Math. Ann. (2015) 363:1399–1400
DOI 10.1007/s00208-015-1285-x

Mathematische Annalen

ERRATUM

Erratum to: Local ill-posedness of the incompressible Euler equations in C^1 and $B^1_{\infty,1}$

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Received: 31 July 2015 / Published online: 16 September 2015

Erratum to: Math. Ann. DOI 10.1007/s00208-015-1213-0

The proof of Theorem 3 in the original article relies on the following bound proved by Kato and Ponce [1], Theorem I, for solutions of the incompressible Euler equations

$$\sup_{[0,T]} \|u(t)\|_{s,p} \leq K(T, \|u_0\|_{s,p})$$

for any $T > 0$ and any divergence free vector field $u_0 \in W^{s,p}(\mathbb{R}^2)$ where $s > 1+2/p$ and $1 < p < \infty$. However, a careful examination indicates that the constant K may also depend directly on p itself and as a result our Lemma 4 is insufficient to guarantee that the above bound is uniform in p. Consequently, the statement of Theorem 3 should be amended as follows

Theorem 3 *Let* $2 < p < \infty$. *Assume that the Euler equations* (1.1)–(1.2) *are well-posed in* X. *Let* $\omega_0 \in C_c^\infty(\mathbb{R}^2)$ *be the initial vorticity defined in* (2.2) *below. Then there*

The online version of the original article can be found under doi:10.1007/s00208-015-1213-0.

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exists $T > 0$ *and a sequence of initial vorticities* $\omega_{0,n} \in C_c^\infty(\mathbb{R}^2)$ *with the following properties, either*

1. *there exists a constant* $C > 0$ *such that* $\|\omega_{0,n}\|_{W^{1,p}} \leq C$ *for all* $n \in \mathbb{Z}_+$ *and*
2. *for any* $M \gg 1$ *there is* $0 < t_0 \leq T$ *such that the solution* $\omega_n(t)$ *of the vorticity equations* (1.4)–(1.5) *with initial data* $\omega_{0,n}$ *satisfies* $\|\omega_n(t_0)\|_{W^{1,p}} \geq M^{1/3}$ *for all sufficiently large* n *and all* p *sufficiently close to* 2 *or we have* $\sup_{0\leq t\leq T} \|\omega_n(t)\|_{W^{s-1,p}} \to \infty$ *as* $p \searrow 2$.

and Theorems 1 and 2 should be reformulated accordingly

Theorem 1 *The 2D incompressible Euler equations* (1.1) *are locally ill-posed in the space* C^1 *provided that the bound* $\sup_{0\leq t\leq T} \|\omega(t)\|_{W^{s-1,p}} \leq K$ *is uniform as* $p \searrow 2$.

Theorem 2 *The 2D incompressible Euler equations are locally ill-posed in the Besov space* $B^1_{\infty,1}$ *provided that the bound* $\sup_{0\leq t\leq T} \|\omega(t)\|_{W^{s-1,p}} \leq K$ *is uniform as* $p \searrow 2$.

Lastly, we observe that if we could find a suitable function space $X \subset C^1$ in which the Euler equations are locally well-posed and the estimates of Lemmas 10, 11 and 12 hold (with appropriate modifications) then the last condition of Theorem 3 would follow. We do not yet know whether this is the case.

Reference

1. Kato, T., Ponce, G.: On nonstationary flows of viscous and ideal fluids in $L^p_s(\mathbb{R}^2)$. Duke Math. J. **55**, 487–499 (1987)