method and lists the main definitions used in the paper. In Section 3, the main contribution of the paper, defuzzification by minimizing feature distance using a scale space approach, is presented. Examples of defuzzification of both synthetic and real 2D and 3D images are given in Section 4, and the positive effect of the suggested scale space approach to defuzzification is illustrated. Concluding remarks are given in Section 5.

2 Background

We give a list of definitions and notions used in the paper and present existing results related to defuzzification. In this paper, we extend our own work on defuzzification based on feature distance minimization, proposed originally in [2]; therefore, most of related work is referenced to our own results. Moreover, Section 2.3 particularly recalls the necessary framework derived in [2] and [3].

2.1 Definitions

A *fuzzy set* S on a reference set X is a set of ordered pairs $S = \{(x, \mu_S(x)) \mid x \in X\}$, where $\mu_S : X \to [0, 1]$ is the *membership function* of S in X. We denote by $\mathcal{F}(X)$ the set of fuzzy sets on a reference set X and by $\mathcal{P}(X)$ the set of crisp subsets of a set (the power set).

Being interested in applications in digital image analysis, we consider digital fuzzy sets, where $X \subset \mathbb{Z}^n$. In addition, when using digital approaches (computers) to represent, store, and analyse images, the (finite) number, $\ell + 1$, of grey-levels available is a natural limitation to the number of membership values that can be assigned to a digital point.

An *α-cut* of a fuzzy set S, for $\alpha \in (0, 1]$, is the set $S_\alpha = \{x \in X \mid \mu_S(x) \geq \alpha\}$. The *support* of a fuzzy set S is the set $\text{Supp}(S) = \{x \in X \mid \mu_S(x) > 0\}$. The *core* of a fuzzy set S is the set $\text{Core}(S) = \{x \in X \mid \mu_S(x) = 1\}$. The *fuzzification principle*, based on the following equation:

$$f(S) = \int_0^1 \hat{f}(S_\alpha)\, d\alpha, \tag{1}$$

can be used to generalize properties $\hat{f}$ defined for crisp sets (here, α-cuts) to fuzzy sets. In order to generalize a function $\hat{f}$, defined on discrete crisp sets, the equation

$$f(S) = \frac{1}{\ell} \sum_{\alpha=1}^{\ell} \hat{f}(S_\alpha)\,, \tag{2}$$

can be used. In this paper, we use Equation (2) to define *perimeter* $P(S)$, and *surface area* $\mathit{Surf}(S)$, of a (2D and 3D) fuzzy set S, respectively (more detailed definition and properties are given in [4]). Equation (2) is also used to define *moments of zero and first order* of a discrete spatial fuzzy set S, denoted by $m_{p,q}(S)$, where for integers p, q it holds $p + q \leq 1$ (in 2D case), and $m_{p,q,r}(S)$, where for integers p, q, r it holds $p + q + r \leq 1$ (in 3D case) (more detailed definitions and properties are described in [5]).

The zero-order moment of a set S is equal to its area $A(S)$ (if $S \in \mathcal{F}(Z^2)$), or its volume $V(S)$ (if $S \in \mathcal{F}(Z^3)$). The centroid of a set S is also defined by the moments of a set S, [6].

2.2 Related Work

Defuzzification is the process of replacing a fuzzy set with an appropriately chosen crisp set. In image analysis, fuzzification of an image is a consequence of the combination of properties of the continuous original, discretization effects, and imaging conditions. Defuzzification should be performed by utilizing the fuzzy representation as a source of valuable information about geometric properties of the object of interest (fuzzy, or crisp), and by defining and following some criteria related to properties which characterize a satisfactory defuzzification result.

In our previous work [2], we present a defuzzification method based on minimizing the feature distance between a fuzzy set and its defuzzification. Feature distance is defined so that the distance between two sets is expressed in terms of the distance between their feature-based vector representations in some defined feature space. A selection of local and global numerical features can be included in the distance measure and considered in defuzzification. By using some optimization procedure, the crisp object fulfilling the minimization criterion is generated.

The results presented in [5,4] show that the precision of estimates for perimeter, area, and higher order moments of a continuous shape, is significantly higher if a fuzzy discrete shape representation, where the membership of a pixel is proportional to the part of its area covered by the observed object, is used instead of a crisp discrete representation. By including these estimates in the feature based representation of the fuzzy set, we generate a crisp digital object which is a good crisp representation of a fuzzy set and, at the same time, highly resembles the original continuous object. Furthermore, it is shown in [5,4], either theoretically or through statistical studies, that a fuzzy approach can provide an alternative to increasing the spatial resolution of the image. This observation motivated the study presented in [3], where we suggested to generate a crisp shape representation of a given fuzzy object at an r times increased spatial resolution. The features, estimated with a high precision from the fuzzy representation, are preserved in the defuzzification by minimizing the corresponding feature distance between the sets. We let each pixel in the fuzzy low resolution representation correspond to a block of $r \times r$ pixels in the crisp high resolution representation. Local (point-size) features of a fuzzy set are compared to the features derived from a block in the defuzzified set. This correspondence is used as a basis for the high-resolution object reconstruction.

2.3 Defuzzification by Feature Distance Minimization

The feature distance measure and the defuzzification method based on its minimization, proposed in [2], are further explored in this paper. The following notation and definitions, introduced in [2], are used in the sequel:

Defuzzification. An optimal defuzzification $\mathcal{D}(A)$ of a fuzzy set $A \in \mathcal{F}(X)$, with respect to the distance d, is

$$\mathcal{D}(A) \in \{C \in \mathcal{P}(X) \mid d(A,C) = \min_{B \in \mathcal{P}(X)} [d(A,B)]\} \; . \tag{3}$$

Distance Measure. Given an injective function Φ from $\mathcal{F}(X)$ to a metric space H, we define a metric on $\mathcal{F}(X)$ by requiring that Φ is an isometry. That is, the *feature distance* between fuzzy sets A and B is

$$d^{\Phi}(A,B) = d(\Phi(A), \Phi(B)) \; . \tag{4}$$

The vector $\Phi(A) \in H$ is understood as a feature representation of the set A.

We use $H \subset \mathbb{R}^n$, with the Minkowski distance of order $p = 1$ as our main choice of distance functions used in H, providing a corresponding feature distance d^{Φ} in $\mathcal{F}(X)$.

By suitably designing the mapping Φ, the distance measure can be tuned to provide defuzzifications where both shape characteristics and membership values are taken into account. This enables defuzzification that fits the individual problem well, and provides a powerful family of defuzzification methods. In this paper, we further explore the appropriate choices of features incorporated in Φ, in order to improve the preservation of relevant characteristics of the fuzzy set.

Optimization. In general, Equation (3) cannot be solved analytically. In addition, the search space $\mathcal{P}(X)$ is too big to be exhaustively traversed. As a consequence, we are forced to rely on heuristic search methods. In [2], two methods, floating search and simulated annealing, are used to find an approximate solution for Equation (3). Simulated annealing, starting from the α-cut closest to the given fuzzy set in terms of the considered distance, showed the most satisfactory behaviour, and is in this paper used as the optimization method of choice. Since the optimization task is a well separated problem, many other search methods can be used to approximatively solve Equation (3).

3 Feature Based Defuzzification Using a Scale Space Approach

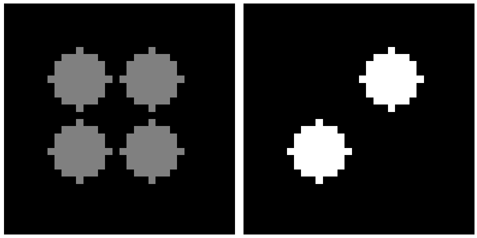
In this section, we present our main contribution, a scale space approach to high-resolution defuzzification based on feature distance minimization using meso-scale features. We give a motivation for using such an approach, and we describe the suggested method, both in the 2D and the 3D case.

3.1 Motivation

Characteristics of a given fuzzy set often have different importance at different scales. When the feature distance is optimized so that point-wise and global features are preserved as well as possible in the defuzzification, it is still possible that meso-scale features are not matched sufficiently well, even though the achieved distance is satisfactory low. The resulting crisp set may, e.g., have area (number of points) perfectly well matched with the area of the fuzzy set, while the pixels are not distributed in an intuitive way over the crisp set. A synthetic example illustrating this is presented in Figure 1. An object, Figure 1(a), is composed of four discrete fuzzy disks, where all non-zero membership values of points are equal to 0.5. Such a (homogeneous) distribution of local features does not provide any information about preferable local distribution of pixels in a defuzzification. A globally optimal solution, Figure 1(b), appears rather non-intuitive, in spite of its well matched features; it is more appealing that defuzzification of individual parts of an image resembles (as much as possible) the individual parts of an image defuzzified as a whole. We conclude that a main problem is that the method may "transport" area from one part of the image to another part.

This observation leads us to investigate the incorporation of features calculated over a range of scales into the distance measure and the defuzzification. Use of a scale space approach provides an appropriate treatment of details in images, where the details are usually relevant only in some range of scales.

An alternative approach would be to include additional higher order moments, to provide a more complete description from a global level. We consider the scale space approach to be more appealing in terms of generality, and also probably in terms of robustness, since higher order moments are in general sensitive to noise.



(a) (b)

Fig. 1. (a) Synthetic image. (b) Defuzzification of (a) based on (local and global) feature distance minimization, as proposed in [2].

A good way to introduce scale dependent defuzzification is to use a resolution pyramid. Representations at different levels in the resolution pyramid correspond to objects made by mapping blocks of pixels (voxels) of an image at a given resolution into one pixel (voxel) in the image at some other (lower) resolution. By comparing the features of interest at corresponding resolution levels, blocks in the fuzzy and the defuzzified image are compared and the features are thereby considered, not only at the local and the global scale, but also at in between meso-scales. In this paper, our choice of feature to observe at meso-scale is area (volume), corresponding to membership values of pixels (voxels) in the representations in the resolution pyramid.

3.2 Scale Space Defuzzification of 2D Fuzzy Sets

For a given fuzzy set $F \in \mathcal{F}(X)$ of size $2^m \times 2^m$ pixels, we generate $m+1$ partitions of the set into square blocks of $2^{m-i} \times 2^{m-i}$ neighbouring pixels, for $i = 0, \ldots, m$. Each partition i consists of 2^{2i} blocks. (If the original image is not of size $2^m \times 2^m$ pixels, we pad it with zeros.) We use a feature representation $\Phi(F)$ consisting of the areas of all the blocks of all the partitions. Obviously, the membership values of all the pixels are included in such a representation, being local areas of one-pixel-size blocks ($i = m$), while the global area of the set is included as the area of the single block of the size $2^m \times 2^m$ (for $i = 0$). In addition, the perimeter of the set F, as well as the coordinates of its centroid, are included in the feature representation.

Weighting of Features. In order to provide that the effect of the total contribution of all measures of one (type of) feature, observed at one particular scale, is approximately the same size as the effect of one global feature, features of multiplicity h are scaled with $\frac{1}{\sqrt[p]{h}}$, where p is the exponent of the Minkowski distance in Equation (4) (we use $p = 1$ throughout this paper).

To compare features calculated at different scales, measures also have to be rescaled with respect to the spatial resolution of the image and the dimensionality of the particular feature. It is taken into account that $P(S) = \mathcal{O}(r_S)$, $A(S) = \mathcal{O}(r_S^2)$, $m_{1,0}(S) = \mathcal{O}(r_S^3)$, $m_{0,1}(S) = \mathcal{O}(r_S^3)$ for a set S inscribed into a grid with spatial resolution r_S. To get resolution invariant global features, we use

$$\tilde{P}(S) = \frac{P(S)}{P(X)}\,, \quad \tilde{A}(S) = \frac{A(S)}{A(X)}\,, \quad \tilde{C_x}(S) = \frac{C_x(S)}{C_x(X)}\,, \quad \tilde{C_y}(S) = \frac{C_y(S)}{C_y(X)}\,,$$

for a fuzzy set $S \in \mathcal{F}(X)$. In this way, it is provided that $\tilde{P}(S) = \mathcal{O}(1)$, $\tilde{A}(S) = \mathcal{O}(1)$, $\tilde{C_x}(S) = \mathcal{O}(1)$, $\tilde{C_y}(S) = \mathcal{O}(1)$ for any grid resolution.

Feature Vector Representation. For a given fuzzy (or crisp) set S of size $2^n \times 2^n$ pixels, let B^i_j represent the jth block of $2^{n-i} \times 2^{n-i}$ pixels, where $j = 1, \ldots, 2^{2i}$, $i = 0, \ldots, n$. Block B^0_1 is equal to the set S and, correspondingly, $\tilde{A}(S) = \tilde{A}(B^0_1)$. The feature representation $\Phi_m(S)$ of S, for $m \le n$, is then

$$\begin{aligned}\Phi_m(S) = \Bigg(& \frac{1}{\sqrt[p]{2^{2m}}}\tilde{A}(B^m_1), \ldots, \frac{1}{\sqrt[p]{2^{2m}}}\tilde{A}(B^m_{2^{2m}}), \\ & \frac{1}{\sqrt[p]{2^{2(m-1)}}}\tilde{A}(B^{m-1}_1), \ldots, \frac{1}{\sqrt[p]{2^{2(m-1)}}}\tilde{A}(B^{m-1}_{2^{2(m-1)}}), \\ & \ldots \\ & \frac{1}{\sqrt[p]{2^0}}\tilde{A}(B^0_1), \\ & \tilde{P}(S), \tilde{C_x}(S), \tilde{C_y}(S)\Bigg)\,. \end{aligned} \tag{5}$$

Resolution Pyramids. We use two resolution pyramids for storing the areas of the blocks B^i_j of the fuzzy original set, and of the crisp defuzzification. Pyramids are built by grouping 2×2 neighbouring (*children*) pixels in the image at the current resolution level, and create one (*parent*) pixel at the next, lower, resolution level, where the value of the parent pixel is assigned to be the sum of the values of the children pixels. The process is repeated at every newly created resolution level, until the lowest possible resolution. The value assigned to the single element at the lowest level in the pyramid is the area of the starting image. To obtain rescaled areas of blocks, used in the feature representation $\Phi(S)$, each value (area of a block) is divided by the number of elements in the block.

Defuzzification. For a given fuzzy set F, containing $2^m \times 2^m$ pixels, a resolution pyramid representation with $m+1$ resolution levels is build. The α-cut of F at minimal distance d^Φ to F is used as the starting configuration for defuzzification. In order to obtain the initial configuration K at 2^r times increased resolution, each pixel in the α-cut is subdivided into 2^{2r} sub-pixels. A resolution pyramid for the crisp set K, with $m+r+1$ resolution levels, is created and defuzzification is performed by minimizing the feature distance

$$d^\Phi(F, K) = d(\Phi_m(F), \Phi_m(K))\,, \tag{6}$$

where d is the Minkowski distance with $p = 1$.

During the search process, when changing one pixel in the crisp set K, all levels of the pyramid representations of K are locally updated.

Simulated Annealing. The simulated annealing search ([2]) was refined by applying a re-annealing scheme where the search was restarted 20 times. An automatic tuning of the temperature was achieved by restarting each annealing at twice the temperature where the current best solution of the previous annealing was accepted; did the previous annealing not provide any improvement, the new temperature was set to be twice the staring temperature of the previous one. Within one annealing, 5 000 random perturbations where tried at each temperature level, before the temperature was lowered so that $T_{new} = 0.995\,T_{old}$. The temperature was successively reduced until 50 000 successive perturbations did not manage to provide any step that gave a reduction in distance, after which a new re-annealing was restarted from the currently best found solution.

3.3 Scale Space Defuzzification of 3D Fuzzy Sets

The defuzzification method, suggested for 2D discrete spatial fuzzy sets is straightforwardly generalized to the 3D case. The features selected to be included in the feature distance are local, meso-scale, and global volumes, obtained by iterative grouping of blocks of $2 \times 2 \times 2$ voxels, and surface area and centroid, as additional global features.

Once when the feature representation is generated, the defuzzification process is exactly the same as in the 2D case.

4 Examples

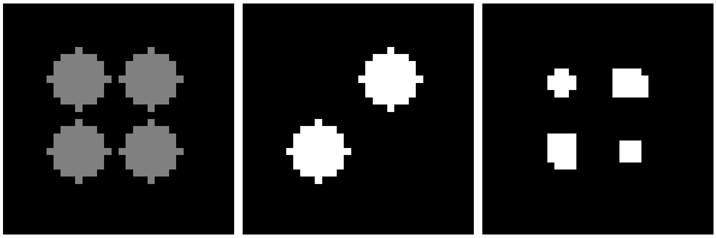
We show three examples of defuzzification using the suggested method. We test its behaviour on the synthetic image, Figure 1(a) and we show two examples of defuzzification of parts of real 3D (medical) images.

4.1 Four Disks

The example presented in Figure 1 is repeated in Figure 2. The result of defuzzification of the object in Figure 2(a), using the proposed scale space approach, is shown in Figure 2(c). Even though the global features are perfectly matched in the solution presented in Figure 2(b) (obtained without meso-scale features), we consider the solution in Figure 2(c) to better preserve the properties of the original set. The contributions of the different features to the overall distance are given in Table 1.

4.2 Bone

An example of defuzzification of a 3D object is presented in Figure 3. The data volume here is a CT image of a bone implant (inserted in a leg of a rabbit). We applied the method to a part of the image ($51 \times 44 \times 59$ voxels) (Figure 3(a) shows a slice through the volume) containing a connected piece of bone area (dark grey), surrounded by a non-bone area (light grey). Figure 3(b) shows a slice through a 3D fuzzy set representing the bone region.



(a) (b) (c)

Fig. 2. **(a)** Four discrete disks of radius 4 and membership 0.5. **(b)** Optimal defuzzification using feature distance without meso-scale area components. **(c)** Defuzzification using feature distance including meso-scale area components.

Table 1. The contribution of the different features to the feature distance, and the total distance, without (Dist 1), and with (Dist 2), the meso-scale area features

Figure	Perimeter	Area	Centroid	Membership	Meso-scale	Dist 1	Dist 2
2(b)	0.0000	0.0000	0.0000	0.0957	0.3828	0.0957	0.4785
2(c)	0.0015	0.0381	0.0000	0.0957	0.1758	0.1353	0.3111

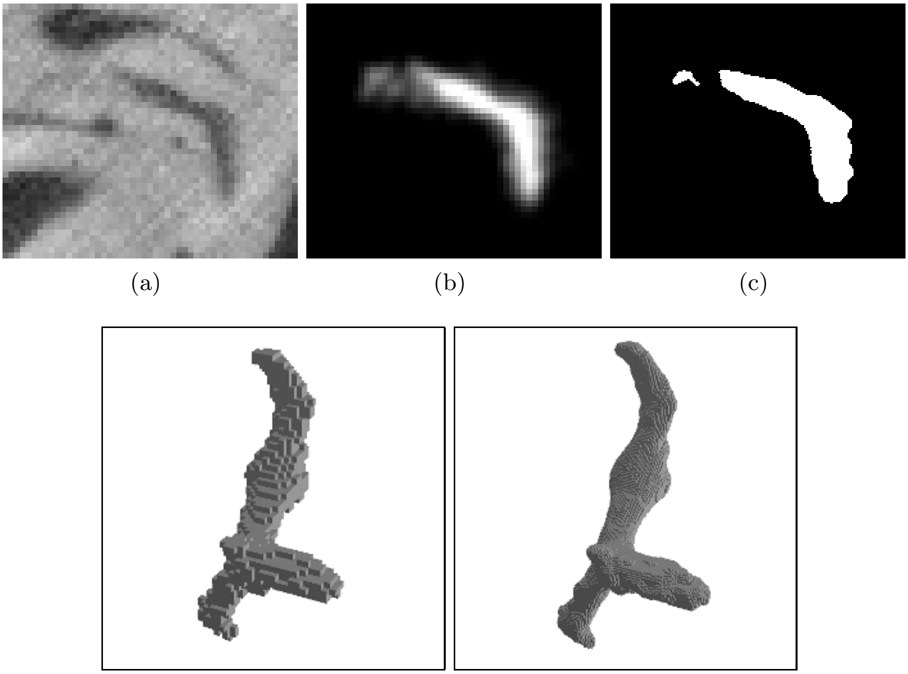
All features are matched well in this example; there are no large regions of high fuzziness, and the global features do not provide any reason for "transportation" of volume as in the example in Section 4.1. Defuzzifications with or without meso-scale features are therefore practically identical.

4.3 Vessels

Fuzzy representations of image objects are especially useful when the spatial resolution is too low to provide a good crisp representation. One such situation can be seen in Figure 4(b), which displays a maximum intensity projection of a part of a rotational b-plane x-ray scan of the arteries of the right half of a human head (provided by Philips Research, Hamburg, Germany), shown in Figure 4(a). A contrast agent is injected into the blood and an aneurism is shown to be present. The intensity values of the image voxels correspond fairly well with partial volume coverage, and are therefore used directly as fuzzy membership values.

This example image violates the sampling theorem; the vessels imaged are not resolved since they are smaller than one voxel thick. This fact causes a number of problems related to information extraction. Using a priori knowledge about the image, it is still possible to obtain a reasonable defuzzification. One such a priori piece of information is the knowledge that the vessel tree is simply connected. Starting from one simply connected component, and preserving topology ([7]) throughout the search, it is provided that the defuzzification is also simply connected.

Centroid position is not an intuitive feature to use for defuzzification of a vessel tree. It may interfere in undesirable ways with the topology preservation during the search procedure, so we exclude centroid from the feature representation in this example.



(a) (b) (c)

(d) $d^{\Phi} = 0.02749$ (e) $d^{\Phi} = 0.01377$

Fig. 3. Defuzzification of a part of a 3D image of a bone implant. (a) Slice through the image volume. The dark grey area is bone, the light parts are non-bone areas. (b) Slice through a fuzzy segmentation of the bone region in the image volume. (c) Slice through a defuzzification, using meso-scale volume features, of the fuzzy segmented image volume. (d) 3D rendering of the α-cut at smallest feature distance to the fuzzy object. (e) 3D rendering of a high resolution defuzzification of the fuzzy segmented object. A four times scaled up version of the best α-cut (d) was used as starting set for the simulated annealing search.

It is clear that high resolution reconstruction is really needed here; any crisp representative at the same resolution as the original image would be a rather bad representation; to preserve the volume of the fuzzy image, many parts of the vessel would not be included in the crisp set.

Performing defuzzification at two times the original resolution, we get the result presented in Figure 4(c). The result is not visually appealing, due to severe under-estimation of the surface area of a crisp thin (less than one voxel thick) structure by the surface area of the fuzzy set. This problem is not present for a crisp object whose fuzzy representation is obtained at sufficiently high resolution and contains points with memberships equal to one in the interior of the object. In the case presented in Figure 4, however, the defuzzification using the inaccurate surface area estimate fails to preserve the vessel structure.

It would be of high interest to have a better surface area estimate for the defuzzification. In the absence of such, we attempt defuzzification without

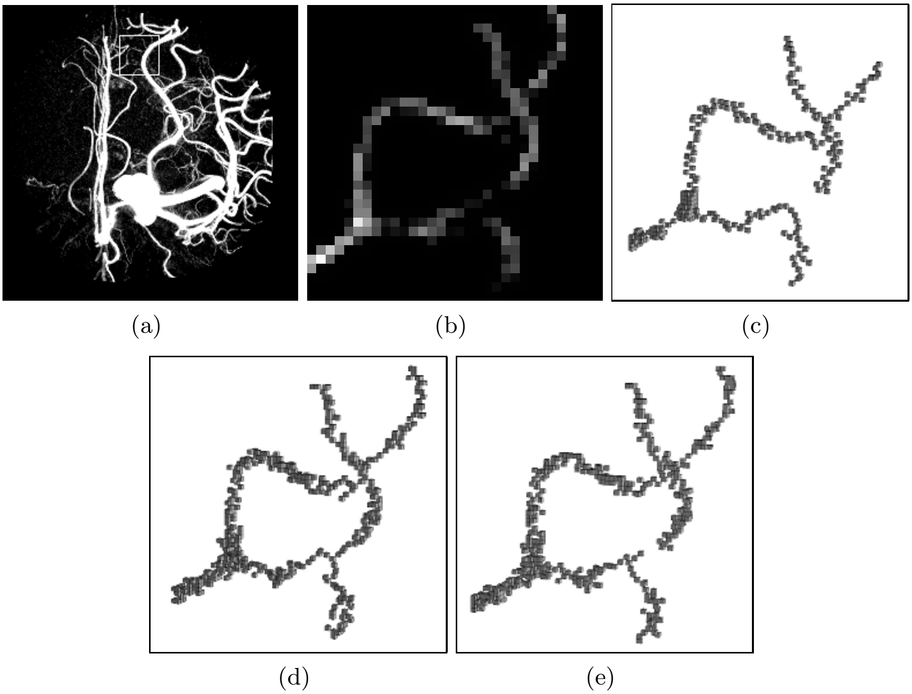


Fig. 4. Defuzzification of a selected part of an angiography 3D image, showing an X-ray scan of the arteries of the right half of a human head. (a) Maximum intensity projection through the image volume; the white square in the upper part of the image indicates the location of the selected part of the volume that is defuzzified in this example. (b) Maximum intensity projection through the selected part the volume. (c) 3D rendering of a defuzzification at twice the resolution using volumes of all scales and surface area. (d) 3D rendering of a defuzzification at twice the resolution using only volumes of all scale. (e) 3D rendering of a defuzzification at twice the resolution using only global and local volumes.

surface area feature. Using only volume based information (at a range of scales) the high resolution reconstruction is fairly unconstrained, which leads to the rather jagged result of Figure 4(d). Dropping the meso-scale feature from the feature representation, we get the result presented in Figure 4(e).

We note that, although not visible in Figure 4, the topology is in deed preserved; all the resulting objects are simply connected. However, the vessels are not always connected in a correct way, so some additional information on how vessels branch and bend may be required in this case.

5 Summary

We have presented an improvement to previously presented work on defuzzification by feature distance minimization. On a synthetic example we show that the combination of global and local features is not always enough to provide an appealing defuzzification. To overcome the described problem, we suggest to use a scale space approach, incorporating into the distance function feature values measured at different scales. The method is extended to work on 3D image volumes and two applications of the method on medical image volumes are provided.

The examples we show indicate possible future work on defuzzification by minimizing feature distance. Better feature estimates, and optionally more appropriate choice of features for thin elongated (tree-like) structures are needed. Using other features but area (volume) at a range of scales is of interest to explore. Thorough evaluation of the results is needed. The simulated annealing search algorithm, already improved by using re-annealing and more careful choice of parameters (temperatures and cooling scheme), could be further explored and better adjusted to particular application needs.

References

1. Udupa, J.K., Grevera, G.J.: Go digital, go fuzzy. Pattern Recognition Letters **23** (2002) 743–754
2. Sladoje, N., Lindblad, J., Nyström, I.: Defuzzification of spatial fuzzy sets by feature distance minimization (2005) Submitted.
3. Lindblad, J., Sladoje, N.: Feature based defuzzification at increased spatial resolution. In Reulke, R., Eckardt, U., Flach, B., Knauer, U., Polthier, K., (eds.): Proc. of 11th International Workshop on Combinatorial Image Analysis (IWCIA 2006). Volume 4040 of LNCS., Berlin, Germany, Springer-Verlag (2006) 131–143
4. Sladoje, N., Nyström, I., Saha, P.: Measurements of digitized objects with fuzzy borders in 2D and 3D. Image and Vision Computing **23** (2005) 123–132
5. Sladoje, N., Lindblad, J.: Estimation of moments of digitized objects with fuzzy borders. In Roli, F., Vitulano, S., (eds.): Proc. of 13th International Conference on Image Analysis and Processing (ICIAP 2005). Volume 3617 of LNCS., Cagliari, Italy, Springer-Verlag (2005) 188–195
6. Zadeh, L.: Fuzzy sets. Information and Control **8** (1965) 338–353
7. Borgefors, G., Nyström, I., Sanniti di Baja, G.: Connected components in 3D neighbourhoods. In: Proc. of 10th Scandinavian Conference on Image Analysis (SCIA 1997), Pattern Recognition Society of Finland (1997) 567–572