ORIGINAL RESEARCH

Experimental and finite element studies on free vibration of skew plates

C. V. Srinivasa · Y. J. Suresh · W. P. Prema Kumar

Abstract The present paper deals with the experimental and finite element studies carried out on free vibration of isotropic and laminated composite skew plates. The natural frequencies were determined using CQUAD8 finite element of MSC/NASTRAN and comparison made between the experimental values and the finite element solution. The effects of the skew angle and aspect ratio on the natural frequencies of isotropic skew plates were studied. The effects of skew angle, aspect ratio, fiber orientation angle and laminate stacking sequence (keeping total number of layers in the laminate constant) on the natural frequencies of antisymmetric laminated composite skew plates were also studied. The experimental values of the natural frequencies are in good agreement with the finite element solution. The natural frequencies generally increase with an increase in the skew angle for any given value of aspect ratio.

Keywords Skew plate · Natural frequency · Aspect ratio · Fibre orientation angle · Finite element analysis

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List of symbols

a	Plate length
b	Plate width
t	Plate thickness
NL	Number of layers in the laminate
E	Modulus of elasticity of the material of isotropic plate
E_1	Young's modulus of the lamina in the longitudinal direction
E_2	Young's modulus of the lamina in the transverse direction
G_{12}	In-plane shear modulus of the lamina
α	Skew angle of the plate
θ	Fiber orientation angle of the lamina
μ	Poisson's ratio of the material of isotropic plate
v_{12}	Major Poisson's ratio of the lamina
ρ	Mass density of the material of the plate
ρ_1	Mass density per unit area
D	Flexural rigidity of isotropic plate, $Et^3/12(1 - \mu^2)$
ω	Natural angular frequency of plate
K_f	Non-dimensional frequency coefficient

Introduction

The skew plates have wide application in civil, marine, aeronautical and mechanical engineering, some of them being wings of aeroplanes, skew bridges, ship hulls and parallelogram slabs in buildings. The exact solutions to free vibration problems of skew plates are mathematically involved and in many complicated cases not available. Hence, most of the available solutions are based on approximate methods, the most commonly used method being the finite element analysis.

The earlier studies on free vibration characteristics of skew plates are those of Barton (1951), Kaul and Cadambe (1956) and Hasegawa (1957) using Rayleigh–Ritz method. Hamada (1959) applied the Lagrangian multiplier technique to obtain the fundamental frequency of rhombic skew plate. Classen (1963) extended the work of Barton (1951) by adopting a Fourier sine series solution scheme in conjunction with the Rayleigh–Ritz method. Conway and Farnham (1965) employed the point matching method to study the free vibration of triangular, rhombic and parallelogram plates. The frequencies were calculated for different values of skew angle for both simply supported and clamped boundary conditions. Laura and Grosson (1968) obtained fundamental frequencies of vibration for simply supported rhombic plates using conformal mapping and Galerkin's method and compared their results with those of Conway and Farnham (1965). The discrepancy has been observed to increase with the skew angle.

Monforton (1968) obtained the fundamental frequencies of clamped rhombic plates using finite element method. The frequencies and mode shapes of clamped skew plates were studied by Durvasula (1969) using Galerkin's method. The deflection function was expressed as a double series of beam characteristic functions in terms of skew coordinates to satisfy the zero deflection and normal slope on all the edges of the plate. The results obtained were compared with the results of Kaul and Cadambe (1956), Hasegawa (1957) and Conway and Farnham (1965). Thangam Babu and Reddy (1971) investigated the free vibration characteristics of orthotropic skew plates with two opposite edges simply supported and the other two edges free. Nair and Durvasula (1973) reported the natural frequencies of isotropic and orthotropic skew plates for simply supported, clamped, free edge boundary conditions and also for combinations of these conditions. Srinivasan and Ramachandran (1975) employed a numerical method to obtain the natural frequencies and mode shapes of orthotropic skew plates. Kuttler and Sigillito (1980) used trial function method to solve the free vibration problem of skew plates.

Mizusuwa et al. (1979, 1980) and Mizusuwa and Kajita (1986, 1987) employed the Rayleigh–Ritz method with B-spline functions and studied the effects of skew angle and location of point supports on natural frequencies of isotropic skew plates. Liew and Lam (1990) used a set of 2-D orthogonal plate functions as admissible deflection functions and studied the flexural vibration of skew plates using Rayleigh–Ritz method. They obtained the free vibration characteristics of rhombic plates with different boundary conditions. Bardell (1992) adopted the hierarchical finite element method to determine the natural frequencies and mode shapes of isotropic skew plates. Liew and Wang (1993) employed the Rayleigh–Ritz method and

obtained the results for rhombic plates with different boundary conditions by varying the location of internal support, skew angle and aspect ratio. Singh and Chakraverthy (1994) evaluated the first five frequencies of the transverse vibration of skew plates under different boundary conditions using orthogonal polynomials. McGee and Butalia (1994) studied the free vibration of thick and thin cantilever skew plates using C^0 continuous isoparametric quadrilateral element.

Kamal and Durvasula (1986) studied the free vibration characteristics of laminated composite plates using modified shear deformation layered composite theory and Rayleigh–Ritz energy approach. Malhotra et al. (1988) studied the free vibration characteristics of rhombic orthotropic plates using a parallelogram orthotropic plate finite element for various boundary conditions and skew angles. Krishnan and Deshpande (1992) employed DKT finite element to determine the effects of fiber orientation angle, skew angle, aspect ratio and length-to-thickness ratio on the fundamental frequencies of single layer Graphite/Epoxy and Glass/Epoxy skew plates.

Krishna Reedy and Palaninathan (1999) used a high precision triangular plate bending element to study the free vibration characteristics of laminated composite skew plates. A consistent mass matrix has been used in the study. Singha and Ganapathi (2004) studied the free flexural vibration characteristics of laminated composite skew plates using finite element approach. Garg et al. (2006) have carried out free vibration studies on isotropic, orthotropic, and layered anisotropic composite and sandwich skew plates using isoparametric finite element model.

Clary (1975) investigated theoretically and experimentally the effect of fiber orientation on the first five flexural modes of free vibration of rectangular unidirectional Boron/Epoxy panels. The theoretical and experimental values of frequencies were in good agreement except in the case of thinner panels where there was appreciable error. Cawley and Adams (1978) used finite element method which included the effect of transverse shear deformation to predict the natural modes of free-free CFRP plates. Maruyama et al. (1983) used the real-time technique of holographic interferometry to determine the natural frequencies and the corresponding mode shapes of transverse vibration of clamped trapezoidal plates. Chakraborty et al. (2000) made studies on free vibration response of FRP composite plates using experimental and numerical techniques. Dutt and Shivanand (2011) studied the free vibration response of woven carbon composite laminated plates with C–F–F–F (one edge clamped and the other three free) and C–F–C–F (two opposite edges clamped and the other two edges free) boundary conditions using a FFT analyzer and compared the results obtained with finite element solution. Today the skew plate problem is widely

recognized as a benchmark in the testing of a newly developed finite element. Very few experimental studies have been made on laminated composite skew plates and the finite element solution experimentally validated. The present paper is an attempt to address this aspect at least in a partial manner.

Test specimens and experimental set-up

Test specimens

In this study, isotropic plates made of Aluminum 7075-T6 were used. The material was supplied by the Rio-Tinto Alcon, Canada (Pechiney Aluminum, France). The laminated composite plate specimens were fabricated by hand lay-up technique using unidirectional Glass fibers, Epoxy-556 Resin and the Hardener (HY951) supplied by Ciba-Geigy India Ltd. The fiber weight percentage is 50:50. The test specimens were prepared in accordance with the relevant ASTM provisions. The material properties of the isotropic plates made of Aluminum 7075-T6 are: $E = 71.7$ GPa, $\mu = 0.33$ and $\rho = 2,800$ kg/m^3 and these were supplied by the manufacturer Rio-Tinto Alcon, Canada. For laminated Glass/Epoxy composite plate, the material constants E_1 and E_2 were evaluated experimentally using Instron Universal Testing Machine as per ASTM Standard D 3039/D 3039 M (2006). The average of three experimental determinations was adopted. For the determination of Poisson's ratio υ_{12}, two strain gages were bonded to the specimen, one in the direction of the loading and the other at right angles to it. The strains were measured in longitudinal and transverse directions using strain indicator. The ratio of transverse to longitudinal strain gives the Poisson's ratio within the elastic range. The average of three experimental determinations was adopted. The shear modulus G_{12} was computed using standard expression available in Jones (1975). The adopted material properties are: $E_1 = 38.07$ GPa, $E_2 = 8.1$ GPa, $G_{12} = 3.05$ GPa, $\upsilon_{12} = 0.22$, $\rho = 2,200$ kg/m^3. The aspect ratio of the test specimens was varied from 1.0 to 2.5.

Experimental set-up

The experimental set-up is shown in Fig. 1. First, the test specimen was placed in the fixture shown in Fig. 1 with two opposite edges fully clamped and the remaining two edges completely free. The piezoelectric accelerometer was directly mounted on the test specimen at the geometric center using an adhesive. The accelerometer was then connected to signal-conditioning unit (Fast Fourier Transform Analyzer), where the signal goes through the charge amplifier and an Analog-to-Digital Converter. The plate

Fig. 1 Experimental Set-up

was excited in a selected point by means of impact hammer. The impact hammer was struck at the selected point five times and the average value of the frequency response function (FRF) was input to the computer through an USB port. Sufficient precautions were taken for ensuring that the strike of the impact hammer was normal to the surface of the plate. The pulse lab software accompanying the equipment was used for recording the signals directly in the memory of the computer. The signal was then read and processed to extract different features including frequencies. The frequencies were measured by moving the cursor to the peaks of the FRF. Five separate experimental determinations were done for the natural frequency of each specimen and then the average value was adopted.

Finite element solution

A finite element analysis was made for obtaining the first three natural frequencies using MSC/NASTRAN software. CQUAD8 (eight-noded isoparametric curved shell element) was employed as it yields better results compared to CQUAD4 element of the said software as revealed by the investigation reported by Srinivasa et al. (2012). Several options exist in the software in respect of real eigenvalue extraction, and the Lanczos method was used in the present study as it combines the best features of the other methods and computes accurate eigenvalues and eigenvectors.

Results and discussion

The results of the present work are presented in tables and figures in terms of non-dimensional frequency coefficient (K_f) defined by $K_f = \frac{\omega a^2}{\pi^2} \sqrt{\frac{\rho t}{D}}$ for isotropic plates and by $K_f = \omega a^2 \sqrt{\frac{\rho_1}{E_2 t^3}}$ for laminated composite plates (Srinivasa et al. 2013).

Table 1 Non-dimensional frequency coefficient for isotropic skew plates

Aspect ratio (a/b)	Mode number	Non-dimensional frequency coefficient (K_f)							
		Skew angle (α)							
		0°		15°		30°		45°	
		Experiment	FEM	Experiment	FEM	Experiment	FEM	Experiment	FEM
1.0	1	2.978 (0.10)	3.102	3.216 (0.11)	3.265	3.732 (0.12)	3.822	4.946 (0.15)	5.066
	2	3.518 (0.12)	3.665	3.735 (0.13)	3.792	4.133 (0.14)	4.233	5.136 (0.15)	5.260
	3	5.795 (0.15)	6.036	6.123 (0.16)	6.216	6.708 (0.17)	6.870	8.383 (0.18)	8.585
1.5	1	3.028 (0.11)	3.080	3.185 (0.11)	3.241	3.548 (0.12)	3.711	4.833 (0.13)	4.761
	2	4.169 (0.13)	4.241	4.313 (0.12)	4.388	4.587 (0.13)	4.798	5.832 (0.14)	5.779
	3	8.343 (0.17)	8.487	8.817 (0.18)	8.970	9.519 (0.19)	9.957	11.483 (0.20)	11.361
2.0	1	3.001 (0.11)	3.071	3.151 (0.11)	3.206	3.538 (0.12)	3.602	4.342 (0.13)	4.418
	2	4.825 (0.12)	4.939	5.000 (0.14)	5.087	5.433 (0.15)	5.532	6.484 (0.15)	6.596
	3	8.270 (0.16)	8.465	8.747 (0.15)	8.898	9.998 (0.16)	10.180	12.554 (0.19)	12.771
2.5	1	3.006 (0.10)	3.064	3.130 (0.10)	3.181	3.417 (0.11)	3.512	4.052 (0.12)	4.150
	2	5.591 (0.12)	5.699	5.764 (0.12)	5.858	6.168 (0.13)	6.339	7.331 (0.13)	7.508
	3	8.281 (0.14)	8.442	8.709 (0.14)	8.850	9.749 (0.15)	10.019	11.972 (0.16)	12.260

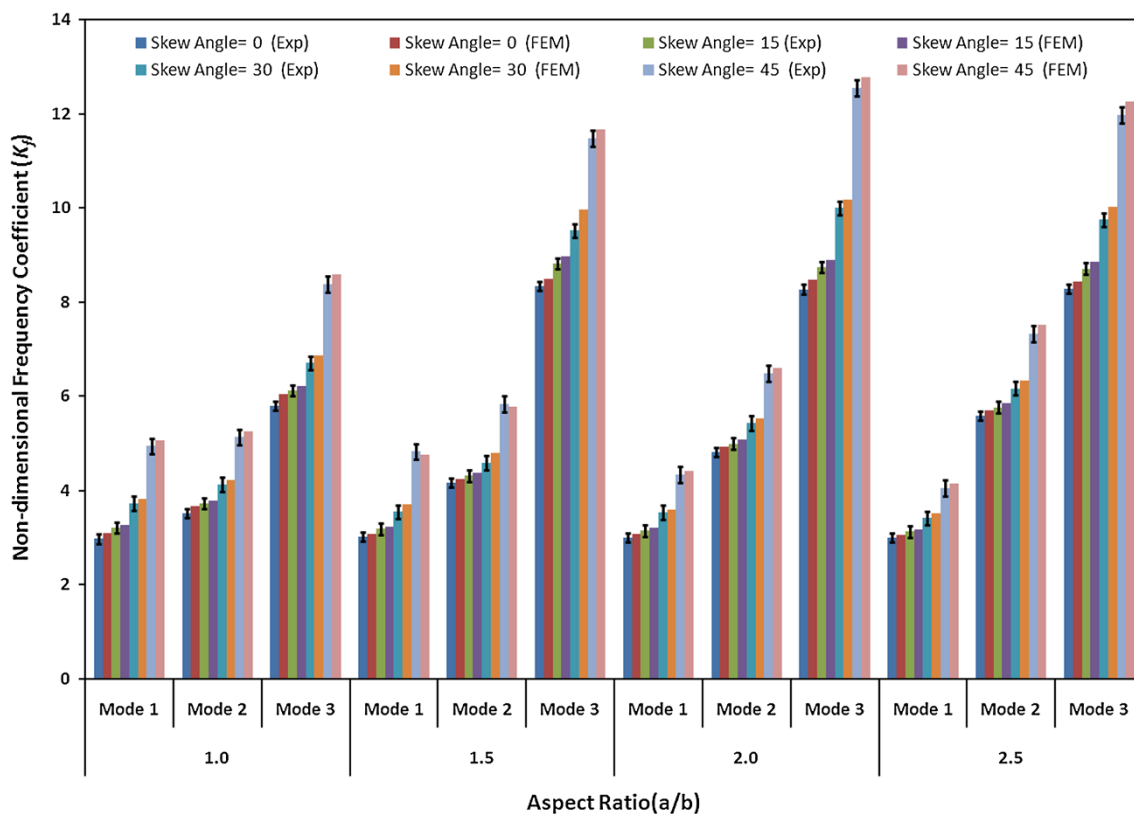The number in parentheses represents the standard deviation

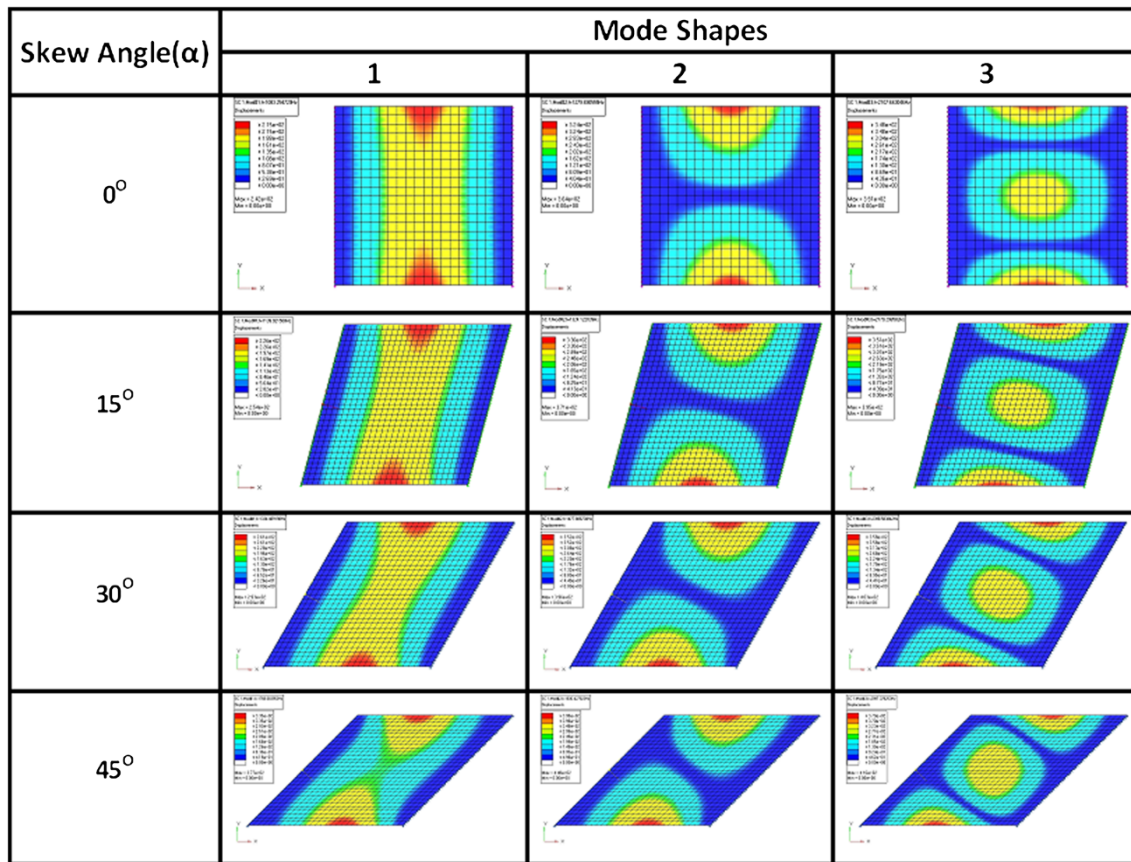**Fig. 2** Variation of K_f with aspect ratio (a/b) for isotropic skew plates

Fig. 3 Mode shapes for isotropic skew plates ($a/b = 1.0$)

Table 2 Non-dimensional frequency coefficient for laminated composite skew plates ($\alpha = 0^o$)

Laminate staking sequence	Mode number	Non-dimensional frequency coefficient (K_f)							
		Aspect ratio (a/b)							
		1.0		1.5		2.0		2.5	
		Experiment	FEM	Experiment	FEM	Experiment	FEM	Experiment	FEM
Angle-ply [+0°/−0°/…/−0°]	1	15.304 (0.13)	15.617	15.180 (0.11)	15.649	14.983 (0.40)	15.607	14.824 (0.40)	15.604
	2	15.990 (0.10)	16.316	16.683 (0.23)	17.199	17.536 (0.13)	18.267	18.613 (0.34)	19.593
	3	19.435 (0.12)	19.832	26.194 (0.15)	27.004	36.684 (0.90)	38.213	40.859 (1.39)	43.010
Angle-ply [+45°/−45°/…/−45°]	1	9.454 (0.13)	9.647	9.260 (0.16)	9.547	9.039 (0.12)	9.416	8.855 (0.12)	9.321
	2	12.851 (0.08)	13.114	15.880 (0.12)	16.371	19.119 (0.34)	19.916	22.488 (0.30)	23.671
	3	22.684 (0.34)	23.147	25.673 (0.12)	26.467	25.069 (0.45)	26.114	24.548 (0.90)	25.840
Angle-ply [+90°/−90°/…/−90°]	1	7.055 (0.07)	7.199	6.996 (0.09)	7.213	6.905 (0.12)	7.193	6.831 (0.11)	7.191
	2	8.448 (0.18)	8.620	9.818 (0.08)	10.121	11.356 (0.22)	11.830	13.021 (0.29)	13.707
	3	19.399 (0.20)	19.795	19.283 (0.018)	19.879	19.031 (0.34)	19.824	18.827 (0.02)	19.818
Cross-ply [0°/90°/…/90°]	1	11.901 (0.16)	12.144	11.805 (0.20)	12.170	11.654 (0.12)	12.139	11.530 (0.34)	12.137
	2	12.777 (0.22)	13.038	13.686 (0.21)	14.109	14.784 (0.25)	15.400	16.090 (0.21)	16.937
	3	19.420 (0.25)	19.817	32.410 (0.30)	33.412	32.120 (0.33)	33.458	31.780 (1.25)	33.452

The number in parentheses represents the standard deviation

Table 3 Non-dimensional frequency coefficient for laminated composite skew plates ($\alpha = 15^\circ$)

Laminate staking sequence	Mode number	Non-dimensional frequency coefficient (K_f)							
		Aspect ratio (a/b)							
		1.0		1.5		2.0		2.5	
		Experiment	FEM	Experiment	FEM	Experiment	FEM	Experiment	FEM
Angle-ply [$+0^\circ/-0^\circ/.../-0^\circ$]	1	15.346 (0.05)	15.821	15.350 (0.20)	15.990	14.997 (0.30)	15.787	14.495 (0.45)	15.420
	2	16.103 (0.15)	16.602	16.502 (0.15)	17.190	17.739 (0.12)	18.672	18.852 (0.56)	20.056
	3	19.876 (0.03)	20.491	26.997 (0.11)	28.122	37.684 (0.18)	39.668	40.917 (1.98)	43.528
Angle-ply [$+45^\circ/-45^\circ/.../-45^\circ$]	1	10.235 (0.07)	10.552	9.688 (0.06)	10.092	9.498 (0.12)	9.998	9.253 (0.90)	9.844
	2	13.044 (0.03)	13.447	16.013 (0.03)	16.680	18.952 (0.40)	19.949	22.047 (1.00)	23.455
	3	22.592 (0.07)	23.291	26.898 (0.08)	28.019	27.056 (0.35)	28.479	25.625 (1.08)	27.260
Angle-ply [$+90^\circ/-90^\circ/.../-90^\circ$]	1	7.368 (0.03)	7.596	7.253 (0.10)	7.556	7.107 (0.10)	7.481	6.976 (0.08)	7.421
	2	8.909 (0.11)	9.184	10.238 (0.11)	10.664	11.722 (0.20)	12.339	13.660 (0.45)	14.532
	3	18.957 (0.02)	19.544	20.157 (0.14)	20.996	19.726 (0.22)	20.765	19.583 (0.08)	20.833
Cross-ply [$0^\circ/90^\circ/.../90^\circ$]	1	12.032 (0.09)	12.404	12.099 (0.18)	12.603	11.740 (0.23)	12.358	11.480 (0.34)	12.212
	2	13.092 (0.02)	13.497	14.051 (0.09)	14.636	15.159 (0.18)	15.957	17.075 (0.88)	18.165
	3	20.156 (0.11)	20.779	31.759 (1.05)	33.082	32.392 (1.57)	34.097	32.422 (1.55)	34.492

The number in parentheses represents the standard deviation

Table 4 Non-dimensional frequency coefficient for laminated composite skew plates ($\alpha = 30^\circ$)

Laminate staking sequence	Mode number	Non-dimensional frequency coefficient (K_f)							
		Aspect ratio (a/b)							
		1.0		1.5		2.0		2.5	
		Experiment	FEM	Experiment	FEM	Experiment	FEM	Experiment	FEM
Angle-ply [$+0^\circ/-0^\circ/.../-0^\circ$]	1	16.014 (0.09)	16.595	15.812 (0.33)	16.557	15.543 (0.23)	16.448	15.314 (0.32)	16.379
	2	16.999 (0.08)	17.616	17.909 (0.25)	18.753	18.955 (0.25)	20.058	20.203 (0.40)	21.608
	3	21.958 (0.35)	22.755	30.296 (0.85)	31.723	41.246 (1.24)	43.646	42.345 (0.90)	45.289
Angle-ply [$+45^\circ/-45^\circ/.../-45^\circ$]	1	12.972 (0.21)	13.442	12.003 (0.30)	12.569	11.117 (0.12)	11.764	10.461 (0.18)	11.188
	2	14.369 (0.25)	14.890	16.867 (0.50)	17.662	19.881 (0.45)	21.038	22.708 (0.20)	24.286
	3	23.764 (0.40)	24.626	33.427 (1.20)	35.002	33.081 (1.34)	35.006	32.189 (0.45)	34.426
Angle-ply [$+90^\circ/-90^\circ/.../-90^\circ$]	1	8.892 (0.15)	9.214	8.474 (0.21)	8.873	8.079 (0.15)	8.549	7.780 (0.08)	8.321
	2	10.655 (0.25)	11.042	11.900 (0.20)	12.461	13.312 (0.12)	14.087	14.902 (0.30)	15.938
	3	20.408 (0.40)	21.149	23.573 (0.40)	24.683	22.761 (0.33)	24.085	22.164 (0.35)	23.705
Cross-ply [$0^\circ/90^\circ/.../90^\circ$]	1	13.020 (0.22)	13.492	12.778 (0.30)	13.380	12.488 (0.28)	13.215	12.242 (0.15)	13.093
	2	14.555 (0.20)	15.082	15.690 (0.35)	16.429	16.832 (0.30)	17.811	18.175 (0.24)	19.439
	3	23.111 (0.35)	23.949	34.105 (1.00)	35.712	34.522 (0.70)	36.531	33.954 (0.80)	36.314

The number in parentheses represents the standard deviation

Isotropic skew plates

The thickness of the isotropic skew plates was taken as 2.0 mm. The aspect ratio was varied from 1.0 to 2.5 and the skew angle from 0° to 45°. Table 1 and Fig. 2 show the variation of non-dimensional frequency coefficient K_f with aspect ratio and skew angle for isotropic skew plates. The variation of first natural frequency with an increase in the aspect ratio is not appreciable up to a skew angle of about

30°. The variation of second and third natural frequencies with an increase in the aspect ratio for a given skew angle is considerable. The first, second and third natural frequencies increase monotonically with an increase in the skew angle for any given value of aspect ratio. The experimental values of the non-dimensional frequency coefficient K_f are in very good agreement with those of the finite element solution. The first three mode shapes obtained by finite element analysis are shown in Fig. 3 for aspect ratio = 1.0.

Table 5 Non-dimensional frequency coefficient for laminated composite skew plates ($\alpha = 45^\circ$)

Laminate staking sequence	Mode number	Non-dimensional frequency coefficient (K_f)							
		Aspect ratio (a/b)							
		1.0		1.5		2.0		2.5	
		Experiment	FEM	Experiment	FEM	Experiment	FEM	Experiment	FEM
Angle-ply $[+0^\circ/-0^\circ/\ldots/-0^\circ]$	1	18.149 (0.20)	18.711	17.719 (0.19)	18.457	17.593 (0.22)	18.137	17.515 (0.11)	17.872
	2	19.284 (0.28)	20.087	20.491 (0.22)	21.570	21.794 (0.45)	23.186	23.288 (0.67)	25.040
	3	26.964 (0.50)	28.087	37.360 (0.80)	39.327	46.893 (2.00)	49.887	46.186 (1.85)	49.662
Angle-ply $[+45^\circ/-45^\circ/\ldots/-45^\circ]$	1	17.949 (0.22)	18.697	16.312 (0.15)	17.171	14.077 (0.33)	14.975	12.561 (0.34)	13.506
	2	18.365 (0.31)	19.130	19.842 (0.25)	20.886	23.149 (0.45)	24.627	26.238 (0.80)	28.213
	3	28.674 (0.54)	29.869	37.477 (1.25)	39.449	43.781 (1.29)	46.576	40.812 (1.56)	43.884
Angle-ply $[+90^\circ/-90^\circ/\ldots/-90^\circ]$	1	13.011 (0.16)	13.554	11.513 (0.12)	12.119	10.253 (0.13)	10.907	9.411 (0.15)	10.119
	2	14.626 (0.13)	15.235	15.792 (0.14)	16.623	17.033 (0.35)	18.121	18.577 (0.40)	19.976
	3	25.705 (0.60)	26.776	30.856 (1.10)	32.480	29.319 (0.90)	31.190	27.562 (0.77)	29.636
Cross-ply $[0^\circ/90^\circ/\ldots/90^\circ]$	1	16.062 (0.17)	16.731	15.304 (0.25)	16.109	14.547 (0.32)	15.475	13.928 (0.33)	14.976
	2	17.989 (0.18)	18.739	19.401 (0.32)	20.422	20.551 (0.40)	21.863	21.952 (0.50)	23.604
	3	29.705 (0.55)	30.943	39.355 (0.75)	41.427	40.157 (0.95)	42.721	38.893 (1.60)	41.820

The number in parentheses represents the standard deviation
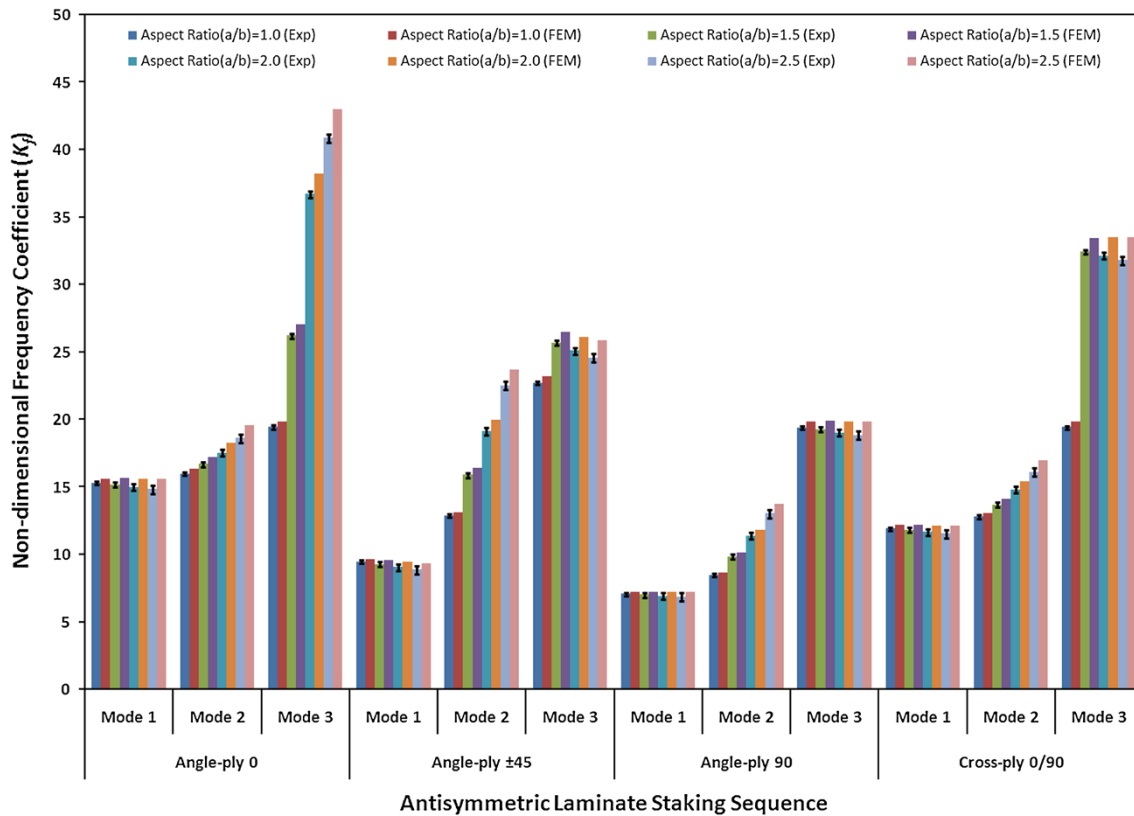

Fig. 4 Variation of K_f with aspect ratio (a/b) for different laminate stacking sequences ($\alpha = 0^\circ$)
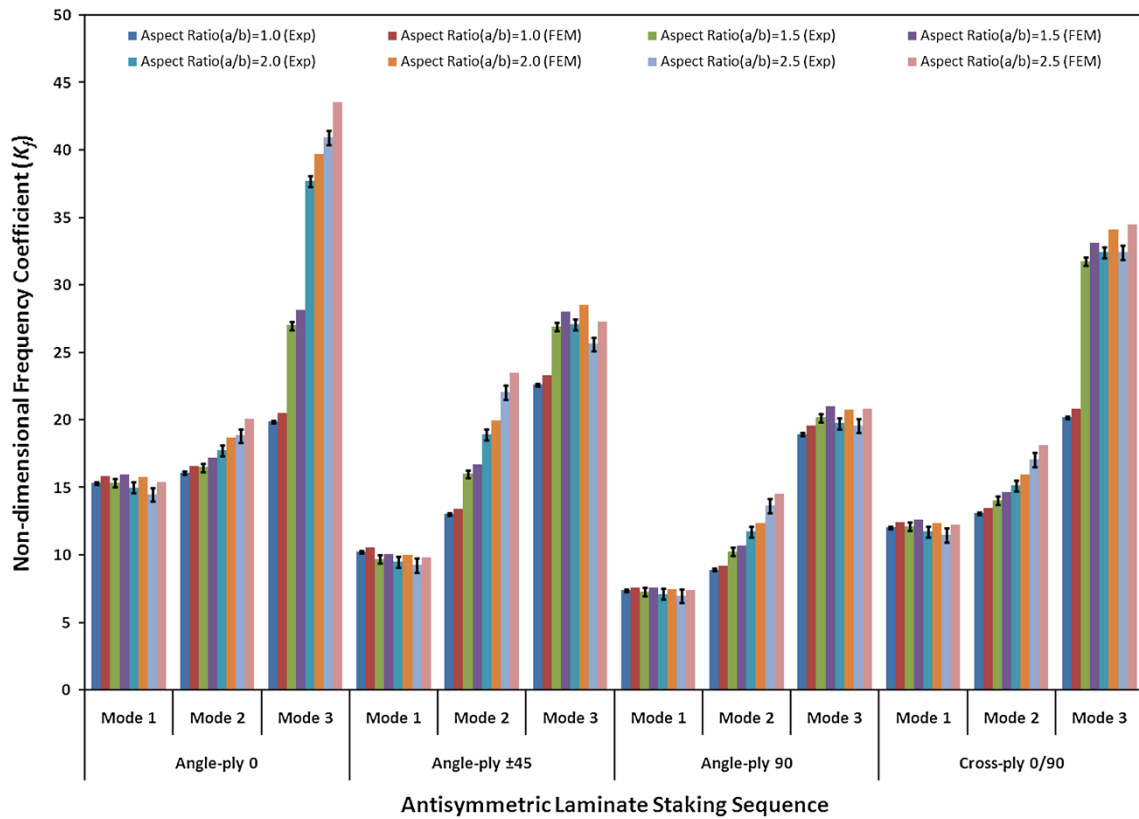
Fig. 5 Variation of K_f with aspect ratio (a/b) for different laminate stacking sequences ($\alpha = 15^{\circ}$)
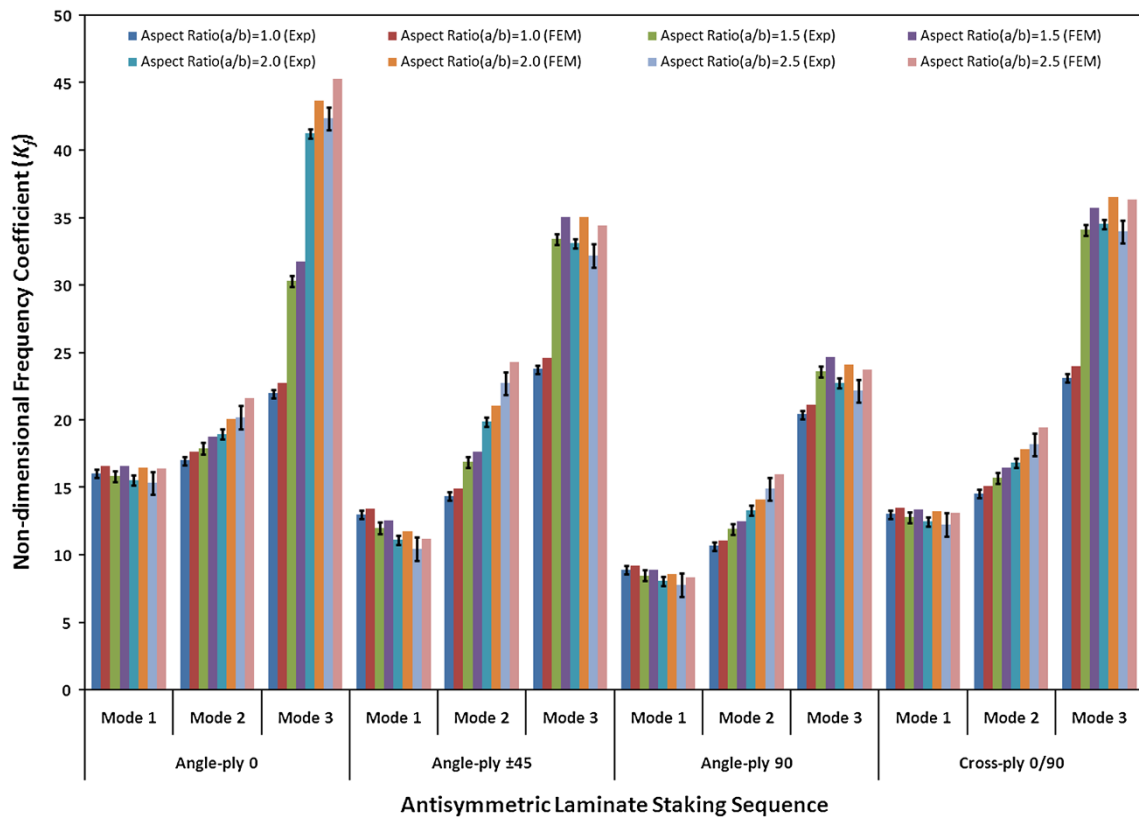

Fig. 6 Variation of K_f with aspect ratio (a/b) for different laminate stacking sequences ($\alpha = 30^{\circ}$)
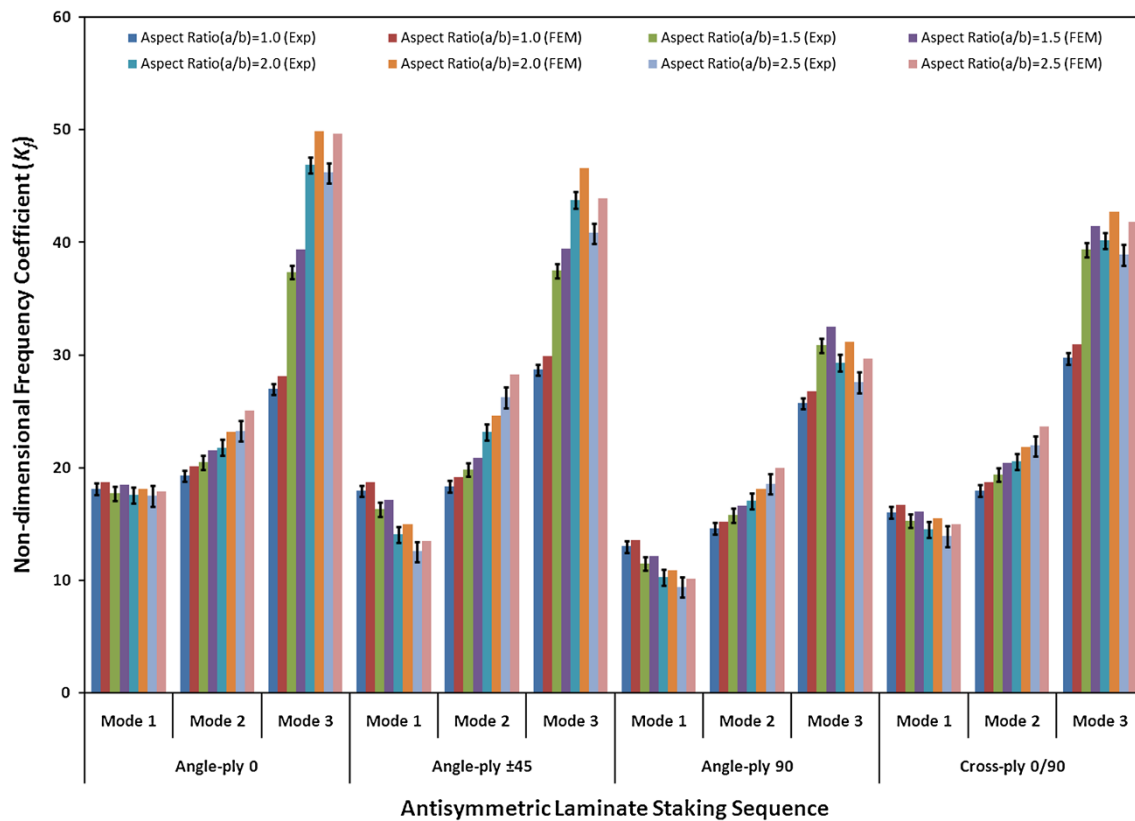
Fig. 7 Variation of K_f with aspect ratio (a/b) for different laminate stacking sequences ($\alpha = 45^{\circ}$)

Laminated composite skew plates

The total thickness of the laminate was maintained constant at 2 mm, the number of layers being 20. The aspect ratio was varied from 1.0 to 2.5 and the skew angle from 0° to 45°. Tables 2, 3, 4, 5 and Figs. 4, 5, 6, 7 show the variation of non-dimensional frequency coefficient K_f with aspect ratio and laminate stacking sequence for various values of skew angle. Four laminate stacking sequences viz., antisymmetric angle-ply $[+0^{\circ}/-0^{\circ}/\ldots/-0^{\circ}]$, antisymmetric angle-ply $[+45^{\circ}/-45^{\circ}/\ldots/-45^{\circ}]$, antisymmetric angle-ply $[+90^{\circ}/-90^{\circ}/\ldots/-90^{\circ}]$ and antisymmetric cross-ply $[0^{\circ}/90^{\circ}/\ldots/90^{\circ}]$ were considered. It is seen that the first natural frequency decreases with an increase in the aspect ratio for all laminate stacking sequences. The second natural frequency increases with an increase in the aspect ratio for all laminate stacking sequences. For laminate stacking sequence $[+0^{\circ}/-0^{\circ}/\ldots/-0^{\circ}]$, the third natural frequency increases with an increase in the aspect ratio except in the case of skew angle = 45° where it increases up to aspect ratio = 2.0 and later decreases. For laminate stacking sequence $[+45^{\circ}/-45^{\circ}/\ldots/-45^{\circ}]$, the third natural frequency first increases and then decreases with an increase in the aspect ratio for all skew angles. For laminate stacking

sequence $[+90^{\circ}/-90^{\circ}/\ldots/-90^{\circ}]$, the third natural frequency decreases monotonically with an increase in the aspect ratio for skew angle = 0 whereas for other skew angles, it first increases and then decreases. For laminate stacking sequence $[0^{\circ}/90^{\circ}/\ldots/90^{\circ}]$, the third natural frequency first increases and later decreases with an increase in the aspect ratio for all the skew angles considered. Other parameters being the same, the antisymmetric angle-ply $[+0^{\circ}/-0^{\circ}/\ldots/-0^{\circ}]$ yields the highest value for the first natural frequency and the antisymmetric angle-ply $[+90^{\circ}/-90^{\circ}/\ldots/-90^{\circ}]$ yields the lowest value. The first natural frequency for the other laminate stacking sequences lies in between the above two extreme values. The first, second and third natural frequencies generally increase with an increase in the skew angle for any given value of aspect ratio for all the laminate stacking sequences. The first three mode shapes obtained by finite element analysis are shown in Fig. 8 for antisymmetric cross-ply $[0^{\circ}/90^{\circ}/\ldots/90^{\circ}]$ for aspect ratio = 1.0. The natural frequency depends on the stiffness of the cross-section of the laminate among other factors such as boundary conditions etc. The stiffness of the cross-section depends upon the contribution made by extensional stiffness, coupling stiffness and bending stiffness terms (Jones 1975).
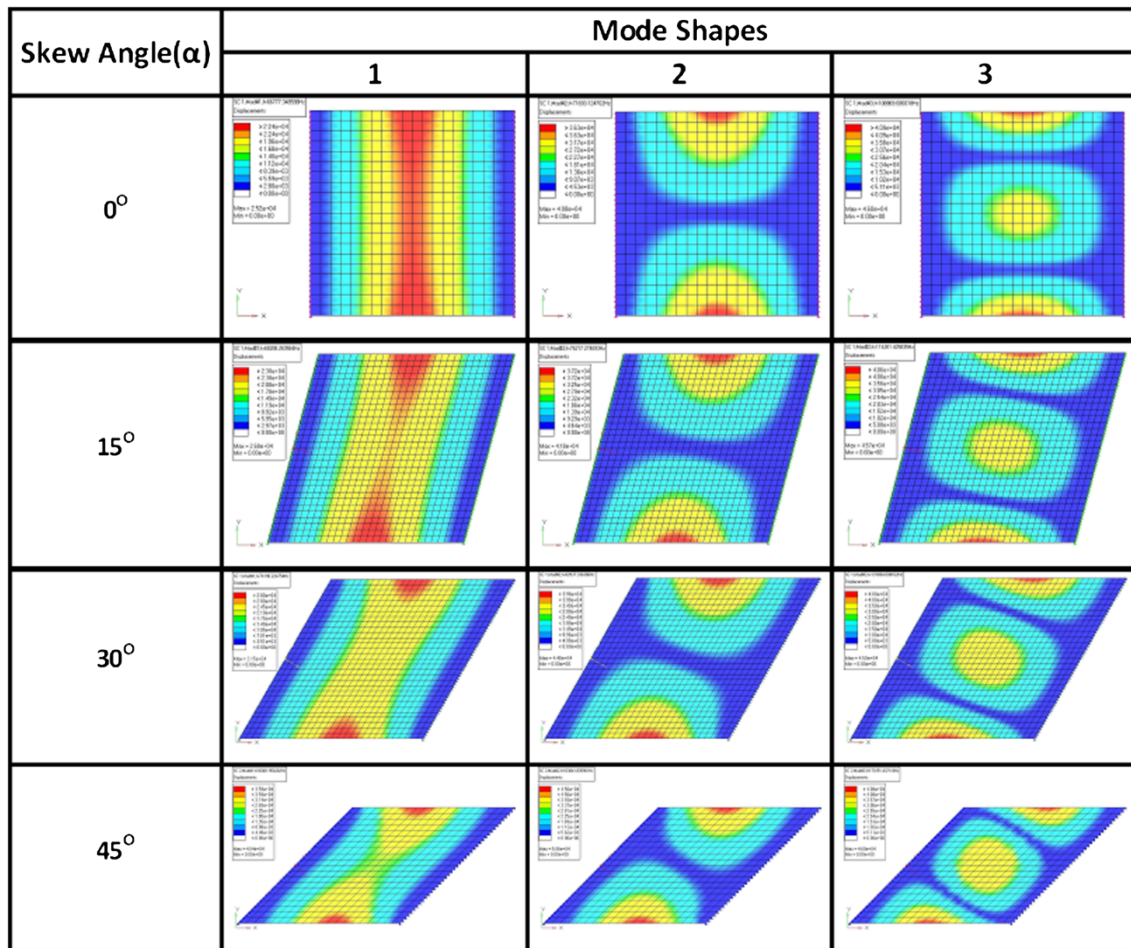
Fig. 8 Mode shapes for laminated composite skew plates ($a/b = 1.0$, NL = 20, antisymmetric cross-ply laminate)

Conclusions

The following conclusions are made based on the above study.

- The experimental values of the first, second and third natural frequencies are in good agreement with those of the finite element solution in the case of both isotropic and laminated composite skew plates.
- In the case of isotropic skew plates, the variation of first natural frequency with an increase in the aspect ratio is not appreciable up to a skew angle of about 30°. The second natural frequency increases with an increase in the aspect ratio for all laminate stacking sequences. The first, second and third natural frequencies increase monotonically with an increase in the skew angle for any given value of aspect ratio.
- In the case of laminated composite skew plates, the first natural frequency decreases with an increase in the aspect ratio for all laminate stacking sequences. The variation of second and third natural frequencies with aspect ratio is considerable in the range of skew angle

considered. Other parameters being the same, the antisymmetric angle-ply $[+0°/-0°/.../-0°]$ yields the highest value for the first natural frequency and the antisymmetric angle-ply $[+90°/-90°/.../-90°]$ yields the lowest value. The first natural frequencies for the other laminate stacking sequences lie in between these two extreme values. The first, second and third natural frequencies generally increase with an increase in the skew angle for any given value of aspect ratio.

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