

Erratum to: Normal subgroups of the algebraic fundamental group of affine curves in positive characteristic

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Let k be an algebraically closed field of characteristic $p > 0$ of countable cardinality. Denote by C a smooth, connected, affine curve defined over k with function field K and by $\pi_1(C)$ its algebraic fundamental group. In the original paper we addressed the question of the structure of the normal closed subgroups of $\pi_1(C)$. More precisely, in Theorem 1.1, we gave a sufficient condition for such a normal closed subgroup N to be isomorphic to a subgroup of a free profinite group of countable rank. There we claimed to prove this under the hypothesis that the group $M = \pi_1(C)/N$ had an infinitely generated Sylow p -subgroup.

However, as remarked in the paragraph after Corollary 1.2 of [1], this hypothesis is not enough. We would like to make this remark more explicit by stating (as noticed by those authors) the additional hypothesis needed to make Theorem 1.1 work. For each integer $g \geq 0$ denote by $P_g(C)$ the intersection of algebraic fundamental groups

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of étale \mathbb{Z}/p -covers $Z \rightarrow C$ such that the genus of the smooth compactification of Z is at least g . This is a subgroup of $\pi_1(C)$. Recall that if \bar{C} denotes the smooth compactification of C and $S = \bar{C} \setminus C$, then $\pi_1(C)$ is isomorphic to $\text{Gal}(K_{\text{un},S}/K)$, where $K_{\text{un},S}$ denotes the maximal Galois extension of K which is étale over C . In this context the subfield $K_g(C)$ of $K_{\text{un},S}$ corresponding to $P_g(C)$ determines an pro-cover $Z_g(C) \rightarrow C$ which dominates every étale \mathbb{Z}/p -cover of C .

Hypothesis 1 Suppose that the normal subgroup N is contained in $P_g(C)$ for some integer $g \geq 0$.

Denote by K_M the subfield of $K_{\text{un},S}$ fixed by N . Under the previous hypothesis we have $K_g(C) \subset K_M$. We change the construction of Lemma 4.5 in the original paper by that of [1, Lemma 2.2]. This gives a \mathbb{Z}/p -cover $T \times_{k[[t]]} k((t))$ of $\bar{C} \times_k k((t))$ which is étale over C . In particular, specializing it to the closed fiber, this will give a \mathbb{Z}/p -cover T_s of \bar{C} which is étale over C . The algebraic fundamental group of this curve and its map to G' and Γ' will give a solution to the embedding problem (4.4) in the original paper. Furthermore, in contrast with the solution obtained in the original article, the function field of T_s will be contained in K_M (since K_M contains $K_g(C)$ for some integer $g \geq 0$). This fixes the gap in the original article.

Observe finally that we could have proved Theorem 1.1 in the original paper without the hypothesis on the Sylow p -subgroup, but with the previous hypothesis. This is covered by [1, Corollary 1.2].

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Reference

1. Bary-Soroker, L., Kumar, M.: Subgroup structure of fundamental groups in positive characteristic. *Commun. Algebra* **41**, 3705–3719 (2013)