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Hodology

Adrian Kent[1,2]

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Abstract
A *hodological* law causes the evolution of the universe to tend to follow particular types of path. I give simple illustrations in toy models and discuss how Kolmogorov complexity characterises the extent to which hodological laws explain, rather than merely describe, data.

Keywords Physical law · Cosmology · Quantum theory · Probabilistic evolution

1 Introduction

If the probabilities we calculate in quantum theory are probabilities of some well-defined, objective, observer-independent features of nature, then a complete formulation of quantum theory has to include a sample space on which these probabilities are defined. The elements of that sample space form configurations of beables, in Bell's terminology. Whatever form they take, if they form part of physics as we understand it they presumably have a mathematical structure. It then makes sense to consider generalisations of quantum theory in which the probabilities depend on that structure as well as the Born rule.

This motivates looking at alternatives [1, 2] to cosmological theories inspired by the standard understanding of quantum theory. Whatever the fine-grained form of the beables, generalised probability laws associated with them could also affect the probabilities of coarse-grained large-scale features of the universe. The very large scale seems perhaps the most promising regime in which to look for empirical evidence of such deviations from quantum theory, since the strongest evidence for quantum theory comes from small scale phenomena, the relationship between

✉ Adrian Kent
A.P.A.Kent@damtp.cam.ac.uk

1 Centre for Quantum Information and Foundations, DAMTP, Centre for Mathematical Sciences, University of Cambridge, Wilberforce Road, Cambridge CB3 0WA, UK

2 Perimeter Institute for Theoretical Physics, 31 Caroline Street North, Waterloo, ON N2L 2Y5, Canada

quantum theory and gravity is not known, and there are other outstanding cosmological puzzles that suggest other lacunae in our understanding.

Theories that are based on quantum theory but guide the universe along paths other than those implied by standard unitary quantum dynamics are not yet part of standard mainstream discourse. They impose constraints, in a statistical sense. However, these theories are qualitatively different from quantum theory applied to constrained systems [3]. They are far more general than theories with independent initial and final boundary conditions. They are also far more general than dynamical collapse models, although dynamical collapse models can be seen as examples and can motivate others. Indeed, the generality they allow may raise a concern that considering such theories takes us out of the domain of science: that they can describe data but cannot explain them in any standard scientific sense.

I explain below why this concern is misplaced, using simple models that show why these "hodological" theories are qualitatively different, illustrate their generality and explain the extent to which they could nonetheless be scientifically useful.

2 Hodology in the Ehrenfest Urn Model

The Ehrenfest urn model [4] nicely illustrates the effect of laws describing a statistical evolution from an initial state. It can be generalized to illustrate cosmological laws with independent initial and final boundary conditions [5]. As we discuss below, it can also be generalized to illustrate hodological laws.

The standard version of the Ehrenfest urn model begins with N labelled balls distributed between two urns (A and B) in some initial configuration (for example, all in urn A, or balls 1 to $\lfloor \frac{N}{2} \rfloor$ in A and the rest in B). The model's state changes in discrete time steps, at each of which one label is chosen randomly, and the corresponding ball switches urn. It is easy to see (analytically or numerically) that low entropy distributions typically evolve quickly towards and then fluctuate around equipartition, spending nearly all the time close to equipartition and returning to low entropy states very infrequently.

We simplify the discussion by considering the model with some number T of time steps that is fixed in advance. One might think of this as a toy model of a universe with a fixed cosmological lifetime between its initial and final state. More generally, one might think of this as a toy model of a universe governed by a law that includes a finite number of statistical constraints on states through which it evolves in between its initial and final state. Such states could be identified by parameters other than cosmological time intervals. The Ehrenfest urn model as given is non-relativistic in spirit, in that it uses a fixed time coordinate, and it also uses a discrete time coordinate. This idealisation allows a simple illustration of hodological theories, their confirmability and refutability, and the sense in which they have explanatory power. However, these idealisations are inessential. The statistical constraints defining a hodological theories can refer to covariant quantities and can be continuous rather than discrete, and these would be the natural types of cosmological hodological theory to consider. The model, and our discussion of hodological theories within the model, can also be extended to allow an indefinite or infinite number of time steps, with statistical constraints chosen so that relative credences are defined for the relevant theories. In this case, to model the implications for human science (rather than that of a hypothetical "timeless" observer who observes the full evolution of the model) it would be natural to consider theories whose relative credences can be quantified from the data given by a finite number of steps.

In our model, we take the numbers of balls N_A and $N_B = N - N_A$ in urns A, B to represent macro-physical variables of interest, and the locations of each labelled ball to represent micro-physical variables. Macrophysically, the possible evolutions from the initial state $N_A = N_A^0$ are thus given by sequences

$$\underline{N_A} = (N_A^0, N_A^1, \dots, N_A^T)\,, \tag{1}$$

where $N_A^i = N_A^{i-1} \pm 1$. The sequence $\underline{N_A}$ has probability

$$\mathrm{Prob}(\underline{N_A}) = \prod_{i=1}^{T} \left(\delta(N_A^i - N_A^{i-1}, 1)\frac{N - N_A^{i-1}}{N} + \delta(N_A^i - N_A^{i-1}, -1)\frac{N_A^{i-1}}{N} \right). \tag{2}$$

We are interested in hodological generalisations of the standard Ehrenfest model that alter, and are defined in terms of, the macrophysics. To define such a model, we modify Eq. (2), reweighting the probabilities by non-negative factors $w(\underline{N_A})$ that depend on the form of the path $\underline{N_A}$ through configuration space. Thus

$$\mathrm{Prob}_{\mathrm{mod}}(\underline{N_A}) = Cw(\underline{N_A})\mathrm{Prob}(\underline{N_A})\,, \tag{3}$$

where C is a normalisation constant chosen so that

$$\sum_{\underline{N_A}} \mathrm{Prob}_{\mathrm{mod}}(\underline{N_A}) = 1. \tag{4}$$

2.1 Simple Examples

Example 1 **(fixed macrophysical path points):** Let $N = 10$, $T = 20$, $N_A^0 = 5$, and take

$$w(\underline{N_A}) = \delta(N_A^{10}, 5)\delta(N_A^{20}, 5). \tag{5}$$

This weighting ensures that the realised evolution path has an equipartition as its initial and final states and also at the midpoint of its evolution. A sample evolution is shown in Fig. 1.

Example 2 **(weighting towards a given macrophysical path):** If, again with $N = 10, T = 20, N_A^0 = 5$, we take

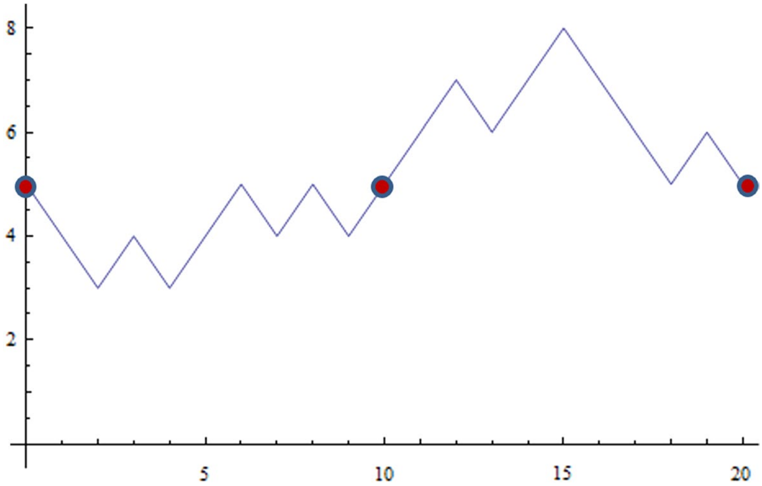

Fig. 1 Single run of $N = 10$ balls, constrained to $N_A = N_B = 5$ at $t = 0, 10$ and 20

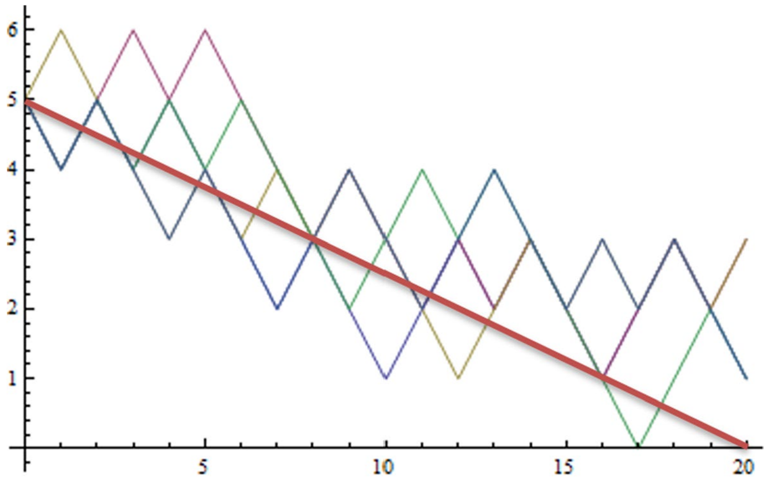

Fig. 2 5 runs of $N = 10$ balls, initial state $N_A = N_B = 5$ at $t = 0$, drawn from an ensemble with evolution probabilities modified by the weight factor (6)

$$w(\underline{N_A}) = \exp\left(-\frac{1}{6}\sum_{t=1}^{20}\left(N_A(t) - (5 - \frac{t}{4})\right)^2\right), \tag{6}$$

then the realised evolution path is likely to be relatively close to the line $N_A(t) = (5 - \frac{t}{4})$. Sample evolutions are shown in Fig. 2.

2.2 Testing Hodological Laws

Suppose now, for the sake of discussion, that we observe a new physical system whose properties are opaque to us, except for one discrete physical parameter that appears to evolve as though following some type of Ehrenfest urn model. That is, there is one observable discrete parameter, N_A, which appears always to lie in the range $0 \leq N_A \leq 10$. We observe its value only at discrete time steps t, after each of which it increases or decreases by 1. Suppose we cannot measure anything else about the system (perhaps it is effectively a black box, or a very distant cosmological object that regularly emits a discrete signal). Suppose also that, while we cannot directly observe the system's internal structure, extrapolating our knowledge of other better understood systems, and examining the evolution statistics of N_A, lead us to the strong hypothesis that it is characterised by some Ehrenfest model, with labelled subsystems playing roles analogous to those of the balls and urns. Suppose also that we have no information or good hypothesis about any interaction with other systems. And suppose that the system goes through repeated runs of 20 time steps, apparently resetting (say after a gap of 10 time steps, so that individual runs are identifiable) after each, with each run starting with $N_A = 5$.

After a while, we will conclude that, so long as we learn nothing more about the system, the only immediately scientifically fruitful theories we can make about it are defined by generalised Ehrenfest urn models of the form (3). We can evaluate these by Bayesian reasoning. Informally, this would run roughly as follows. First, if our physical theories (the new system aside) take their current form, defined by initial states and standard evolution laws, then before we examine the data we would assign a high prior weight to the standard Ehrenfest probability law (2), i.e. to $Cw(\underline{N_A}) = 1$ for all paths $\underline{N_A}$. We might assign a lower prior weight to the hypothesis that any modification of the form (3) gives a better theory, and we would almost certainly assign low prior weights to specific modified laws like (5) and (6). But since the system is novel and mysterious, we should and probably would be undogmatic: every specific law L would be assigned a non-zero prior weight $\mathrm{Prob}_{\mathrm{prior}}(L)$.

Suppose that on the first run we observed an evolution of the form of Fig. 1. According to the standard Ehrenfest probability law (2), the probability of equipartition of 10 balls at $t = 10$, given initial equipartition, is $\frac{964533}{1953125} \approx \frac{1}{2}$. The probability of equipartition at both $t = 10$ and $t = 20$, given initial equipartition, is thus $\approx \frac{1}{4}$.

Bayesian hypothesis testing, given data D, assigns the posterior probability weight

$$\mathrm{Prob}_{\mathrm{post}}(L) = \frac{\mathrm{Prob}(D \mid L)\mathrm{Prob}_{\mathrm{prior}}(L)}{\sum_i \mathrm{Prob}(D \mid L_i)\mathrm{Prob}_{\mathrm{prior}}(L_i)}, \tag{7}$$

where the sum is over the set (which we assume countable) of all laws considered.

After the resulting Bayesian reweighting, our posterior weights for some of our modified laws would thus be smaller or zero, and our weight for (5) would (for sensible values of $\mathrm{Prob}_{\mathrm{prior}}(L_i)$) be somewhat larger. If our prior confidence in the law defined by Eq. (2) was high, our posterior confidence would still be high. However, if we saw M runs, all of which produced evolutions with equipartition at $t = 10$ and $t = 20$, the numerator in our posterior weight for (2) will be scaled by $(\frac{1}{4})^M$, while the corresponding expression for Eq. (5) remains unchanged. If the evolutions appear to be otherwise random, then our posterior weights for Eq. (5) increase with M, tending to 1 for large M. In other words, we would eventually become very confident that the system is in fact governed by Eq. (5),

Suppose instead that we saw an evolution of the type illustrated by Fig. 2. According to the standard Ehrenfest urn model, the probability of an evolution as close as these to the line $N_A(t) = (5 - \frac{t}{4})$ is roughly 1 in 50,000. Even after a single run, unless our prior weight for any law other than (2) was significantly less than 2×10^{-5}, we would significantly lose confidence in (2) and begin considering alternative laws seriously. After a small number of runs, we would likely arrive at something like Eq. (6) as our best fit to the data.

Since known physical laws are based on probabilistic or deterministic evolution from initial conditions, we might think a system apparently described by a modified Ehrenfest urn model such as (5) or (6) must very likely have some additional internal mechanism and associated variables hidden from us, so that the complete system is described by a more conventional law. We might then continue to search for ways of observing the hidden variables and obtaining a better and more detailed model. Still, unless and until we succeeded, the relevant modified Ehrenfest urn model would be our best description. And we might not succeed: there need not necessarily be any internal mechanism that gives any deeper explanation.

Formally, these calculations can be underpinned by the theory of Solomonoff induction and the principle of minimum description length (MDL) for hypothesis identification [6]. Roughly speaking, according to the MDL principle, the best hypothesis to fit the data is the one that minimizes the sum of the length of the program required to frame the hypothesis and the length of the string required to characterize the data given the hypothesis. The latter is approximately the Shannon entropy $S(H)$ of the probability distribution on paths in variable space implied by the hypothesis H. The former is the length $L(H)$ of a program mapping $\approx S(H)$ bit strings to paths that, according to hypothesis H, are typical. If H_0 is given by (2), H_1 by (5) and H_2 by (6), then for a single run

$$S(H_0) - S(H_1) \approx 2\,, \qquad S(H_0) - S(H_2) \approx 16. \tag{8}$$

For M runs, $L(H_i)$ is fixed, while

$$S(H_0) - S(H_1) \approx 2M\,, \qquad S(H_0) - S(H_2) \approx 16M. \tag{9}$$

Hence, if H_1 or H_2 fit the data, their description length becomes less than that of H_0 for large M, and they become preferred to H_0; if no more refined hypothesis fits the data then they become the MDL hypothesis. The same is true of any hypothesis H such that $S(H_0) - S(H) > 0$.

3 Discussion

The Ehrenfest urn model illustrates how a model with probabilistic microdynamics can be modified by macrodynamical laws that guide macroscopic variables towards particular paths. Such laws themselves may be either deterministic [as in example (5)] or probabilistic [as in example (6)]. It also illustrates how standard scientific inference can identify such laws, if they offer a sufficiently compressed description of the observed data.

A common initial objection is that hodological laws can explain nothing, because they can describe everything. This confuses the class of all hodological laws with its individual members. The class of hodological laws indeed is defined to include arbitrarily complex laws. For example, one could defined a cosmological law that specifies some initial conditions together with a list of the present observed positions of every observed star and planet. Such a law would indeed be scientifically uninteresting, because it simply lists the observed data rather than explaining it. More formally, such a law would not be selected by the MDL principle, because the program required to frame it includes a very long uncompressed list of data. However, as our toy examples illustrate, the class also includes relatively simple hodological laws that are stated by short programs. If such a law is selected by the MDL principle, it has genuine explanatory power.

The second toy example above illustrates the distinction well. Suppose we observed one of the evolutions displayed in Fig. 2. A hodological law describing precisely the observed evolution would be valid, but uninteresting, unexplanatory, and unsupported by the MDL principle. On the other hand, the hodological law encapsulated in (6) is relatively simple and fits the data well. In this sense it is explanatory, and it would be supported by the MDL principle, in that its credence would be significantly enhanced by the data. If similarly confirmed in a sufficiently long run of trials, it would be effectively selected as the best explanation. A similar law applied to a system with sufficiently many states and a sufficiently long time run would, likewise, be selected in a single trial.

All these points apply when we consider a microdynamics given by any version of quantum theory that makes probabilistic predictions about the microdynamics underpinning the physics of a macroscopic system, including in principle the evolution of the universe. As we have seen, one can model the evolution of a physical system via an Ehrenfest urn model without committing to identifying specific subsystems as balls and urns, or even committing to the belief that such subsystems necessarily exist. Similarly, one can search for modified macrodynamical laws in nature while remaining agnostic about precisely which events, beables, or histories define the fundamental sample space for quantum theory.[1]

[1] Examples of relevant versions of quantum theory include theories with some form of Copenhagen collapse rule, quantum theory supplemented by mass-energy beables determined mathematically by (fictitious) asymptotically late time measurements, a consistent history version of quantum theory defined via some appropriate set selection rule, an one-world version of quantum theory defined by some appropriate selection rule for Everettian branches, or some version of de Broglie-Bohm theory.

The task is more complicated, because there are many more possibly relevant variables and types of law. Nonetheless, the methodology of Solomonoff induction applies.

A systematic search for modified macrodynamical laws that might fit observation better than standard quantum theory should be a major goal of cosmological science, since in a strong sense it would necessarily advance our knowledge. Null results, excluding all laws of a given type up to a given degree of complexity, would solidify and parametrise our confidence in the standard paradigm. New hodological laws that were strongly empirically confirmed would qualitatively and radically change our understanding of nature.

Cosmological models in which initial and final states and unitary evolution laws are all independently fixed ([5], cf. [7]) already provide a clear, if still under-appreciated, counter-example. However, if taken as showing that time-neutral quantum cosmology is the single natural generalisation of standard quantum cosmology, they risk reinforcing the intuition that cosmologies are necessarily defined by boundary conditions and dynamics, against which we have argued here (and previously, e.g. [1]).

The idea that a hodological law could constitute as complete an explanation as possible of some fundamental aspect of physics, such as the evolution of the universe, runs contrary to some strongly held intuitions. I would argue that this is because these intuitions conflate familiar *types* of scientific explanation with the very *definition* of scientific explanation. Our intuitions derive from the types of law or model that have become familiar in classical and quantum physics and in cosmology. In these laws and models, the most complete explanation available is given by a description of the initial state and evolution equations that are time-independent and (if probabilistic) Markovian, or have appropriately analogous properties in the relativistic context. Many scientists do not see the fact that nature is described by laws and models of this type as something that itself needs explanation or indeed that could be further explicable, but instead see it as the necessary basis of science. In fact, though, if the fundamental theory of nature is probabilistic, there would ultimately no more or less of a puzzle if it were described by a simple hodological law than a comparably simple standard law involving an initial state and standard dynamics. In either case, there would be strong regularities that allow the apparently astronomical complexity of the universe to be reduced to comparatively simple rules. In either case, one could ask why those simple rules apply, and whether there is any deeper mechanism that enforces them. No further explanation or deeper mechanism is logically or scientifically necessary, though, in either case.

Rather, on the view argued here, science is ultimately the search for simple hypotheses that most compellingly explain all the available data. The fact that initial condition theories have been so successful should certainly affect our priors at this point. It is presently reasonable to have strong credence that successor theories will have the same form. However, it is not reasonable to elevate this into dogma. Like any other scientific hypothesis, it is open to question, and sufficiently compelling data could make it reasonable to shift credence to alternative hodological hypotheses. The application of Solomonoff induction requires a choice of model, and our illustrative models are admittedly simple. Nonetheless they make the point: with sufficient data, the evidence for hodological models can overcome any finite degree of scepticism.

It is worth mentioning in this context the dark energy problem (see e.g. [8]), a notoriously puzzling feature of cosmological evolution that currently has no compelling theoretical explanation. It is somewhat tempting to speculate that a suitable hodological model might possibly give a more succinct description of the phenomenon than standard models. Any such claim would need a very careful analysis of the relevant data and the associated uncertainties. It would nonetheless be worthwhile to include the possibility in future extensions of Bayesian analyses [9].

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References

1. Kent, A.: Beyond boundary conditions: general cosmological theories. In: COSMO-97, pp. 562–564. World Scientific (1998). (arxiv:0905.0632)
2. Kent, A.: Beable-guided quantum theories: generalizing quantum probability laws. Phys. Rev. A **87**(2), 022105 (2013)
3. Dirac, P.A.M.: Lectures on quantum mechanics, vol. 2. Courier Corporation, Chelmsford (2001)
4. Ehrenfest, P., Ehrenfest, T.: Über zwei bekannte einwände gegen das Boltzmannsche H-Theorem. Physikalische Zeitschrift **8**, 311–314 (1907)
5. Gell-Mann, M., Hartle, J.B.: Time symmetry and asymmetry in quantum mechanics and quantum cosmology. Phys. Orig. Time Asymmetry **1**, 311–345 (1994)
6. Li, M., Vitányi, P., et al.: An introduction to Kolmogorov complexity and its applications, vol. 3. Springer, New York (2008)
7. Gold, T.: The arrow of time. Am. J. Phys. **30**(6), 403–410 (1962)
8. Peebles, P.J.E., Ratra, B.: The cosmological constant and dark energy. Rev. Mod. Phys. **75**, 559–606 (2003). https://doi.org/10.1103/RevModPhys.75.559
9. Lonappan, A.I., Kumar, S., Dinda, B.R., Sen, A.A.: Bayesian evidences for dark energy models in light of current observational data. Phys. Rev. D **97**, 043524 (2018). https://doi.org/10.1103/PhysRevD.97.043524